

KEY

TO

THE NATIONAL ARITHMETIC;

CONTAINING

Full Solutions to nearly all the Problems.

DESIGNED FOR THE

USE OF TEACHERS AND PRIVATE STUDENTS.

BY JOHN HERBERT SANGSTER, M.A.,

MATHEMATICAL MASTER AND LECTURER IN CHEMISTRY AND NATURAL
PHILOSOPHY IN THE NORMAL SCHOOL FOR UPPER CANADA.



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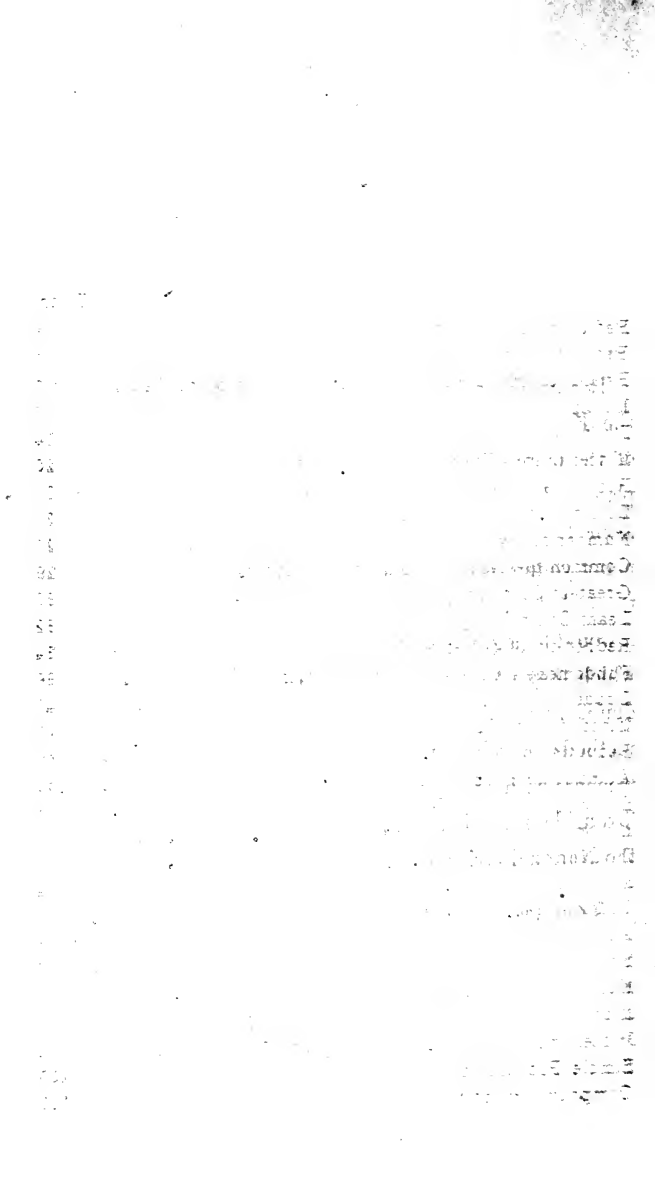
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vince of Canada.

PREFACE.

It was the original intention of the author to give, in the Key, merely a series of brief hints upon the solutions of the more difficult problems. He was led to modify this plan and to issue the work in its present form, chiefly from the consideration that as there are in the country many young persons who, from various causes, are unable to avail themselves of the advice and assistance of a teacher, it would be a great boon to these to have access to a book to which they might refer with the certainty of having every doubt removed as to the correctness of their work and methods of solution. He offers the work to his fellow-teachers with the hope that they will accord it the same favourable reception that they have so kindly given to the National Arithmetic.

TORONTO, *May*, 1861.



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KEY TO NATIONAL ARITHMETIC.

Page 50.

(1)	(2)	(3)	(4)	(5)
d.	£	£ s.	£ s.	£ s.
23328	348	38 10	58 13	58 13
4	20	20	20	20
<u>93312 f.</u>	<u>6960 s.</u>	<u>770 s.</u>	<u>1173 s.</u>	<u>1173 s.</u>
		12	12	12
		<u>9240 d.</u>	<u>14076 d.</u>	<u>14076 d.</u>
				4
				<u>56304 f.</u>

(6)	(7)	(8)	(9)
£ s. d.	£ s. d.	cwt. qrs. lbs.	cwt. qrs. lbs.
59 13 6½	63 0 9	16 2 16	14 3 16
20	20	4	4
<u>1193 s.</u>	<u>1260 s.</u>	<u>66 qrs.</u>	<u>59 qrs.</u>
12	12	25	25
<u>14322 d.</u>	<u>15129d.</u>	<u>346</u>	<u>311</u>
4		132	118
<u>57291 f.</u>		<u>1666 lbs.</u>	<u>1491 lbs.</u>

(10)	(11)	(12)	(13)	(14)
lbs. oz. dwt. grs.	lbs. oz. dwt. grs.	grs. yrs.	mile.	yrs. d. h. m.
3 5 12 16	7 11 15 14	20	1	46 21 8 56
12	12	365	8	365
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
41 oz.	95	7300 dys.	8 fur.	251
20	20	24	40	276
<hr/>	<hr/>	<hr/>	<hr/>	138
832 dwt.	1915 dwt.	29200	320 per.	<hr/>
24	24	14600	5½	16811 days.
<hr/>	<hr/>	<hr/>	<hr/>	24
3344	7674	175200 hrs	1600	<hr/>
1664	3830		160	67252
<hr/>	<hr/>		1760 yds.	33622
19984 grs.	45974 grs.		3	403472 hrs.
			5280 ft.	60
				24208376 min.

(15)	(16)	(17)	(18)	(19)	(20)
sq. per.	a. r. per.	sq. miles.	cub. ft.	pks.	pks.
74	46 3 12	767	767	767	797
30½	4	640	1728	2	2
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
2220	187 r.	30680	6136	1534 gals	1594 gals
18½	40	4602	1534	4	4
<hr/>	<hr/>	<hr/>	5369	<hr/>	<hr/>
2238½ sq. yds.	7492 per.	490880 sq. a.	767	6136 qts.	6376 qts.
	30½		<hr/>		2
	224760		1325376 cub. in.		<hr/>
	1873				12752 pts.
	226633 sq. yds.				

Page 51.

(1)	(2)	(3)
f.	grs.	yds.
4)32756	24)23547	5½)397024
<hr/>	<hr/>	2 2
12)8189 d.	20)981 dwt. 3 grs.	<hr/>
<hr/>	<hr/>	11)794048
20)682s. 5d.	12)49 oz. 1 dwt. 3 grs.	<hr/>
<hr/>	<hr/>	40)72186r. 2hf-yds.=1yd.
£34 2s. 5d.	4 lbs. 1 oz. 1 dt. 3 g.	8)1804 fur. 26 r. 1 yd.
		225 m. 4 f. 26 r. 1 y.

(4) sec. 60)28635 <hr/> 60)477 m. 15 sec. <hr/> 7 hrs. 57 m. 15 sec.	(5) lbs. 25)1666 <hr/> 4)66 qrs. 16 lbs. <hr/> 16 cwt. 2 q. 16 lbs.	(6) lbs. 25)1491 <hr/> 4)59 qrs. 16 lbs. <hr/> 14 cwt. 3 q. 16 lbs.
(7) grs. 24)115200 <hr/> 20)4800 dwt. <hr/> 12)240 oz. <hr/> 20 lbs.	(8) oz. 16)107520 <hr/> 6720 lbs.	(9) cub. in. 1728)1674674 <hr/> 969 ft. 242 in. <hr/> 4)2301 qrs. <hr/> 575 yds. 1 qr.
(11) ft. 3)183810 <hr/> 5½)61270 yds. 2) 2 <hr/> 11)122540 <hr/> 40)11140 per. <hr/> 8)278 fur. 20 per. <hr/> 3)34 m. 6 fur. 20 per. <hr/> 11 lea. 1 m. 6 fur. 20 per.	(12) cub. in. 1728)138297 <hr/> 27)80 ft. 57 in. <hr/> 2 c. yds. 26 c. ft. 57 c. in.	(13) cub. ft. 128)67893 <hr/> 530 cord 53c.ft.
(14) sec. 60)3561829 <hr/> 60)59363 m. 49 sec. <hr/> 24)989 h. 23 m. 49 s. <hr/> 7)41 d. 5 h. 23 m. 49 s. <hr/> 5 wks. 6 days 5 hrs. 23 min. 49 sec.	(15) qts. 4)1597 <hr/> 2)399 gals. 1 qt. <hr/> 4)199 pks. 1 gal. 1 qt. <hr/> 49 bush. 3 pecks 1 gal. 1 qt.	(16) c. ft. 8)1000 <hr/> 125 cords

$$\begin{array}{r} (17) \\ \text{seconds.} \\ 60)10000 \\ \hline \end{array}$$

$$60)166' 40''$$

$$\hline 2^{\circ} 46' 40''$$

$$\begin{array}{r} (18) \\ \text{sq. links.} \\ 10000)70000 \\ \hline \end{array}$$

$$7 \text{ sq. ch.}$$

$$\begin{array}{r} (19) \\ \text{grs.} \\ 20)11521 \\ \hline \end{array}$$

$$3)576 \text{ scr. 1 gr.}$$

$$8)192 \text{ dr. 1 gr.}$$

$$12)24 \text{ oz. 1 gr.}$$

$$\hline 2 \text{ lbs. 1 gr.}$$

(20)

sq ft.

$$9)26025$$

$$30\frac{1}{4})2891 \text{ yds. 6 ft.}$$

$$\begin{array}{r} 4 \quad) \quad 4 \\ \hline \end{array}$$

$$121)11564 \text{ quarter yards.}$$

$$95 \text{ per. 69 quar. yds. 6 ft. =}$$

$$40)95 \text{ per. 17 yds. 8 ft. 36 in.}$$

$$\hline 2 \text{ r. 15 sq. p. 17 sq. y. 8 sq. ft. 36 sq. in.}$$

Page 53.

(1)

$$£3 \times 400 = 1200 \text{ cents.}$$

$$7\text{s.} \times 20 = 140 \text{ "}$$

$$1\frac{1}{4}\text{d.} = 5 \text{ far.} \times 5 \div 12 = 2\frac{1}{12} \text{ " } 3\frac{1}{2}\text{d.} = 14 \text{ far.} \times 5 \div 12 = .05\frac{5}{6}$$

$$£3 \text{ 7s. } 1\frac{1}{4}\text{d.} = 1342\frac{1}{12} \text{ cts.}$$

$$£29 \text{ 18s. } 3\frac{1}{2}\text{d.} = \$119.65\frac{5}{6}$$

(3)

$$11\frac{1}{4}\text{d.} = 45 \text{ far.} \times 5 \div 12 = 18\frac{3}{4} \text{ cts.}$$

(4)

$$£69 \times 400 = \$276.00$$

$$15\text{s.} \times 20 = 3.00$$

$$6\text{d.} = 24 \text{ far.} \times 5 \div 12 = .10$$

$$\hline £69 \text{ 15s. } 6\text{d.} = \$279.10$$

(2)

$$£29 \times 400 = \$116.00$$

$$18\text{s.} \times 20 = 3.60$$

$$3\frac{1}{2}\text{d.} = 14 \text{ far.} \times 5 \div 12 = .05\frac{5}{6}$$

$$\hline £29 \text{ 18s. } 3\frac{1}{2}\text{d.} = \$119.65\frac{5}{6}$$

(5)

$$18\text{s.} \times 20 = \$3.60$$

$$8\frac{1}{2}\text{d.} = 34 \text{ far.} \times 5 \div 12 = .14\frac{1}{6}$$

$$\hline 18\text{s. } 8\frac{1}{2}\text{d.} = \$3.74\frac{1}{6}$$

(6)

$$£17 \times 400 = \$68.00$$

$$16s. \times 20 = 3.20$$

$$5\frac{1}{4}d. = 23 \text{ far.} \times 5 \div 12 = .09\frac{7}{12}$$

$$£17 \ 16s. \ 5\frac{1}{4}d. = \$71.29\frac{7}{12}$$

(7)

$$£87 \times 400 = \$348.00$$

(8)

$$15s. \times 20 = \$3.00$$

$$15s. \ 11\frac{1}{4}d. = \$3.19\frac{7}{12}$$

(9)

$$£16 \times 400 = \$64.00$$

$$6s. \times 20 = 1.20$$

$$2d. = 8 \text{ far.} \times 5 \div 12 = .03\frac{1}{3}$$

$$£16 \ 6s. \ 2d. = \$65.23\frac{1}{3}$$

(10)

$$£2 \times 400 = \$8.00$$

$$9s. \times 20 = 1.80$$

$$11d. = 44 \text{ far.} \times 5 \div 12 = .18\frac{1}{3}$$

$$£2 \ 9s. \ 11d. = \$9.98\frac{1}{3}$$

Page 90.

(25)

$$36 = 12 \times 3$$

$$\$169.78$$

$$12$$

$$2037.36$$

$$3$$

$$\$6112.08$$

(26)

$$121 = 11 \times 11$$

$$796342.3$$

$$11$$

$$8759765.3$$

$$11$$

$$96357418.3$$

(27)

$$144 = 12 \times 12$$

$$\$33460$$

$$12$$

$$401520$$

$$12$$

$$\$4818240$$

(28)

$$648 = 12 \times 9 \times 6$$

$$735$$

$$12$$

$$8820$$

$$9$$

$$79380$$

$$6$$

$$476280$$

(29)

$$18 = 6 \times 3$$

$$£ \quad s. \quad d.$$

$$3 \quad 7 \quad 6$$

$$6$$

$$20 \quad 5 \quad 0$$

$$3$$

$$60 \quad 15 \quad 0$$

(30)

$$22 = 11 \times 2$$

$$£ \quad s. \quad d.$$

$$5 \quad 14 \quad 6\frac{1}{2}$$

$$11$$

$$62 \quad 19 \quad 11\frac{1}{2}$$

$$2$$

$$125 \quad 19 \quad 11$$

(31)

$$810 = 10 \times 9 \times 9$$

$$£ \quad s. \quad d.$$

$$3 \quad 4 \quad 7$$

$$10$$

$$32 \quad 5 \quad 10$$

$$9$$

$$290 \quad 12 \quad 6$$

$$9$$

$$2615 \quad 12 \quad 6$$

(32)

$$54 = 9 \times 6$$

$$\text{cwt. qrs. lbs. oz.}$$

$$11 \quad 3 \quad 14 \quad 7$$

$$9$$

$$107 \quad 0 \quad 4 \quad 15$$

$$6$$

$$642 \quad 1 \quad 4 \quad 10$$

(33)

$$49 = 7 \times 7$$

bush. pks. gal. qt. pt.

26	3	1	1	1
				7

188	1	1	2	1
				7

1319	0	1	1	1
------	---	---	---	---

(34)

$$63 = 9 \times 7$$

yds. qrs. na. in.

2	2	2	2
			9

24	0	2	0
			7

168	3	2	0
-----	---	---	---

(35)

$$288 = 12 \times 12 \times 2$$

dys. hrs. min. sec.

5	17	33	11
			12

68	18	38	12
			12

825	7	38	24
			2

1650	15	16	48
------	----	----	----

(40)

$$83 = 3 + 10 \times 8$$

£ s. d. £ s. d.

12	2	4	×	3	=	36	7	0
								10

121	3	4	×	8	=	969	6	8
-----	---	---	---	---	---	-----	---	---

1005	13	8
------	----	---

(41)

$$999 = 10 \times 10 \times 10 - 1$$

£ s. d.

963	0	0½
		10

9630	0	7½
		10

96300	6	3
		10

963003	2	6
963	0	0½

962040	2	5½
--------	---	----

(42)

$$3178 = 8 + 10 \times 7 + 10 \times 10 \times 1 + 10 \times 10 \times 10 \times 3$$

£ s. d.

3	6	5½	×	8	=
					10

33	4	4½	×	7	=
					10

332	3	9	×	1	=
					10

3321	17	6	×	3	=	9965	12	6
------	----	---	---	---	---	------	----	---

10556	18	4½
-------	----	----

(43)

$$678 = 8 + 10 \times 7 + 10 \times 10 \times 6$$

bush. pk. gal.

16	3	1	×	8	=
					10

168	3	0	×	7	=
					10

1687	2	0	×	6	=
------	---	---	---	---	---

10125	0	0
-------	---	---

11441	1	0
-------	---	---

(44)

$$247 = 7 + 10 \times 4 + 10 \times 10 \times 2$$

m.	fur.	rds.	yds.		m.	fur.	rds.	yds.
23	6	33	4	$\times 7 =$	166	7	36	$0\frac{1}{2}$
					10			

238	4	17	$1\frac{1}{2}$	$\times 4 =$	954	1	29	$0\frac{1}{2}$
					10			

2385	4	12	4	$\times 2 =$	4771	0	25	$2\frac{1}{2}$
					5892 2 10 $3\frac{1}{2}$			

(45)

$$721 = 1 + 10 \times 2 + 10 \times 10 \times 7$$

S.	deg.	min.	sec.		S.	deg.	min.	sec.
3	16	30	45	$\times 1 =$	3	16	30	45
					10			

35	15	7	30	$\times 2 =$	71	0	15	0
					10			

355	1	15	0	$\times 7 =$	2485	8	45	0
					2559 25 30 45			

(52)

7071
556

(53)

15607
3094

(54)

39948123
6007

(55)

2778588
9867

42426

62428

279636861

19450116

35355

140463

23968873800

16671528

35355

468210

239968374861

22228704

3931476

48288058

25007292

27416327796

(61)

3.2517
.023

(62)

64.001
340

(63)

482000
.37

(64)

3782.4
.00917

(65)

87.96
220

97551

2560040

3374000

264768

175920

65034

192003

1446000

37824

17592

.0747891

21760.340

178340.00

340416

19351.20

34.684608

(66)	(67)	(68)	(69)
$216 = 6 \times 6 \times 6$	$\$61135.37$	255226	$176 = 11 \times 8 \times 2$
	229	143	
$\$83469$	<u>55021833</u>	<u>765678</u>	203736
6	12227074	1020904	11
<u>500814</u>	12227074	255226	<u>2241096</u>
6	<u>\$13999999.73</u>	<u>36497318</u>	8
<u>3004884</u>			<u>17928768</u>
6			2
<u>\$18029304</u>			<u>35857536</u>

(70)	(71)	(72)	(73)
116700	3721	$297 = 11 \times 9 \times 3$	$35 = 7 \times 5$
235	73	32000	9344000
<u>583500</u>	<u>11163</u>	11	7
350100	26047	<u>352000</u>	<u>65408000</u>
233400	<u>271633</u>	9	5
<u>27424500</u>		<u>3168000</u>	<u>327040000</u>
		3	
		<u>9504000</u>	

(74)

(75)

$$749 = 9 + 10 \times 4 + 10 \times 10 \times 7$$

$$999998 = 1000000 - 2$$

lbs.	oz.	drs.	scr.	gr.	lbs.	oz.	drs.	scrs.	grs.	
123	4	7	2	$17 \times 9 =$	1110	8	7	1	13	1698732
				10						1000000
<u>1234</u>	<u>1</u>	<u>7</u>	<u>1</u>	$10 \times 4 =$	<u>4936</u>	<u>7</u>	<u>6</u>	<u>0</u>	<u>0</u>	1698732000000
				10						3397464
<u>12341</u>	<u>7</u>	<u>3</u>	<u>0</u>	$0 \times 7 =$	<u>86391</u>	<u>3</u>	<u>5</u>	<u>0</u>	<u>0</u>	1698728602536
					<u>92438</u>	<u>8</u>	<u>2</u>	<u>1</u>	<u>13</u>	

(76)

$$\begin{array}{r}
 640 = 10 \times 8 \times 8 \\
 \text{bush. pk. gal. qt. pt.} \\
 123 \quad 1 \quad 1 \quad 1 \quad 1 \\
 \hline
 10 \\
 1234 \quad 0 \quad 1 \quad 3 \quad 0 \\
 \hline
 8 \\
 9873 \quad 3 \quad 0 \quad 0 \quad 0 \\
 \hline
 8 \\
 78990 \quad 0 \quad 0 \quad 0 \quad 0
 \end{array}$$

(77)

$$\begin{array}{r}
 89 \\
 .73 \\
 \hline
 267 \\
 623 \\
 \hline
 \$64.97
 \end{array}$$

(78)

$$\begin{array}{r}
 1143 = 3 + 10 \times 4 + 10 \times 10 \times 1 + 10 \times 10 \times 10 \times 1 \\
 \text{yds. qrs. na. in.} \quad \text{yds. qrs. na. in.} \\
 7 \quad 3 \quad 2 \quad 1 \times 3 = 23 \quad 2 \quad 3 \quad 0\frac{1}{4} \\
 \hline
 10 \\
 79 \quad 0 \quad 0 \quad 1 \times 4 = 316 \quad 0 \quad 1 \quad 1\frac{1}{4} \\
 \hline
 10 \\
 790 \quad 1 \quad 0 \quad 1 \times 1 = 790 \quad 1 \quad 0 \quad 1 \\
 \hline
 10 \\
 7902 \quad 3 \quad 0 \quad 1 \times 1 = 7902 \quad 3 \quad 0 \quad 1 \\
 \hline
 9032 \quad 3 \quad 2 \quad 0
 \end{array}$$

(79)

$$\begin{array}{r}
 1634.5789 \\
 635000 \\
 \hline
 81728945060 \\
 49037367 \\
 98074734 \\
 \hline
 1037957601.5
 \end{array}$$

(80)

$$\begin{array}{r}
 \$968.49 \\
 3.4 \\
 \hline
 387396 \\
 290547 \\
 \hline
 \$3292.866
 \end{array}$$

$$\begin{array}{r}
 \$3292.866 \\
 3.7 \\
 \hline
 23050062 \\
 9878598 \\
 \hline
 \$12183.6042
 \end{array}$$

$$\begin{array}{r}
 \$12183.6042 \\
 3292.866 \\
 968.49 \\
 \hline
 \$16444.9602
 \end{array}$$

Page 110.

(18)

6423)798965(124²⁵¹³₆₄₂₃6423

15666

12846

28205

25692

2513

(19)

£ s. d.

12)176 14 6

14 14 6¹/₂

(20)

741)56789(764¹³₇₄₁5187

4919

4446

473

(21)

7894)6785158(8594¹³₇₈₉₄63152

46995

39470

75258

71046

4212

(22)

£ s. d. £ s. d.

317)4728 16 2(14 18 4⁵⁴₃₁₇ 429)\$97896·64(\$228·19¹³₄₂₉317

1558

1268

290

20

5816

317

2646

2536

110

12

1322

1268

54

(24)

6)970763

161793·8333+

(25)

9)71234

7914⁸/₉

(23)

858

1209

858

3516

3432

84·6

42·9

41·74

38·61

3·13

(26)

1209 47600)977076(20²⁵²⁷⁶_{1209 47600}95200

25076

(27)

lbs.	oz.	drs.	scr.	grs.	lbs.	oz.	drs.	scr.	grs.
498)7289	6	4	2	13	(14	7	5	0	12 $\frac{37}{8}$
498									
2309									
1992									
317									
12									
3810									
3486									
324									
8									
2596									
2490									
106									
3									
320									
20									
6413									
5976									
437									

(29)

9712)7867674	(810 $\frac{954}{9712}$
77696	
9807	
9712	
954	

(28)

£	s.	d.	s.	d.
487)157	16	7	(6	5 $\frac{3}{4}$ $\frac{59}{487}$
20				

(30)

m.	fur.	rds.	m.	fur.	rds.
37)422	3	38	(11	3	14
407					
15					
8					
123					
111					
12					
40					
518					
37					
148					
148					

Page 112.

(32)

25=5×5
5)3766
5)753... 1
150... 3
3×5+1=16
150 $\frac{16}{5}$

(33)

42=7×6
7)26406
6)3772... 2
628... 4
4×7+2=30
628 $\frac{30}{42}$

(34)

96=12×8
12)25431
8)2119... 3
264... 7
7×12+3=87
264 $\frac{87}{96}$

(35)

24=12×2
£ s. d.
12)24 17 6
2)2 1 5 $\frac{1}{2}$
1 0 8 $\frac{1}{2}$

(36)

$$\begin{array}{r}
 49 = 7 \times 7 \\
 \text{£ s. d.} \\
 7 \overline{) 740 \ 13 \ 4} \\
 \hline
 7 \overline{) 105 \ 16 \ 2\frac{1}{4} \dots 1} \\
 \hline
 15 \ 2 \ 3\frac{3}{4} \dots \frac{1}{49}
 \end{array}$$

(37)

$$\begin{array}{r}
 56 = 8 \times 7 \\
 \text{£ s. d.} \\
 8 \overline{) 547 \ 12 \ 4} \\
 \hline
 7 \overline{) 68 \ 9 \ 0\frac{1}{2}} \\
 \hline
 9 \ 15 \ 6\frac{3}{4} \dots \frac{10}{56}
 \end{array}$$

(38)

$$\begin{array}{r}
 35 = 7 \times 5 \\
 7 \overline{) 6789436} \\
 \hline
 5 \overline{) 969919 \dots 3} \\
 \hline
 193983 \dots 4 \\
 4 \times 7 + 3 = 31 \\
 193983\frac{31}{5}
 \end{array}$$

(39)

$$\begin{array}{r}
 147 = 7 \times 7 \times 3 \\
 7 \overline{) 753293} \\
 \hline
 7 \overline{) 107613 \dots 2} \\
 \hline
 3 \overline{) 15373 \dots 2} \\
 \hline
 5124 \dots 1 \\
 1 \times 7 \times 7 + 2 \times 7 + 2 = 65 \\
 5124\frac{65}{47}
 \end{array}$$

(40)

$$\begin{array}{r}
 81 = 9 \times 9 \\
 \text{lbs. oz. dwt. grs.} \\
 9 \overline{) 1798 \ 6 \ 11 \ 9} \\
 \hline
 9 \overline{) 199 \ 10 \ 1 \ 6 \dots 3} \\
 \hline
 22 \ 2 \ 9 \ 0 \dots 6 \\
 6 \times 9 + 3 = 57 \\
 22 \text{ lbs. } 2 \text{ oz. } 9 \text{ dwt. } 0\frac{57}{81} \text{ grs.}
 \end{array}$$

(43)

$$\begin{array}{r}
 \text{£ s. d.} \quad \text{£ s. d.} \\
 491 \ 12 \ 0\frac{1}{4} \) \ 8968 \ 13 \ 7\frac{1}{2} \\
 \underline{20} \qquad \qquad \underline{20} \\
 9832 \qquad \qquad 179373 \\
 \underline{12} \qquad \qquad \underline{12} \\
 117984 \qquad \qquad 2152483 \\
 \underline{4} \qquad \qquad \underline{4} \\
 471937 \) \ 8609934 \ (18\frac{114068}{471937} \\
 \underline{471937} \\
 3890564 \\
 \underline{3775496} \\
 115068
 \end{array}$$

(44)

$$\begin{array}{r}
 \text{m. fur. rds.} \quad \text{m. fur. rds.} \\
 17 \ 5 \ 27 \) \ 1027 \ 1 \ 6 \\
 \underline{8} \qquad \qquad \underline{8} \\
 141 \qquad \qquad 8217 \\
 \underline{40} \qquad \qquad \underline{40} \\
 5667 \) \ 328686 \ (58 \\
 \underline{28335} \\
 45336 \\
 \underline{45336}
 \end{array}$$

$$\begin{array}{r}
 \text{(45)} \\
 \begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 57 \quad 0 \quad 7\frac{1}{2} \quad) \\
 20 \\
 \hline
 1140 \\
 12 \\
 \hline
 13687 \\
 4 \\
 \hline
 54750 \quad)
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{(47)} \\
 \begin{array}{r}
 \text{a.} \quad \text{r. per.} \\
 91 \quad 0 \quad 6 \quad) \\
 4 \\
 \hline
 364 \\
 40 \\
 \hline
 14566 \quad)
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{(57)} \\
 756.98 \div 76.73612 = \\
 7673612) 75698000 (9.864 + \\
 69062508 \\
 \hline
 6635492.0 \\
 6138889.6 \\
 \hline
 496602.40 \\
 460416.72 \\
 \hline
 36185.680 \\
 30694.448 \\
 \hline
 5491.232
 \end{array}$$

$$\begin{array}{r}
 \text{(46)} \\
 \begin{array}{r}
 \text{dwt. grs.} \quad \text{lbs.} \quad \text{oz.} \quad \text{dwt. grs.} \\
 5 \quad 9 \quad) \\
 24 \\
 \hline
 129
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{(56)} \\
 47.655 \div 4.5 = \\
 45) 476.55 (10.59 \\
 45 \\
 \hline
 26.5 \\
 22.5 \\
 \hline
 4.05 \quad \text{'} \\
 4.05 \quad \text{' }
 \end{array}$$

$$\begin{array}{r}
 \text{(58)} \\
 47.5782975 \div 26.175 = \\
 26175) 47578.2975 (1.8177 \\
 26175 \\
 \hline
 21403.2 \\
 20940.0 \\
 \hline
 463.29 \\
 261.75 \\
 \hline
 201.547 \\
 183.225 \\
 \hline
 18.3225 \\
 18.3225
 \end{array}$$

(59)

(60)

$$1 \div 7.6345 =$$

$$76345) 10000.0(0.1309 +$$

$$7634.5$$

$$2365.50$$

$$2290.35$$

$$75.1500$$

$$68.7105$$

$$75.347 \div 0.3829 =$$

$$3829) 753470(196.7798 +$$

$$3829$$

$$37057$$

$$34461$$

$$25960$$

$$22974$$

$$2986.0$$

$$2680.3$$

$$305.70$$

$$268.03$$

$$37.670$$

$$34.461$$

$$3.2090$$

$$3.0632$$

$$.1458$$

(61)

$$.0002 \div .000000008 =$$

$$8) 200000$$

$$25000$$

Page 116.

(61)

(62)

$$95) \$3300000(\$34736.8421$$

$$126) \$3860000(\$30634.9206$$

$$285$$

(63) dys.

$$378$$

$$28800) 95270400(3308$$

$$450$$

$$86400$$

$$800$$

$$380$$

$$756$$

$$88704$$

$$700$$

$$86400$$

$$440$$

$$665$$

$$378$$

$$230400$$

$$350$$

$$230400$$

$$620$$

$$285$$

$$504$$

days.

$$650$$

$$365\frac{1}{4}) 3308$$

$$116.0$$

$$570$$

$$4 \quad 4$$

$$113.4$$

yrs. days.

$$80.0$$

$$1461) 13232(9 \quad 20\frac{3}{4}$$

$$2.60$$

$$76.0$$

$$13149$$

$$2.52$$

$$4.00$$

$$4) 83$$

$$.800$$

$$3.80$$

$$.756$$

$$20\frac{3}{4}$$

$$.044$$

$$.200$$

$$.190$$

$$.100$$

$$.095$$

$$.005$$

(64)

$$35781628) \$1145012096(\$32$$

$$107344884$$

$$71563256$$

$$71563256$$

(65)		(66)	(67)
27475271)	\$3764112127(\$137	9)\$972	108)\$972(\$9
	27475271	<u> </u>	972
	<u> </u>	\$108	
	101658502		(70)
	82425813	(69)	1728)1000(.578 oz.
	<u> </u>	792)340480(429 $\frac{1}{9}$ $\frac{2}{3}$ oz.	864.0
	192326897	3168	<u> </u>
	192326897	<u> </u>	136.00
(68)		2368	120.96
294)\$8526(\$29		1584	<u> </u>
588	(71)		(72)
<u> </u>	m. fur.	7840	19)4750(250lbs. 15.040
2646	33 2	7128	38
2646	8	<u> </u>	<u> </u>
	<u> </u>	95	1.216
	266	712	95
	40		(74)
	<u> </u>	bush. pk. gal. qt. pt. bush. pk. gal. qt. pt.	
	10640	297)729 1 1 1 1 (2 1 1 2 1 $\frac{2}{11}$	
	5 $\frac{1}{2}$	594	
	<u> </u>	<u> </u>	
	53200	135	
	5320	4	
	<u> </u>	<u> </u>	
1155)58520(50 $\frac{7}{11}$ $\frac{10}{5}$		541	
5775		297	
<u> </u>		<u> </u>	
770		244	
50 $\frac{7}{11}$ $\frac{10}{5}$ =50 $\frac{2}{3}$.		2	
		<u> </u>	
(73)		489	
978.634÷96.34762 =		297	
9634762)97863400(10.157		<u> </u>	
9634762		192	
<u> </u>		4	
1515780.0		<u> </u>	
963476.2		769	
<u> </u>		594	
552303.80		<u> </u>	
481738.10		175	
<u> </u>		2	
70565.700		<u> </u>	
67443.334		351	
<u> </u>		297	
3122.366		<u> </u>	
		54	
		$\frac{54}{297}$ = $\frac{2}{11}$.	

(75)									
lbs. oz. dr.				cwt. qr.		lbs. oz. dr.			
9	7	8)	179	3	4	16	0	
16				4					(76)
						m. fur. rds.		m.	
151				719		93	4	7	25000
16				25		8			8
914				3599		748			200000
151				1438		40			40
2424				17979		29927	29927	8000000	dys. hrs. (267 7½ ⁸⁸ ₉₉ ²⁵ ₂₇)
				16				59854	
				107890				201460	
				17979				179562	
				287680				218980	
				16				209489	
				1726080				9491	
				287680				24	
2424				4602880		(1898 ²⁶⁶ ₃₀₃)		37964	
				2424				18982	
				21788				227784	
				19392				209489	
				23968				18295	
				21816					
				21520					
				19392					
				2128		$\frac{2128}{2424} = \frac{266}{303}$			

DCCIX, M̄VCCCLXXVI, M̄XCMXCIX, L̄XXXVMIV,
MMMCMXLVMMDXCVI.

(4)
72=8×9
lbs. oz.
749 10
8

5997 0
9

53973 0

(5)
17=7+10×1
s. d. £ s. d.
4 7¼×7=1 12 6¼
10

£2 6 5½×1=2 6 5½

3 18 11¾

(6)
30)2850000000Q
----- dys. hrs.
24)95000000(3958333 8
72

230 365¼)3958333(10837
216 4 4

140 1461)15833332
120 1461

200 12233
192 11688

80 5453
72 4383

80 10702
72 10227

80 4)475 quarter days.
72 ----- days. hrs.
----- 118¾=118 18
8 rem. Add 8

119 2
10837 yrs. 119 days, 2 hrs.

(7)
£729×400 = \$2916·00
170×20 = 3·40
6¼d.=25 far.×5÷12= 10·⁵/₁₂

\$2919·50⁵/₁₂
B

(8)
\$10000
9876·23

\$123·77

$$\begin{array}{r}
 \text{in.} \\
 12)7964327 \\
 \hline
 12)663693-11 \left. \vphantom{12)663693-11} \right\} 119 \text{ in.} \\
 \hline
 55307-9 \\
 9)55307 \text{ ft. 119 in.} \\
 \hline
 \end{array}$$

$$30\frac{1}{4}) 6145 \text{ yds. 2 ft. 119 in.}$$

$$\begin{array}{r} 4 \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r}
 121)24580 \\
 11)24580 \\
 \hline
 \end{array}
 \quad
 \begin{array}{l}
 203 \text{ p. } 4\frac{1}{4} \text{ y.} = 203 \text{ p. 4 yds. 2 ft. 36 in.} \\
 \text{Add} \quad \quad \quad 2 \text{ ft. 119 in.} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 11)2234-6 \left. \vphantom{11)2234-6} \right\} 17 \text{ qr. yds.} \\
 \hline
 203-1
 \end{array}$$

$$40)203 \text{ p. 4 yds. 5 ft. 11 in.}$$

$$4)5 \text{ fur. 3 p. 4 yds. 5 ft. 11 in.}$$

$$1 \text{ m. 1 f. 3 p. 4 yds. 5 ft. 11 in.}$$

(12)

$$429 = 9 + 10 \times 2 + 10 \times 10 \times 4$$

$$\text{wks. dys. hrs. min.} \quad \text{wks. dys. hrs. min.}$$

$$6 \quad 4 \quad 3 \quad 17 \times 9 = 59 \quad 2 \quad 5 \quad 33$$

$$10$$

$$\begin{array}{r} 65 \quad 6 \quad 8 \quad 50 \times 2 = 131 \quad 5 \quad 17 \quad 40 \\ \hline \end{array}$$

$$10$$

$$\begin{array}{r} 659 \quad 0 \quad 16 \quad 20 \times 4 = 2636 \quad 2 \quad 17 \quad 20 \\ \hline \end{array}$$

$$2827 \quad 3 \quad 16 \quad 33$$

wks.

$$52)2827(54 \text{ yrs. 19 wks. 3 dys. 16 hrs. 33 min.}$$

$$260$$

$$227$$

$$208$$

$$19 \text{ wks.}$$

(15)

(16)

tons.
324
20
cwt. qr. lbs. —
13 2 14 6480
4 4

\$136
\$136 × 4 = 544 — 95 = 449
1902
2487

54 25920
25 25

\$9237 — \$2487 = \$6750

284 129600
108 51840

(17)
yds. qrs. na. yds. qrs. na.
3 1 2) 39 2 3
4 4

1364) 648000 (475 $\frac{100}{1364}$ =
5456 475 $\frac{25}{341}$ hds.

13 158
4 4

(14)
78·96 10240
·00042 9548

54) 635 (11 $\frac{11}{4}$
54

15792 6920
31584 6820
·0331632 100

95
54
41

(18)

(19)

(21)

a. a. a. r. per.
25 732 96 3 17
197 674 4
156 — —
97 58 387
199 40
674 15497) \$7764·0 (\$0·501

(20)

\$
20 \$312
75 275
97 —
83 \$ 37

lbs. oz. dwt. grs.
12)36 8 14 16
3 0 14 13 $\frac{1}{2}$

(22)

a. r. per.
6 3 12
7 2 0
9 0 13
5 2 36
29 0 21

7748·5
15·500
15·497
3

(23)

(24)

(25)

5	lbs. oz. dwt. grs.	$\pounds 972 \times 400 = \$3888 \cdot 00$
7	5 9 8 0	$11s. \times 20 = 2 \cdot 20$
9	3 2 16 16	$11\frac{1}{4}d. = 45 \text{ far.} \times 5 \div 12 = \cdot 18\frac{3}{4}$
—	4 6 17 0	
21)294(14	1 8 19 22	$\$3890 \cdot 38\frac{1}{2}$
21		
—	15 4 1 14	
84		
84		

(26)

(27)

(28)

lbs. oz. drs. scr. grs.	56	cwt. qr. lbs.
179 3 3 1 14	25	6 2 11
12	—	5 3 16
—	280	8 0 7
2151 oz.	112	3 1 17
8	—	
—	1400	24 0 1 = 2401 lbs.
17211 drs.	2	$\cdot 15$
3	—	
—	2800 sq. ft. in roof.	12005
51634 scr.	6	2401
20	—	
—	16800	$\$360 \cdot 15$
1032694 grs.		

(29)

(30)

29	\$	\$
57	139468	370129
—	98579	238047
203	—	
145	$\$238047$	$\$132082$
—		
1653		
$\cdot 15$		
—		
8265		
1653		
—		
$\$247 \cdot 95$		

(31)

£ s. d.	£ s. d.
9 19 11½	1694 16 0½
20	20
— ¾ = 6¾	— 1½ = 2¾
199	33896
12	12
2399	406752
84	84
9659	1627030
19192	3254016
201579	34167190(169·49
	201579
	1400929
	1209474
	1914550
	1814211
	100339·0
	80631·6
	19707·40
	18142·11
	1565·29

(34)

cwt. qr. lbs.	lbs.
2 0 17	
3 2 15	
2 1 20	
5 3 17	
14 0 19	1419
	·37½
	9933
	4257
	709½
	\$532·12½

(32)

£19 × 400	= \$76·00
19s. × 20	= 3·80
11½d = 47far. × 5 ÷ 12	= ·19½
	\$79·99½

(33)

cwt. qr. lbs.	cwt. qr. lbs.	lbs.
3 2 11	12 0 0	
4 1 15	8 0 1	
8 0 1	3 3 24	399
		·15
		1995
		399
		\$59·85

(36)

43·2 ÷ 76·8437	=
768437)432000·0(0·562	
384218·5	
47781·50	
46106·22	
1675·280	
1536·874	
138·406	

(37)

$$\begin{array}{r}
 123\cdot4 \div \cdot 000000066 = \\
 123400000000 \div 66 \\
 6) 123400000000 \\
 \hline
 11) 20566666666\cdot666 \\
 \hline
 1869696969\cdot69
 \end{array}$$

(38)

$$\begin{array}{r}
 \$63\cdot29 \quad \$2789\cdot27 \\
 17 \quad 1075\cdot93 \\
 \hline
 44303 \quad \$1713\cdot34 \\
 \hline
 6329 \\
 \hline
 \$1075\cdot93
 \end{array}$$

(39)

$$\begin{array}{r}
 £29 \times 400 = \$116\cdot00 \quad \$278\cdot43 \\
 6s. \times 20 = 1\cdot20 \quad 417\cdot16 \\
 11\frac{1}{4}d. = 47far. \times 5 \div 12 = \cdot 19\frac{7}{12} \quad 11\cdot27 \\
 \hline
 117\cdot39\frac{7}{12} \quad 2110\cdot40 \\
 \hline
 723\cdot15 \\
 \hline
 117\cdot39\frac{7}{12} \\
 \hline
 173) 3657\cdot80\frac{7}{12} \\
 12 \quad 12 \\
 \hline
 2076) 43893\cdot67(21\cdot1433
 \end{array}$$

(40)

$$\begin{array}{r}
 2076) 491544(236\frac{402}{19} \\
 4152 \\
 \hline
 7634 \\
 6228 \\
 \hline
 14064 \\
 12456 \\
 \hline
 1608 \\
 \hline
 1608 = 402 \\
 2076 = 519
 \end{array}$$

$$\begin{array}{r}
 4152 \\
 \hline
 2373 \\
 2076 \\
 \hline
 297\cdot6 \\
 207\cdot6 \\
 \hline
 90\cdot00 \\
 83\cdot04 \\
 \hline
 6\cdot960 \\
 6\cdot228 \\
 \hline
 \cdot 7320 \\
 \cdot 6228 \\
 \hline
 \cdot 1092
 \end{array}$$

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(3)	(4)	(5)	(6)
2)11368	2)2934	3)1011	2)1000
<u>2)5684</u>	<u>3)1467</u>	<u>337</u>	<u>2)500</u>
2)2842	3)489	3 × 337	2)250
<u>7)1421</u>	<u>163</u>		<u>5)125</u>
7)203	2 × 3 ² × 163		5)25
<u>29</u>			<u>5</u>
2 ³ × 7 ² × 29			2 ³ × 5 ³
(7)	(8)	(9)	(10)
2)1024	2)32320	7)707	2)1118
<u>2)512</u>	<u>2)16160</u>	<u>101</u>	<u>13)559</u>
2)256	2)8080	7 × 101	43
<u>2)128</u>	<u>2)4040</u>		2 × 13 × 43
2)64	2)2020		
<u>2)32</u>	<u>2)1010</u>		
2)16	5)505		
<u>2)8</u>	<u>101</u>		
2)4	2 ⁶ × 5 × 101		
<u>2</u>			
2 ¹⁰			

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$$\begin{array}{l}
 (3) \\
 100 = 2^2 \times 5^2 \\
 \begin{array}{l}
 1..2..4 \\
 1..5..25 \\
 \hline
 1..2..4..5..10..20..25..50..100
 \end{array}
 \end{array}$$

(4)

$$810 = 3^4 \times 2 \times 5$$

1..3..9..27..81

1..2

1..3..9..27..81..2..6..18..54..162

1..5

$$1..3..9..27..81..2..6..18..54..162..5..15..45..135..405..10..30..90..270..810 =$$

$$1..2..3..5..6..9..10..15..18..27..30..45..54..81..90..135..162..270..405..810.$$

(5)

$$920 = 2^3 \times 5 \times 23.$$

1..2..4..8

1..5

1..2..4..8..5..10..20..40

1..23

$$1..2..4..8..5..10..20..40..23..46..92..184..115..230..460..920 =$$

$$1..2..4..5..8..10..20..23..40..46..92..115..184..230..460..920.$$

(6)

$$25000 = 5^5 \times 2^3$$

1..5..25..125..625..3125

1..2..4..8

$$1..5..25..125..625..3125..2..10..50..250..1250..6250..4..20..100..500..2500..12500..8..40..200..1000..5000..25000 =$$

$$1..2..4..5..8..10..20..25..40..50..100..125..200..250..500..625..1000..1250..2500..3125..5000..6250..12500..25000.$$

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(2)

$$88200 = 2^3 \times 3^2 \times 5^2 \times 7^2$$

$$3+1=4$$

$$2+1=3$$

$$2+1=3$$

$$2+1=3$$

$$4 \times 3 \times 2 \times 3 = 108$$

(3)

$$3500 = 2^2 \times 5^3 \times 7$$

$$2+1=3$$

$$3+1=4$$

$$1+1=2$$

$$3 \times 4 \times 2 = 24$$

(4)

$$6336=2^6 \times 3^2 \times 11$$

$$6+1=7$$

$$2+1=3$$

$$1+1=2$$

$$7 \times 3 \times 2=42$$

(5)

$$824=2^3 \times 103$$

$$3+1=4$$

$$1+1=2$$

$$4 \times 2=8$$

(6)

$$49000=2^3 \times 5^3 \times 7^2$$

$$3+1=4$$

$$3+1=4$$

$$2+1=3$$

$$4 \times 4 \times 3=48$$

(7)

$$81000=2^3 \times 3^4 \times 5^3$$

$$3+1=4$$

$$4+1=5$$

$$3+1=4$$

$$4 \times 5 \times 4=80$$

(8)

$$75600=2^4 \times 3^3 \times 5^2 \times 7$$

$$4+1=5$$

$$3+1=4$$

$$2+1=3$$

$$1+1=2$$

$$5 \times 4 \times 3 \times 2=120$$

(9)

$$256000=2^{10} \times 5^2$$

$$10+1=11$$

$$2+1=3$$

$$11 \times 3=33$$

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(2)

$$21=7 \times 3$$

$$18=2 \times 3 \times 3$$

$$27=3 \times 3 \times 3$$

$$36=4 \times 3 \times 3$$

(3)

$$21=3 \times 7$$

$$77=11 \times 7$$

$$42=2 \times 3 \times 7$$

$$35=5 \times 7$$

(4)

$$26=2 \times 13$$

$$52=2 \times 2 \times 13$$

$$91=7 \times 13$$

$$143=11 \times 13$$

3 is common to all. 7 is common to all. 13 is common to all.

(5)

$$82=41 \times 2$$

$$118=59 \times 2$$

$$146=73 \times 2$$

2 is common to all.

Page 130.

(2)

$$\begin{array}{r}
 296)407(1 \\
 \underline{296} \\
 111)296(2 \\
 \underline{222} \\
 74)111(1 \\
 \underline{74} \\
 37)74(2 \\
 \underline{74} \\
 \text{G. C. M.} = 37.
 \end{array}$$

(3)

$$\begin{array}{r}
 308)506(1 \\
 \underline{308} \\
 198)308(1 \\
 \underline{198} \\
 110)198(1 \\
 \underline{110} \\
 88)110(1 \\
 \underline{88} \\
 22)88(4 \\
 \underline{88} \\
 \text{G. C. M.} = 22.
 \end{array}$$

(4)

$$\begin{array}{r}
 74)84(1 \\
 \underline{74} \\
 10)74(7 \\
 \underline{70} \\
 4)10(2 \\
 \underline{8} \\
 2)4 \\
 \underline{2} \\
 \text{G. C. M.} = 2.
 \end{array}$$

(5)

$$\begin{array}{r}
 1825)2555(1 \\
 \underline{1825} \\
 730)1825(2 \\
 \underline{1460} \\
 365)730(2 \\
 \underline{730} \\
 \text{G. C. M.} = 365.
 \end{array}$$

(6)

$$\begin{array}{r}
 556)672(1 \\
 \underline{556} \\
 116)556(4 \\
 \underline{464} \\
 92)116(1 \\
 \underline{92} \\
 24)92(3 \\
 \underline{72} \\
 20)24(1 \\
 \underline{20} \\
 4)20 \\
 \underline{5} \\
 \text{G. C. M.} = 4.
 \end{array}$$

Page 131.

$$\begin{array}{r} (9) \\ 110)140(1 \\ \underline{110} \\ 30)110(3 \\ \underline{90} \end{array}$$

$$\begin{array}{r} 20)30(1 \\ \underline{20} \end{array}$$

$$\begin{array}{r} 10)680 \quad 10)20 \\ \underline{\quad} \quad \underline{\quad} \\ 68 \quad 2 \end{array}$$

Therefore 10 is their G. C. M.

$$\begin{array}{r} (11) \\ 468)922(1 \\ \underline{468} \\ 454)468(1 \\ \underline{454} \\ 14)454(32 \\ \underline{42} \\ 34 \\ \underline{28} \\ 6)14(2 \\ \underline{12} \\ 2)6 \\ \underline{\quad} \\ 3 \end{array}$$

375 is not divisible by 2,
and therefore their G. C. M. is 1.

$$\begin{array}{r} (10) \\ 1326)3094(2 \\ \underline{2652} \\ 442)1326(3 \\ \underline{1326} \end{array}$$

Also 4420 is divisible by 442 ;
therefore it is their G. C. M.

$$\begin{array}{r} (12) \\ 204)1190(5 \\ \underline{1020} \\ 170)204(1 \\ \underline{170} \\ 34)170(5 \\ \underline{170} \\ 17)2006(118 \\ \underline{17} \\ 30 \\ \underline{17} \\ 136 \\ \underline{136} \\ \text{G. C. M.}=17. \end{array}$$

Page 132.

$$\begin{array}{l} (15) \\ 56=2^3 \times 7 \\ 84=2^2 \times 3 \times 7 \\ 140=2^2 \times 5 \times 7 \\ 168=2^3 \times 3 \times 7 \end{array}$$

The greatest factors which are common are 2^2 and 7 ;
therefore the G. C. M.= $2^2 \times 7=28$.

(16)

$$241920 = 2^8 \times 3^3 \times 5 \times 7$$

$$380160 = 2^8 \times 3^3 \times 5 \times 11$$

$$69120 = 2^9 \times 3^3 \times 5$$

$$103680 = 2^8 \times 3^4 \times 5$$

The greatest factors which are common are 2^8 , 3^3 and 5 ;
therefore the G. C. M. $= 2^8 \times 3^3 \times 5 = 34560$.

(17)

$$10800 = 2^4 \times 3^3 \times 5^2$$

$$28040 = 2^3 \times 5 \times 701$$

$$2160 = 2^4 \times 3^3 \times 5$$

The greatest factors which are common are 2^3 and 5 ;
therefore the G. C. M. $= 2^3 \times 5 = 40$.

 Page 133.

(2)

$$6 = 2 \times 3$$

$$7 = 7$$

$$42 = 2 \times 3 \times 7$$

$$9 = 3^2$$

$$10 = 2 \times 5$$

$$630 = 2 \times 3^2 \times 5 \times 7$$

$$2 \times 3^2 \times 5 \times 7 = 630.$$

(3)

$$1 = 1$$

$$2 = 2$$

$$3 = 3$$

$$4 = 2^2$$

$$5 = 5$$

$$6 = 2 \times 3$$

$$7 = 7$$

$$8 = 2^3$$

$$9 = 3^2$$

$$3^2 \times 2^3 \times 5 \times 7 = 2520.$$

(4)

$$6 = 2 \times 3$$

$$9 = 3^2$$

$$12 = 2^2 \times 3$$

$$15 = 3 \times 5$$

$$18 = 2 \times 3^2$$

$$21 = 3 \times 7$$

$$30 = 2 \times 3 \times 5$$

$$2^2 \times 3^2 \times 5 \times 7 = 1260.$$

(5)

$$670 = 2 \times 5 \times 67$$

$$100 = 2^2 \times 5^2$$

$$335 = 5 \times 67$$

$$25 = 5^2$$

$$2^2 \times 5^2 \times 67 = 6700.$$

(6)

$$8 = 2^3$$

$$10 = 2 \times 5$$

$$18 = 2 \times 3^2$$

$$27 = 3^3$$

$$36 = 2^2 \times 3^2$$

$$44 = 2^2 \times 11$$

$$396 = 2^2 \times 3^2 \times 11$$

$$2^3 \times 3^3 \times 5 \times 11 = 11880.$$

(9)

$$2)12..10..24 \quad 2)14..21..3..2..63 \quad 2)18..12..39..216..234$$

$$2) \begin{array}{r} 6.. \\ \hline 5..12 \end{array} \quad 3) \begin{array}{r} 7.. \\ \hline 21..3..1..63 \end{array} \quad 2) \begin{array}{r} 9.. \\ \hline 6..39..108..117 \end{array}$$

$$3) \begin{array}{r} 3.. \\ \hline 5..6 \end{array} \quad 7) \begin{array}{r} 7.. \\ \hline 7..1..1..21 \end{array} \quad 3) \begin{array}{r} 9.. \\ \hline 3..39..54..117 \end{array}$$

$$\begin{array}{r} 1.. \\ \hline 5..2 \end{array} \quad \begin{array}{r} 1.. \\ \hline 1..1..1..3 \end{array} \quad 3) \begin{array}{r} 3.. \\ \hline 1..13..18..39 \end{array}$$

$$2 \times 2 \times 3 \times 5 \times 2 = 120 \quad 2 \times 3 \times 7 \times 3 = 126 \quad 13) \begin{array}{r} 1.. \\ \hline 1..13..6..13 \end{array}$$

$$\begin{array}{r} 1.. \\ \hline 1..1..6..1 \end{array} \quad 2 \times 2 \times 3 \times 3 \times 13 \times 6 = 2808$$

(12)

$$2)8..18..15..20..70$$

$$2) \begin{array}{r} 4.. \\ \hline 9..15..10..35 \end{array}$$

$$3) \begin{array}{r} 2.. \\ \hline 9..15..5..35 \end{array}$$

$$5) \begin{array}{r} 2.. \\ \hline 3..5..5..35 \end{array}$$

$$\begin{array}{r} 2.. \\ \hline 3..1..1..7 \end{array} \quad 2 \times 2 \times 3 \times 5 \times 2 \times 3 \times 7 = 2520.$$

(13)

$$2)24..16..18..20$$

$$2) \begin{array}{r} 12.. \\ \hline 8..9..10 \end{array}$$

$$2) \begin{array}{r} 6.. \\ \hline 4..9..5 \end{array}$$

$$3) \begin{array}{r} 3.. \\ \hline 2..9..5 \end{array}$$

$$\begin{array}{r} 1.. \\ \hline 2..3..5 \end{array} \quad 2 \times 2 \times 2 \times 3 \times 2 \times 3 \times 5 = 720.$$

(14)

$$2)60..50..144..35..18$$

$$2) \begin{array}{r} 30.. \\ \hline 25..72..35..9 \end{array}$$

$$3) \begin{array}{r} 15.. \\ \hline 25..36..35..9 \end{array}$$

$$3) \begin{array}{r} 5.. \\ \hline 25..12..35..3 \end{array}$$

$$5) \begin{array}{r} 5.. \\ \hline 25..4..35..1 \end{array}$$

$$\begin{array}{r} 1.. \\ \hline 5..4..7..1 \end{array} \quad 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 4 \times 7 = 25200.$$

(15)

$$2)27..54..81..14..63$$

$$3) \begin{array}{r} 27.. \\ \hline 27..81..7..63 \end{array}$$

$$3) \begin{array}{r} 9.. \\ \hline 9..27..7..21 \end{array}$$

$$3) \begin{array}{r} 3.. \\ \hline 3..9..7..7 \end{array}$$

$$7) \begin{array}{r} 1.. \\ \hline 1..3..7..7 \end{array}$$

$$\begin{array}{r} 1.. \\ \hline 1..3..1..1 \end{array} \quad 2 \times 3 \times 3 \times 3 \times 7 \times 3 = 1134.$$

(19)

$$300 \overline{) 3000} \quad 200 \overline{) 2000} \quad 150 \overline{) 1500} \quad 75 \overline{) 750} \quad 125 \overline{) 1250}$$

$$300 \times 10 = 3000.$$

(20)

$$165 \overline{) 2060} \quad 15 \overline{) 150} \quad 10 \overline{) 100} \quad 63 \overline{) 630} \quad 27 \overline{) 270}$$

$$165 \times 21 \times 12 = 41580.$$

(21)

$$\begin{array}{r|l}
 144 & 12 \dots 132 \dots 144 \dots 60 \dots 96 \dots 1728 \\
 12 & 11 \qquad \qquad 5 \quad 2 \quad 12 \\
 55 & 11 \qquad \qquad 5 \\
 \hline
 & 144 \times 12 \times 55 = 95040.
 \end{array}$$

Page 138.

(3)	(4)	(5)	(6)
12)592835	5)3700	11)10000	6)1000000
<u>12)49402...e</u>	<u>5)740...0</u>	<u>11)909...1</u>	<u>6)166666...4</u>
<u>12)4116...t</u>	<u>5)148...0</u>	<u>11)82...7</u>	<u>6)27777...4</u>
<u>12)343...0</u>	<u>5)29...3</u>	<u>7...5</u>	<u>6)4629...3</u>
<u>12)28...7</u>	<u>5)5...4</u>	<u>7571.</u>	<u>6)771...3</u>
<u>2...4</u>	<u>1...0</u>		<u>6)128...3</u>
2470te.	104300.		<u>6)21...2</u>
			<u>3...3</u>
			33233344.
(7)	(8)	(9)	(10)
8)10000	12)12345654321	9)10000	2)300
<u>8)1250...0</u>	<u>12)1028804526...9</u>	<u>9)1111...1</u>	<u>2)150...0</u>
<u>8)156...2</u>	<u>12)85733710...6</u>	<u>9)123...4</u>	<u>2)75...0</u>
<u>8)19...4</u>	<u>12)7144475...t</u>	<u>9)13...6</u>	<u>2)37...1</u>
<u>2...3</u>	<u>12)595372...e</u>	<u>1...4</u>	<u>2)18...1</u>
23420.	<u>12)49614...4</u>	<u>14641.</u>	<u>2)9...0</u>
	<u>12)4134...6</u>		<u>2)4...1</u>
	<u>12)344...6</u>		<u>2)2...0</u>
	<u>12)28...8</u>		<u>1...0</u>
	<u>2...4</u>		
	248664et69.		100101100

(14)

(15)

(16)

IX

V

V

IV

8)37704

7)444

7)4321

9)1212201

8)4311..5

7)32..5

7)313..5

9)23121..0

8)480..1

2..3

7)21..6

9)1101..0

8)54..4

235.

1..4

9)21..0

6..1

1465.

1..0

61415.

10000.

(19)

(20)

(21)

(22)

IV

III

IX

VI

20212331

101202220

1522365

33233344

4

3

9

6

—

—

—

—

8

3

14

21

4

3

9

6

—

—

—

—

34

10

128

128

4

3

9

6

—

—

—

—

137

32

1154

771

4

3

9

6

—

—

—

—

550

96

10389

4629

4

3

9

6

—

—

—

—

2203

290

93507

27777

4

3

9

6

—

—

—

—

8815

872

841568

166666

4

3

6

—

—

—

—

35261

2618

1000000

3

7854

(24)

IX
3)132713
3)40834..0
3)13271..1
3)4083..1
3)1327..0
3)408..1
3)132..2
3)40..2
3)13..0
3)4..0
1..1

IX
12)132713
12)10207..9
12)682..t
12)51..8
3..t

IX
8)132713
8)14757..1
8)1652..0
8)184..6
8)21..5
2..3

IX	III		XII	VIII
132713 =	11002210110 =		3t8t9 =	235601
<u>9</u>	<u>3</u>		<u>12</u>	<u>8</u>
12	4	332	46	19
<u>9</u>	<u>3</u>	<u>3</u>	<u>12</u>	<u>8</u>
110	12	997	560	157
<u>9</u>	<u>3</u>	<u>3</u>	<u>12</u>	<u>8</u>
997	36	2991	6730	1262
<u>9</u>	<u>3</u>	<u>3</u>	<u>12</u>	<u>8</u>
8974	110	8974	80769 den.	10096
<u>9</u>	<u>3</u>	<u>3</u>		<u>8</u>
80769 denary.	332	26923		80769 denary.
		<u>3</u>		
		80769 denary.		

(25)

XII	XII	XII	XII
9) <u>t2t290</u>	6) <u>t2t290</u>	4) <u>t2t290</u>	2) <u>t2t290</u>
9) <u>117978..0</u>	6) <u>185856..0</u>	4) <u>268683..0</u>	2) <u>515146..0</u>
9) <u>1624t..2</u>	6) <u>34e4e..0</u>	4) <u>78180..3</u>	2) <u>268683..0</u>
9) <u>2032..4</u>	6) <u>69t9..5</u>	4) <u>1e050..0</u>	2) <u>134341..1</u>
9) <u>284..2</u>	6) <u>1179..3</u>	4) <u>5913..0</u>	2) <u>78180..1</u>
9) <u>37..1</u>	6) <u>233..3</u>	4) <u>1533..3</u>	2) <u>3t0t0..0</u>
4..7	6) <u>46..3</u>	4) <u>439..3</u>	2) <u>1e050..0</u>
	6) <u>9..0</u>	4) <u>10e..1</u>	2) <u>e626..0</u>
	1..3	4) <u>32..3</u>	2) <u>5913..0</u>
		4) <u>9..2</u>	2) <u>2t67..1</u>
		2..1	2) <u>1533..1</u>
			2) <u>877..1</u>
			2) <u>439..1</u>
			2) <u>21t..1</u>
			2) <u>10e..0</u>
			2) <u>65..1</u>
			2) <u>32..1</u>
			2) <u>17..0</u>
			2) <u>9..1</u>
			2) <u>4..1</u>
			2) <u>2..0</u>
			1..0

(Continued on next page.)

(25 continued.)

XII	IX	VI	IV	II
$2290 = 4712420 = 130333500 = 21231330030 = 1001101101111100001100$				
<u>12</u>	<u>9</u>	<u>6</u>	<u>4</u>	<u>2</u>
122	43	9	9	2
<u>12</u>	<u>9</u>	<u>6</u>	<u>4</u>	<u>2</u>
1474	388	54	38	4
<u>12</u>	<u>9</u>	<u>6</u>	<u>4</u>	<u>2</u>
17690	3494	327	155	9
<u>12</u>	<u>9</u>	<u>6</u>	<u>4</u>	<u>2</u>
212289	31450	1965	621	19
<u>12</u>	<u>9</u>	<u>6</u>	<u>4</u>	<u>2</u>
2547468	283052	11793	2487	38
	9	6	4	2
	2547468	70763	9951	77
		6	4	2
		424578	39804	155
		6	4	2
	2547468	159216	310	159216
		4	2	2
		636867	621	318433
		4	2	2
		2547468 den.	1243	636867
				2
				1273734
				2
				2547468 den.

Page 142.

(31)	(32)	(33)	(34)
VI	XII	III	VIII
252	62te)32e75721(62te	201210	57264
252	31556	102221	675
<hr/>	<hr/>	<hr/>	<hr/>
544	161e7	21212	354604
2224	1059 t		513354
544	<hr/>		434070
<hr/>	58192		<hr/>
122024	52512		51117344
	<hr/>		
	58801		
	58801		
 (35)	 (36)	 (37)	 (38)
II	VII	VII	XII
101	2143)142613(50·5254	65432	7t348
1001	14111	43210	5e6t4
1111	<hr/>	1444	<hr/>
1011	1503·0	65001	1t864
1000	1411·1	54321	
1111	<hr/>	<hr/>	
10101	61·60	326041	
<hr/>	43·16		
1010100	<hr/>		
	15·410		
	14·111		
	<hr/>		
	1·2660		
	1·1635		
	<hr/>		
	·1022		
 (39)	 (40)		
XII	II		
34t7	100101)1010100001(10010 ¹¹¹ 10010T		
6666	100101		
<hr/>	<hr/>		
18536	101000		
18536	100101		
18536	<hr/>		
18536	111		
<hr/>			
1t36e296			

Page 146.

(45)

4 ft.	7'	6''	10'''			
9	7	11	11			
<hr/>						
		4	2	11'''	3'''	2'''
	4	2	11	3	2	
2	8	4	11	10		
41	8	1	6			
<hr/>						
44	9	1	8	0	5	2

(46)

19 ft.	10'	3''				
11	2	7				
<hr/>						
		11	6	11'''	9'''	
3	3	8	6			
218	4	9				
<hr/>						
222	8	0	5	9		

(47)

9''	7'''	4'''				
7	3	11'''				
<hr/>						
			8'''	9'''	8'''	8'''
	2	4	10	0		
5	7	3	4			
<hr/>						
5	10	4	11	8	8	

(48)

9½ in.	=	9'	9''			
9'	9''					
5	7	4'''				
<hr/>						
		3	3'''	0'''		
	5	8	3			
4	0	9				
<hr/>						
4	6	8	6			

(49)

7 ft.	4'	11''			
3	2	2			
<hr/>					
	1	2	9'''	10'''	
1	2	9	10		
22	2	9			
<hr/>					
23	6	9	7	10	

(51)

15 ft.					
1	2'				
<hr/>					
2	6				
15					
<hr/>					
17	6				
	8				
<hr/>					
11	8	0''	=		

11½ cub.ft.=11 cub.ft. 1152 cub.in. ——— cub.ft.

(52)

XII					
45·6					
t·3					
<hr/>					
1146					
3870					
<hr/>					
398·46					
2					

774·90=1096 9'

(53)

10 ft.					
5					
<hr/>					
50 sq.ft.					
7					
<hr/>					
—c'rds.c.ft.					
128)350(2	94				
256					
<hr/>					
94 cub.ft.					

XII

774=1096 com. scale.

(54)		(55)		(56)
4 ft.				25 ft. = 300 in.
5 $\frac{1}{4}$		XII		20 " = 240 "
—		4·78		2 ft. 6 in. = 30 "
20		9·6		
1		—		8
—		23 t 0		4
21 sq. ft.		3590		—
70		—		32
—		38·0 t		300
128) 1470 (11 $\frac{3}{4}$ cords		2·e		240
128		—		—
—		34492		64 = 8 × 8
190		7418		72000
128		— cub.ft.		30
—		t8·652 = 128 6' 5" 2'''		8) 2160000
62		t8 duoden. = 128 den.		8) 270000
$\frac{62}{128} = \frac{31}{64}$		—		—
				33750

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(1)

£93×400	=	\$372·00	£276×400	=	\$1104·00
14s.×20	=	2·80	19s.×20	=	3·80
7½d.=30 f.×5÷12	=	·12½	10½d.=42 f.×5÷12	=	·17½
		<hr/>			<hr/>
£93 14s. 7½d.	=	\$374·92½	£276 19s. 10½d.	=	\$1107·97½
£275×400	=	\$1100 00			\$729·18
4s.×20	=	·80			710·50
11¼d.=47 f.×5÷12	=	·19⅞			166·78
		<hr/>			374·92½
£275 4s. 11¼d.	=	\$1100·99⅞			1107·97½
					497·81
					1100·99⅞
					<hr/>
					\$4688·16⅞

$$\begin{array}{r}
 (2) \\
 576 = 6 + 10 \times 7 + 10 \times 10 \times 5 \\
 \begin{array}{r}
 \text{m. fur. per. yds. ft. in.} \qquad \qquad \text{m. fur. per. yds. ft. in.} \\
 47 \ 6 \ 17 \ 4 \ 2 \ 7 \times 6 = 286 \ 6 \ 27 \ 1 \ 2 \ 0 \\
 \hline
 478 \ 0 \ 18 \ 4 \ 1 \ 10 \times 7 = 3346 \ 3 \ 11 \ 4 \ 2 \ 4 \\
 \hline
 4780 \ 4 \ 28 \ 2 \ 0 \ 4 \times 5 = 23902 \ 7 \ 21 \ 4 \ 3 \ 2 \\
 \hline
 \qquad \qquad \qquad 27536 \ 1 \ 21 \ 0 \ 1 \ 6
 \end{array}
 \end{array}$$

(3)

$$243000=2^3 \times 3^5 \times 5^3$$

$$3+1=4$$

$$5+1=6$$

$$3+1=4$$

$$4 \times 6 \times 4 = 96$$

(4)

(5)

v		viii		79·342÷·00006378=	
8)4234434		5)713427		6378)7934200000(1243994·98275	
<u> </u>		<u> </u>		6378	
8)241110..4		5)133721..2		<u> </u>	
<u> </u>		<u> </u>		15562	
8)13423..1		5)22303..2		12756	
<u> </u>		<u> </u>		<u> </u>	
8)1024..1		5)3532..1		28060	
<u> </u>		<u> </u>		25512	
8)32..3		5)570..2		<u> </u>	
<u> </u>		<u> </u>		25480	
2..1		5)113..1		19134	
		<u> </u>		<u> </u>	
		5)17..0		63460	
		<u> </u>		57402	
		3..0		<u> </u>	
				60580	
				57402	
				<u> </u>	
				31780	
				25512	
				<u> </u>	
				6268·0	
				5740·2	
				<u> </u>	
				527·80	
				510·24	
				<u> </u>	
				17·560	
				12·756	
				<u> </u>	
				4·8040	
				4·4646	
				<u> </u>	
				·33940	
				·31890	
				<u> </u>	
				·02050	

(7)

40	5.7.9.11.15.18.20.21.22.24.28.30.33.35.36.40.42.44.45.48.50.
21	7.9.11. 3. 5 21.11. 3. 7. 3.33. 7. 5 21.11. 5. 6. 5.
33	3 11 3 11. 11 3 11. 3. 2. 5.
10	2. 5.

$$40 \times 21 \times 33 \times 10 = 277200.$$

(9)

$$9999993000 = 10000000000 - 7000.$$

$$64276 \cdot 3427 \times 10000000000 = 642763427000000$$

$$64276 \cdot 3427 \times 7000 = \underline{\quad\quad\quad} 449934398 \cdot 9$$

$$\underline{\quad\quad\quad} 642762977065601 \cdot 1$$

(10)

$$\begin{array}{r}
 \text{IX} \\
 5) 78263 \\
 \hline
 5) 15230 \dots 3 \\
 \hline
 5) 2760 \dots 0 \\
 \hline
 5) 511 \dots 4 \\
 \hline
 5) 102 \dots 0 \\
 \hline
 5) 17 \dots 3 \\
 \hline
 3 \dots 1
 \end{array}$$

$$\begin{array}{r}
 \text{IX} \\
 11) 78263 \\
 \hline
 11) 6430 \dots 3 \\
 \hline
 11) 526 \dots 6 \\
 \hline
 11) 43 \dots 0 \\
 \hline
 3 \dots 6
 \end{array}$$

$$\begin{array}{r}
 \text{IX} \\
 7) 78263 \\
 \hline
 7) 11160 \dots 3 \\
 \hline
 7) 1407 \dots 5 \\
 \hline
 7) 177 \dots 3 \\
 \hline
 7) 23 \dots 4 \\
 \hline
 3 \dots 0
 \end{array}$$

$$\begin{array}{r}
 \text{V} \\
 7) 3130403 \\
 \hline
 7) 214200 \dots 3 \\
 \hline
 7) 13220 \dots 5 \\
 \hline
 7) 1101 \dots 3 \\
 \hline
 7) 41 \dots 4 \\
 \hline
 3 \dots 0
 \end{array}$$

$$\begin{array}{r}
 \text{XI} \\
 7) 36063 \\
 \hline
 7) 5640 \dots 3 \\
 \hline
 7) 884 \dots 5 \\
 \hline
 7) 128 \dots 3 \\
 \hline
 7) 14 \dots 4 \\
 \hline
 3 \dots 0
 \end{array}$$

(12)

$$\begin{array}{rcl}
 £672 \times 400 & = & \$2688 \cdot 00 \\
 7s. \times 20 & = & 1 \cdot 40 \\
 7d. = 28f. \times 5 \div 12 = & & \cdot 11\frac{2}{3} \\
 \hline
 £672 \ 7s. \ 7d. & = & \$2689 \cdot 51\frac{2}{3}
 \end{array}$$

(13)

$$\begin{array}{r}
 891)243000(272 \\
 \underline{1782} \\
 6480 \\
 \underline{6237}
 \end{array}$$

(13 continued.)

$$81)37800(466$$

$$\underline{324}$$

$$540$$

$$\underline{486}$$

$$54)81(1$$

$$\underline{54}$$

$$27)54(2$$

$$\underline{54}$$

$$2430$$

$$\underline{1782}$$

$$648)891(1$$

$$\underline{648}$$

$$243)648(2$$

$$\underline{486}$$

$$27)35100$$

$$\underline{1300}$$

$$162)243(1$$

$$\underline{162}$$

$$81)162(2$$

$$\underline{162}$$

Therefore G. C. M. = 27.

(17)

(18)

(19)

$$£ \ s. \ d. \ 2)276000$$

$$178 \ 16 \ 4\frac{3}{4} \quad \underline{\hspace{1cm}} \quad 6 \text{ ft. } 2' \ 7'' \ 9''' \ 10''''$$

$$97 \ 15 \ 11\frac{1}{2} \ 2)138000 \ 13 \ 11 \ 11 \ 11 \ 7$$

$$693 \ 19 \ 11\frac{3}{4}$$

$$216 \ 11 \ 9\frac{1}{2} \ 2)69000 \quad \quad \quad 3 \ 7 \ 6'''' \ 6'''''' \ 8'''''''' \ 10''''''''$$

$$678 \ 14 \ 7\frac{1}{2} \quad \quad \quad 5 \ 8 \ 5 \ 2 \ 0 \ 2$$

$$197 \ 13 \ 11\frac{3}{4} \ 2)34500 \quad 5 \ 8 \ 5 \ 2 \ 0 \ 2$$

$$117 \ 6 \ 5 \quad \quad \quad 5 \ 8 \ 5 \ 2 \ 0 \ 2$$

$$91 \ 1 \ 1\frac{1}{2} \ 2)17250 \ 80 \ 10 \ 5 \ 7 \ 10$$

$$2272 \ 0 \ 3\frac{1}{4} \ 3)8625 \ 87 \ 1 \ 1 \ 3 \ 0 \ 10 \ 8 \ 10 \ 10$$

$$\underline{5)2875}$$

$$\underline{5)575}$$

$$\underline{5)115}$$

$$\underline{23}$$

$$2^5 \times 3 \times 5^3 \times 23$$

(20)	(21)	(22)
XII	IV	VIII
713t96)7te9·047(·011436	3333333	10000
<u>713t·96</u>	<u>4</u>	<u>8 .</u>
97t·2t7	15	8
<u>713·t96</u>	<u>4</u>	<u>8</u>
266·4110	63	64
<u>245·3720</u>	<u>4</u>	<u>8</u>
21·05300	255	512
<u>19·3e846</u>	<u>4</u>	<u>8</u>
3·862760	1023	4096
<u>3·67e490</u>	<u>4</u>	
·1t3290	4095	
	<u>4</u>	
	16383	

(23)

$$74002702 \div 144 = 513907 \text{ ft. } 94 \text{ in.}$$

$$512907 \text{ ft. } \div 9 = 57100 \text{ yards } 7 \text{ ft.}$$

$$57100 \text{ yds. } \div 30\frac{1}{4} = 1887 \text{ per. } 18\frac{1}{4} \text{ yds.}$$

$$1887 \text{ per. } 18 \text{ yds. } 2 \text{ ft. } 36 \text{ in.}$$

$$\text{Add } 7 \text{ ft. } 94 \text{ in.}$$

$$40)1887 \text{ per. } 19 \text{ yds. } 0 \text{ ft. } 130 \text{ in.}$$

$$4)47 \text{ r. } 7 \text{ per. } 19 \text{ yds. } 0 \text{ ft. } 130 \text{ in.}$$

$$11 \text{ a. } 3 \text{ r. } 7 \text{ per. } 19 \text{ yds. } 0 \text{ ft. } 130 \text{ in.}$$

(24)

$$\begin{array}{r|l} 1728 & 240..780..1260..1728 \\ 65 & 5..65..15 \\ 3 & 3 \end{array}$$

$$1728 \times 65 \times 3 = 336960.$$

(28)	(29)	(30)
762·4978	723426	lbs. oz. drs. scr.
63·423	938·9126141	129 0 0 0
<hr/>	<hr/>	63 4 7 2
22874934	722487·0873859	<hr/>
15249956		65 7 0 1
30499912		
22874934		
45749868		
<hr/>		
48359·8979694		

(31)

$$1064=2^3 \times 7 \times 19.$$

1..2..4..8

1..7

1..2..4..8..7..14..28..56

1..19

1..2..4..8..7..14..28..56..19..38..76..152..133..266..532..1064=

1..2..4..7..8..14..19..28..38..56..76..133..152..266..532..1064

(32)

30 ft. 6 in. = 366 in. 366

20 ft. 11 in. = 251 in. 251

2 ft. 7 in. = 31 in. —

366

1830

732

— in.

31)91866(2963 $\frac{1}{3}$

62

—

298

279

—

2963 $\frac{1}{3}$ ÷ 36 = 82 $\frac{5}{18}$ yds.

196

186

—

106

93

—

13

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(30)

$$\frac{2}{5}, \frac{5}{7}, \frac{8}{9}, \frac{3}{5}, \frac{5}{8} = \frac{2 \times 7 \times 9 \times 5 \times 18}{5 \times 7 \times 9 \times 5 \times 18}, \frac{5 \times 5 \times 9 \times 5 \times 18}{5 \times 7 \times 9 \times 5 \times 18}, \frac{8 \times 5 \times 7 \times 5 \times 18}{5 \times 7 \times 9 \times 5 \times 18},$$

$$\frac{3 \times 5 \times 7 \times 9 \times 18}{5 \times 7 \times 9 \times 5 \times 18}, \frac{5 \times 5 \times 7 \times 9 \times 5}{5 \times 7 \times 9 \times 5 \times 18} = \frac{11340}{28350}, \frac{21250}{28350}, \frac{25200}{28350}, \frac{17010}{28350}, \frac{7875}{28350}$$

(31)

$$\frac{8}{11}, \frac{12}{13}, \frac{5}{14} = \frac{8 \times 13 \times 14}{11 \times 13 \times 14}, \frac{12 \times 11 \times 14}{11 \times 13 \times 14}, \frac{5 \times 11 \times 13}{11 \times 13 \times 14} = \frac{1456}{2002}, \frac{1848}{2002}, \frac{715}{2002}$$

(32)

$$\frac{6}{7}, \frac{4}{11}, \frac{5}{13}, \frac{4}{7}, \frac{1}{2} = \frac{6 \times 11 \times 13 \times 7 \times 2}{7 \times 11 \times 13 \times 7 \times 2}, \frac{4 \times 7 \times 13 \times 7 \times 2}{7 \times 11 \times 13 \times 7 \times 2},$$

$$\frac{5 \times 7 \times 11 \times 7 \times 2}{7 \times 11 \times 13 \times 7 \times 2}, \frac{4 \times 7 \times 11 \times 13 \times 2}{7 \times 11 \times 13 \times 7 \times 2}, \frac{1 \times 7 \times 11 \times 13 \times 7}{7 \times 11 \times 13 \times 7 \times 2} =$$

$$\frac{12012}{14014}, \frac{5096}{14014}, \frac{5390}{14014}, \frac{8008}{14014}, \frac{7007}{14014}.$$

(33)

$$\frac{6}{11}, \frac{4}{7}, \frac{8}{13} = \frac{6 \times 7 \times 13}{11 \times 7 \times 13}, \frac{4 \times 11 \times 13}{11 \times 7 \times 13}, \frac{8 \times 11 \times 7}{11 \times 7 \times 13} = \frac{546}{1001}, \frac{572}{1001}, \frac{616}{1001}$$

(34)

$$\frac{5}{6}, \frac{4}{7}, \frac{4}{5}, \frac{2}{11} = \frac{5 \times 7 \times 5 \times 11}{6 \times 7 \times 5 \times 11}, \frac{4 \times 6 \times 5 \times 11}{6 \times 7 \times 5 \times 11}, \frac{4 \times 6 \times 7 \times 11}{6 \times 7 \times 5 \times 11},$$

$$\frac{2 \times 6 \times 7 \times 5}{6 \times 7 \times 5 \times 11} = \frac{1925}{2310}, \frac{1320}{2310}, \frac{1848}{2310}, \frac{420}{2310}.$$

(35)

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{2}{7} = \frac{1 \times 3 \times 5 \times 7}{2 \times 3 \times 5 \times 7}, \frac{2 \times 2 \times 5 \times 7}{2 \times 3 \times 5 \times 7}, \frac{3 \times 2 \times 3 \times 7}{2 \times 3 \times 5 \times 7},$$

$$\frac{2 \times 2 \times 3 \times 5}{2 \times 3 \times 5 \times 7} = \frac{105}{210}, \frac{140}{210}, \frac{126}{210}, \frac{60}{210}.$$

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(38)

$$\frac{4}{5}, \frac{3}{8}, \frac{4}{6}, \frac{3}{4}, \frac{7}{15}.$$

The least common multiple of 5, 8, 6, 4, 15 is 120.

The multiplier for both terms of the first fraction is $\frac{120}{5} = 24$; for the second $\frac{120}{8} = 15$; for the third $\frac{120}{6} = 20$; for the fourth $\frac{120}{4} = 30$; for the fifth $\frac{120}{15} = 8$.

Multiplying by these numbers, we obtain $\frac{96}{120}, \frac{45}{120}, \frac{80}{120}, \frac{90}{120}$, and $\frac{56}{120}$.

(39)

$$\frac{6}{11}, \frac{2}{3}, \frac{4}{7}, \frac{48}{77}, \frac{13}{33}.$$

The least common multiple of 11, 3, 7, 77 and 33 is 231.

The multiplier for both terms of the first fraction is $\frac{231}{11} = 21$; for the second, $\frac{231}{3} = 77$; for the third, $\frac{231}{7} = 33$; for the fourth, $\frac{231}{77} = 3$; and for the fifth, $\frac{231}{33} = 7$.

Multiplying by these numbers, we obtain $\frac{126}{231}, \frac{154}{231}, \frac{132}{231}, \frac{54}{231}$, and $\frac{133}{231}$.

(40)

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10}, \frac{13}{15}, \frac{7}{16}, \frac{37}{80}.$$

The least common multiple of 2, 3, 5, 6, 8, 10, 15, 16 and 80 is 240.

The multiplier for both terms of the first fraction is $\frac{240}{2} = 120$; for the second, $\frac{240}{3} = 80$; for the third, $\frac{240}{5} = 48$; for the fourth, $\frac{240}{6} = 40$; for the fifth, $\frac{240}{8} = 30$; for the sixth, $\frac{240}{10} = 24$; for the seventh, $\frac{240}{15} = 16$; for the eighth, $\frac{240}{16} = 15$; and for the ninth, $\frac{240}{80} = 3$.

Multiplying by these numbers, we obtain $\frac{120}{240}, \frac{160}{240}, \frac{144}{240}, \frac{200}{240}, \frac{210}{240}, \frac{216}{240}, \frac{208}{240}, \frac{105}{240}$, and $\frac{111}{240}$.

(41)

$$\frac{3}{5}, \frac{7}{10}, \frac{6}{25}, \frac{11}{30}, \frac{13}{45}, \frac{23}{60}.$$

The least common multiple of 5, 10, 25, 30, 45, and 60 is 900.

The multiplier for both terms of the first fraction is $\frac{900}{6} = 180$; for the second, $\frac{900}{10} = 90$; for the third, $\frac{900}{25} = 36$; for the fourth, $\frac{900}{30} = 30$; for the fifth, $\frac{900}{45} = 20$; and for the sixth, $\frac{900}{60} = 15$.

Multiplying by these numbers, we obtain $\frac{540}{900}, \frac{630}{900}, \frac{216}{900}, \frac{330}{900}, \frac{260}{900}$, and $\frac{345}{900}$.

(42)

$$\frac{19}{20}, \frac{7}{30}, \frac{11}{40}, \frac{1}{50}.$$

The least common multiple of 20, 30, 40, and 50 is 600.

The multiplier for both terms of the first fraction is $\frac{600}{20} = 30$; for the second, $\frac{600}{30} = 20$; for the third, $\frac{600}{40} = 15$; and for the fourth, $\frac{600}{50} = 12$.

Multiplying by these numbers, we obtain $\frac{570}{600}, \frac{140}{600}, \frac{165}{600}$ and $\frac{12}{600}$.

(43)

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{11}{12}, \frac{15}{16}, \frac{23}{24}.$$

The least common multiple of 2, 3, 4, 6, 8, 12, 16, and 24 is 48.

The multiplier for both terms of the first fraction is $\frac{48}{2} = 24$; for the second, $\frac{48}{3} = 16$; for the third, $\frac{48}{4} = 12$; for the fourth, $\frac{48}{6} = 8$; for the fifth, $\frac{48}{8} = 6$; for the sixth, $\frac{48}{12} = 4$; for the seventh, $\frac{48}{16} = 3$; and for the eighth, $\frac{48}{24} = 2$.

Multiplying by these numbers, we obtain $\frac{24}{48}, \frac{32}{48}, \frac{36}{48}, \frac{40}{48}, \frac{42}{48}, \frac{44}{48}, \frac{45}{48}$, and $\frac{46}{48}$.

(44)

$$\frac{5}{7}, \frac{11}{12}, \frac{2}{15}, \frac{8}{27}, \frac{9}{35}, \frac{17}{40}.$$

The least common multiple of 7, 12, 15, 27, 35 and 40 is 7560.

The multiplier for both terms of the first fraction is $\frac{7560}{5} = 1512$; for the second, $\frac{7560}{12} = 630$; for the third, $\frac{7560}{15} = 504$; for the fourth, $\frac{7560}{27} = 280$; for the fifth, $\frac{7560}{35} = 216$; for the sixth, $\frac{7560}{40} = 189$.

Multiplying by these numbers, we obtain $\frac{5400}{7560}, \frac{6330}{7560}, \frac{1008}{7560}, \frac{2240}{7560}, \frac{1944}{7560}$, and $\frac{3213}{7560}$.

(45)

$$\frac{1}{15}, \frac{7}{8}, \frac{4}{3}, \frac{11}{12}, \frac{6}{11}, \frac{19}{20}, \frac{6}{7}, \frac{29}{35}.$$

The least common multiple of 15, 8, 3, 12, 11, 20, 7, and 35 is 9240.

The multiplier for both terms of the first fraction is $\frac{9240}{15} = 616$; for the second, $\frac{9240}{8} = 1155$; for the third, $\frac{9240}{3} = 3080$; for the fourth, $\frac{9240}{12} = 770$; for the fifth, $\frac{9240}{11} = 840$; for the sixth, $\frac{9240}{20} = 462$; for the seventh, $\frac{9240}{7} = 1320$; for the eighth, $\frac{9240}{35} = 264$.

Multiplying by these numbers, we obtain $\frac{8624}{9240}, \frac{8085}{9240}, \frac{12320}{9240}, \frac{8470}{9240}, \frac{5040}{9240}, \frac{8778}{9240}, \frac{7920}{9240}$, and $\frac{7656}{9240}$.

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(47)

$$\frac{4}{7} \text{ of } \frac{3}{5} \text{ of } \frac{6}{11} \text{ of } \frac{35}{72} = \frac{4 \times 3 \times 6 \times 35}{7 \times 5 \times 11 \times 72} = \frac{2520}{27720} = \frac{1}{11}.$$

(48)

$$\frac{2}{3} \text{ of } \frac{4}{9} \text{ of } \frac{6}{7} \text{ of } \frac{81}{100} \text{ of } \frac{25}{24} = \frac{2 \times 4 \times 6 \times 81 \times 25}{3 \times 9 \times 7 \times 100 \times 24} = \frac{97200}{453600} = \frac{1}{4}.$$

(49)

$$\frac{21}{35} \text{ of } \frac{6}{11} \text{ of } \frac{77}{36} = \frac{21 \times 6 \times 77}{35 \times 11 \times 36} = \frac{7}{10}.$$

(50)

$$\frac{2}{5} \text{ of } \frac{4}{7} \text{ of } \frac{3}{11} \text{ of } \frac{13}{17} = \frac{2 \times 4 \times 3 \times 13}{5 \times 7 \times 11 \times 17} = \frac{312}{6545}.$$

Page 161.

(53)

$$\frac{5}{9} \text{ of } \frac{6}{7} \text{ of } \frac{3}{4} \text{ of } \frac{1}{16} = \frac{5 \times 6 \times 2 \times 3}{9 \times 7 \times 3 \times 16} = \frac{5 \times \overset{2}{\cancel{6}} \times 2 \times 3}{\underset{3}{9} \times 7 \times \underset{8}{\cancel{3}} \times \underset{4}{\cancel{16}}} = \frac{5}{3 \times 7 \times 4} = \frac{5}{84}.$$

(54)

$$\frac{2}{3} \text{ of } \frac{5}{9} \text{ of } \frac{1}{132} \text{ of } \frac{6}{11} \text{ of } \frac{1}{13} \text{ of } \frac{1}{17} = \frac{2 \times 5 \times 18 \times 6 \times 11 \times 13}{3 \times 9 \times 132 \times 11 \times 13 \times 17} =$$

$$\frac{2 \times 5 \times \overset{2}{\cancel{18}} \times \overset{2}{\cancel{6}} \times 11 \times 13}{\underset{33}{9} \times \underset{1}{9} \times \underset{11}{132} \times 11 \times 13 \times 17} = \frac{2 \times 5}{33 \times 17} = \frac{10}{561}.$$

(55)

$$\frac{2}{7} \text{ of } \frac{4}{11} \text{ of } 5\frac{1}{2} = \frac{2 \times 4 \times 11}{7 \times 11 \times 2} = \frac{2 \times 4 \times 11}{7 \times 11 \times 2} = 4.$$

(56)

$$\frac{1}{9} \text{ of } \frac{8}{13} \text{ of } \frac{1}{200} \text{ of } \frac{50}{169} \text{ of } \frac{1}{17} \text{ of } \frac{1}{6} = \frac{1 \times 8 \times 117 \times 50 \times 13 \times 13}{9 \times 13 \times 200 \times 169 \times 17 \times 6} =$$

$$\frac{1 \times \overset{2}{\cancel{8}} \times \overset{9}{\cancel{117}} \times \underset{4}{\cancel{50}} \times \underset{13}{\cancel{13}} \times 13}{9 \times \underset{4}{\cancel{13}} \times \underset{13}{\cancel{200}} \times \underset{13}{\cancel{169}} \times 17 \times \underset{3}{\cancel{6}}} = \frac{1}{17 \times 3} = \frac{1}{51}.$$

(57)

$$\frac{3}{11} \text{ of } \frac{4}{7} \text{ of } \frac{9}{19} \text{ of } \frac{33}{47} \text{ of } \frac{38}{72} \text{ of } \frac{47}{7} = \frac{3 \times 4 \times 9 \times 33 \times 38 \times 47}{11 \times 7 \times 19 \times 47 \times 72 \times 7} =$$

$$\frac{3 \times \underset{11}{\cancel{4}} \times \underset{3}{\cancel{9}} \times \underset{11}{\cancel{33}} \times \underset{2}{\cancel{38}} \times \underset{2}{\cancel{47}}}{11 \times 7 \times 19 \times \underset{11}{\cancel{47}} \times \underset{2}{\cancel{72}} \times 7} = \frac{3 \times 3}{7 \times 7} = \frac{9}{49}.$$

(58)

$$\frac{1}{4} \text{ of } \frac{3}{11} \text{ of } \frac{154}{1} = \frac{4 \times 3 \times 154}{7 \times 11 \times 1} = \frac{4 \times 3 \times \overset{2}{\cancel{14}} \overset{11}{\cancel{14}}}{7 \times 11 \times 1} = \frac{2 \times 4 \times 3}{1} = 24.$$

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(61)

$$\frac{\frac{14}{45}}{1\frac{7}{25}} = \frac{\frac{14}{45}}{\frac{42}{25}} = \frac{14 \times 25}{45 \times 42} = \frac{\overset{5}{\cancel{14}} \times \overset{5}{\cancel{25}}}{\underset{9}{\cancel{45}} \times \underset{3}{\cancel{42}}} = \frac{5}{9 \times 3} = \frac{5}{27}.$$

(62)

$$\frac{1\frac{1}{2}}{7\frac{7}{18}} = \frac{\frac{11}{2}}{1\frac{43}{18}} = \frac{11 \times 18}{12 \times 143} = \frac{11 \times \overset{3}{\cancel{18}}}{\underset{2}{\cancel{12}} \times \underset{13}{\cancel{143}}} = \frac{3}{2 \times 13} = \frac{3}{26}.$$

(63)

$$\frac{15\frac{3}{5}}{7\frac{4}{6}} = \frac{\frac{78}{5}}{\frac{39}{6}} = \frac{78 \times 5}{5 \times 39} = \frac{\overset{2}{\cancel{78}} \times \overset{5}{\cancel{5}}}{\underset{5}{\cancel{5}} \times \underset{39}{\cancel{39}}} = 2.$$

(64)

$$\frac{11\frac{3}{5}}{12\frac{8}{5}}, \frac{3\frac{1}{4}}{9}, \frac{\frac{2}{7}}{\frac{3}{8}} = \frac{\frac{35}{5}}{\frac{68}{5}}, \frac{\frac{13}{4}}{\frac{9}{1}}, \frac{\frac{2}{7}}{\frac{3}{8}} = \frac{35 \times 5}{3 \times 68}, \frac{13 \times 1}{9 \times 4}, \frac{2 \times 5}{7 \times 3} = \frac{175}{204}, \frac{13}{36}, \frac{10}{21}.$$

(65)

$$\frac{7\frac{1}{2}}{15\frac{3}{4}}, \frac{5\frac{7}{8}}{16}, \frac{2\frac{2}{3}}{3\frac{2}{7}} = \frac{\frac{7}{2}}{\frac{63}{4}}, \frac{\frac{47}{8}}{16}, \frac{\frac{12}{5}}{\frac{24}{7}} = \frac{\overset{7}{\cancel{7}} \times \overset{4}{\cancel{4}}}{\underset{3}{\cancel{12}} \times \underset{9}{\cancel{63}}}, \frac{47 \times \overset{2}{\cancel{16}}}{8 \times 3}, \frac{12 \times 7}{5 \times \underset{2}{\cancel{24}}} = \frac{1}{3 \times 9}, \frac{47 \times 2}{3}, \frac{7}{5 \times 2} = \frac{1}{27}, 31\frac{1}{3}, 1\frac{7}{5}.$$

(66)

$$\frac{16\frac{2}{3}}{11\frac{2}{3}}, \frac{6\frac{1}{3}}{13}, \frac{17}{18\frac{1}{3}}, \frac{21\frac{3}{5}}{10\frac{2}{7}}, \frac{\frac{1}{2}}{4\frac{3}{5}} = \frac{\frac{50}{3}}{\frac{35}{3}}, \frac{\frac{31}{5}}{\frac{13}{1}}, \frac{\frac{17}{1}}{\frac{55}{3}}, \frac{\frac{108}{6}}{\frac{72}{7}}, \frac{\frac{1}{2}}{\frac{23}{5}} = \frac{10}{50 \times 3}, \frac{31}{3 \times 35}, \frac{17}{7}$$

$$\frac{31 \times 1}{5 \times 13}, \frac{17 \times 3}{55 \times 1}, \frac{108 \times 7}{72 \times 5}, \frac{1 \times 5}{2 \times 23} = \frac{10}{7}, \frac{31}{65}, \frac{51}{55}, \frac{21}{10}, \frac{5}{46} = 1\frac{3}{7}, 3\frac{1}{5}, 5\frac{1}{5}, 2\frac{1}{10}, 5\frac{5}{6}.$$

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(69)

$$\frac{\frac{4}{5}}{5} \text{ of } \frac{1}{16\frac{1}{4}} = \frac{1}{20} \text{ of a lb.}$$

(70)

$$\frac{\frac{2}{3}}{3} \text{ of } \frac{3}{7} \text{ of } \frac{1}{12\frac{1}{2}} \text{ of } \frac{1}{20} = \frac{1}{7 \times 6 \times 20} = \pounds \frac{1}{840}.$$

(71)

$$\frac{2}{9} \text{ of } \frac{35}{4\frac{1}{2}} \text{ of } \frac{1}{8} = \frac{5}{9 \times 2} = \frac{5}{18} \text{ wk.}$$

(72)

$$\frac{5}{11} \text{ of } \frac{81}{5} \text{ of } \frac{4}{4} \text{ of } \frac{1}{5} = \frac{81}{11 \times 4 \times 5} = \frac{81}{220} \text{ Eng. Ell.}$$

(73)

$$\frac{3}{7} \text{ of } \frac{4}{11} \text{ of } \frac{1}{5\frac{1}{2}} = \frac{3}{7} \text{ of } \frac{4}{11} \text{ of } \frac{2}{11} = 3\frac{4}{47} \text{ per.}$$

(74)

$$\frac{2}{3} \text{ of } \frac{4}{7} \text{ of } 21 \frac{1}{14} \text{ of } \frac{1}{8} = \frac{2 \times 4 \times 295 \times 1}{3 \times 7 \times 14 \times 8} = \frac{295}{294} = 1 \frac{1}{294} \text{ c.}$$

(75)

$$\frac{3}{19} \text{ of } \frac{4}{17} \text{ of } 9 \frac{1}{2} \text{ of } \frac{1}{40} \text{ of } \frac{1}{4} = \frac{3 \times 4 \times 19 \times 1 \times 1}{19 \times 17 \times 2 \times 40 \times 4} = \frac{3}{17 \times 2 \times 40} = \frac{3}{1360} \text{ a.}$$

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(78)

$$\frac{14}{79} \text{ of } \frac{4}{1} \text{ of } \frac{2}{1} \text{ of } \frac{4}{1} = \frac{448}{79} \text{ qt.}$$

(79)

$$\frac{2}{9} \text{ of } \frac{4}{1} \times \frac{2}{1} \times \frac{4}{1} \times \frac{5}{1} \times \frac{3}{2} = \frac{2 \times 4 \times 4 \times 5}{3} = \frac{160}{3}.$$

(80)

$$\frac{7}{9} \times \frac{2}{1} \times \frac{2}{1} \times \frac{4}{1} \times \frac{2}{1} \times \frac{2}{1} \times \frac{3}{2} = \frac{7 \times 2 \times 2 \times 4 \times 2}{3} = \frac{224}{3}.$$

(81)

$$\frac{17}{22} \times \frac{6}{11} \times \frac{8}{1} \times \frac{3}{1} = \frac{17 \times 6 \times 8 \times 3}{11} = \frac{2448}{11} \text{ scr.}$$

(82)

$$\frac{1}{5000} \times \frac{2}{3} \times \frac{3}{4} \times \frac{6}{11} \times \frac{2}{7} \times \frac{2}{1} \times \frac{4}{1} = \frac{2 \times 6 \times 2 \times 2 \times 4}{625 \times 7} = \frac{192}{4375} \text{ dr.}$$

(84)

$$\begin{array}{r}
 \text{bush. pk. gal. qt. pt.} \\
 11) 3 \quad 0 \quad 0 \quad 0 \quad 0 \\
 \hline
 1 \quad 0 \quad 0 \quad 1 \frac{5}{11}
 \end{array}$$

$$\begin{array}{r}
 \text{lbs. oz. dr.} \\
 7) 6 \quad 0 \quad 0 \\
 \hline
 13 \quad 11 \frac{3}{7}
 \end{array}$$

(85)

$$\begin{array}{r}
 \text{yds. qr. na. in.} \\
 13) 7 \quad (2 \quad 0 \quad 1 \frac{6}{13} \\
 4 \\
 \hline
 28 \text{ qrs.} \\
 26 \\
 \hline
 2 \\
 4 \\
 \hline
 8 \text{ na.} \\
 2 \frac{1}{4} \\
 \hline
 18 \\
 13 \\
 \hline
 5
 \end{array}$$

(87)

$$\begin{array}{r}
 \text{fur. per. yds. ft. in.} \\
 9) 8 \quad 0 \quad 0 \quad 0 \quad 0 \\
 \hline
 35 \quad 3 \quad 0 \quad 2
 \end{array}$$

$$\begin{array}{r}
 \text{£ s. d.} \\
 7) 4 \quad 0 \quad 0 \\
 \hline
 11 \quad 5 \frac{1}{7}
 \end{array}$$

(86)

$$\begin{array}{r}
 \text{lbs. oz. dwt. grs.} \\
 9) 8 \quad 0 \quad 0 \quad 0 \\
 \hline
 10 \quad 13 \quad 8
 \end{array}$$

$$\begin{array}{r}
 \text{sq. m. a. r. pr. yds. ft. in.} \\
 113) 11 \quad (62 \quad 1 \quad 8 \quad 4 \quad 2 \quad 79 \frac{1}{113} \\
 640 \\
 \hline
 7040 \text{ a.} \\
 678
 \end{array}$$

$$\begin{array}{r}
 260 \qquad 484 \text{ yds.} \\
 226 \qquad 452 \\
 \hline
 34 \qquad 32 \\
 4 \qquad 9 \\
 \hline
 136 \text{ r.} \qquad 288 \text{ ft.} \\
 113 \qquad 226 \\
 \hline
 23 \qquad 62 \\
 40 \qquad 144 \\
 \hline
 920 \text{ per.} \qquad 248 \\
 904 \qquad 248 \\
 \hline
 16 \qquad 62 \\
 30 \frac{1}{4} \qquad 8928 \text{ in.} \\
 \hline
 480 \qquad 791 \\
 4 \qquad 1018 \\
 \hline
 484 \text{ yds.} \qquad 1017 \\
 \hline
 1
 \end{array}$$

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(90)

$$6 \text{ bus. } 1 \text{ pk. } 1 \text{ gal. } 1 \text{ qt. } 1 \text{ pt.} = 411 \text{ pts.}$$

$$50 \text{ bush.} = 3200 \text{ pts.}$$

And the required fraction is $\frac{411}{3200}$.

(91)

$$35 \text{ per. } 9 \text{ ft. } 2 \text{ in.} = 7040 \text{ in.}$$

$$1 \text{ fur.} = 7920 \text{ in.}$$

The required fraction is $\frac{7040}{7920} = \frac{88}{99} = \frac{8}{9}$.

(92)

$$7 \text{ hrs. } 12 \text{ min.} = 432 \text{ min.}$$

$$1 \text{ day} = 1440 \text{ min.}$$

Therefore the fraction is $\frac{432}{1440} = \frac{3}{10}$.

(93)

$$2 \text{ sq. yds. } 2 \text{ ft. } 120 \text{ in.} = 3000 \text{ in.}$$

$$3 \text{ sq. per. } 13\frac{1}{4} \text{ yds. } 1 \text{ ft. } 72 \text{ in.} = 135000 \text{ in.}$$

And the fraction is $\frac{3000}{135000} = \frac{1}{45}$.

(94)

$$7 \text{ oz. } 7 \text{ drs. } 2 \text{ scr. } 14 \text{ grs.} = 3834 \text{ grs.}$$

$$21 \text{ lbs.} = 120960 \text{ grs.}$$

The fraction is $\frac{3834}{120960} = \frac{426}{14400} = \frac{71}{2400}$.

(95)

$$9 \text{ min. } 48 \text{ sec.} = 588 \text{ sec.}$$

$$1 \text{ day} = 86400 \text{ sec.}$$

The required fraction is $\frac{588}{86400} = \frac{49}{7200}$.

(96)

$$16 \text{ bush. } 1 \text{ pk. } 1 \text{ pt.} = 1041 \text{ pts.}$$

$$69 \text{ bush.} = 4416 \text{ pts.}$$

Therefore the fraction is $\frac{1041}{4416} = \frac{347}{1472}$.

(97)

$$3 \text{ qrs. } 3\frac{1}{9} \text{ na.} = 15\frac{1}{9} = 1\frac{16}{9} \text{ na.}$$

$$1 \text{ Eng. ell} = 20 \text{ na.}$$

$$\text{And the fraction is } \frac{1\frac{16}{9}}{20} = \frac{1\frac{16}{9}}{20} = \frac{34}{45}.$$

(98)

$$13 \text{ dwt. } 7 \text{ grs.} = 319 \text{ grs.}$$

$$1 \text{ lb. Troy} = 5760 \text{ grs.}$$

$$\text{The required fraction is } \frac{319}{5760}.$$

(99)

$$4800 \text{ cub. ft.}$$

$$54 \text{ cords} = 6912 \text{ cub. ft.}$$

$$\text{Therefore the fraction is } \frac{4800}{6912} = \frac{400}{576} = \frac{50}{72} = \frac{25}{36}.$$

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(6)

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = 2\frac{1}{3}.$$

(7)

$$\frac{1}{12} + \frac{6}{12} + \frac{7}{12} + \frac{9}{12} + \frac{11}{12} + \frac{5}{12} = \frac{32}{12} = 3\frac{2}{3} = 3\frac{4}{6} = 3\frac{1}{2}.$$

(8)

$$4\frac{3}{7} + 11\frac{4}{7} + 16\frac{2}{7} + 21\frac{3}{7} + 19\frac{6}{7} = 4 + 11 + 16 + 21 + 19 + (\frac{3}{7} + \frac{4}{7} + \frac{2}{7} + \frac{3}{7} + \frac{6}{7}) = 71 + \frac{18}{7} = 73\frac{4}{7}.$$

(9)

$$16\frac{2}{3} + 11\frac{1}{3} + 18\frac{4}{3} + 17\frac{1}{3} + 112\frac{2}{3} = 16 + 11 + 18 + 17 + 112 + (\frac{2}{3} + \frac{1}{3} + \frac{4}{3} + \frac{1}{3} + \frac{2}{3}) = 174 + \frac{10}{3} = 174 + 3\frac{1}{3} = 177\frac{1}{3}.$$

(10)

$$4\frac{1}{4} + 1\frac{1}{4} + \frac{7}{4} = 4 + 1 + (\frac{1}{4} + \frac{1}{4} + \frac{7}{4}) = 5 + (\frac{9}{4} + \frac{14}{4} + \frac{8}{4}) = 5 + \frac{31}{4} = 6\frac{3}{4}.$$

(11)

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{6}{7} + \frac{7}{8} + \frac{8}{9}.$$

These fractions reduced to their least common denominator become $\frac{1260}{2520} + \frac{1680}{2520} + \frac{1890}{2520} + \frac{2016}{2520} + \frac{2100}{2520} + \frac{2160}{2520} + \frac{2205}{2520} + \frac{2240}{2520} = \frac{15551}{2520} = 6\frac{131}{2520}.$

(12)

$\frac{3}{4} + \frac{5}{6} + \frac{4}{5}$ when reduced to their least common denominator become $\frac{45}{60} + \frac{50}{60} + \frac{48}{60} = \frac{143}{60} = 2\frac{23}{60}$.

(13)

$$\frac{4}{5} + \frac{5}{6} + \frac{6}{7} + \frac{3}{8} + \frac{8}{11}.$$

These fractions when reduced to their least common denominator become $\frac{7392}{9240} + \frac{7700}{9240} + \frac{7920}{9240} + \frac{3465}{9240} + \frac{6720}{9240} = \frac{33197}{9240} = 3\frac{5477}{9240}$.

(14)

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}.$$

These fractions when reduced to their least common denominator become $\frac{210}{420} + \frac{140}{420} + \frac{105}{420} + \frac{84}{420} + \frac{70}{420} + \frac{60}{420} = \frac{669}{420} = 1\frac{83}{140}$.

(15)

$$16\frac{3}{4} + 47\frac{2}{3} + 21\frac{7}{8} + 1\frac{7}{8} + 19\frac{1}{2} = 16 + 47 + 21 + 19 + (\frac{3}{4} + \frac{2}{3} + \frac{7}{8} + \frac{7}{8} + \frac{1}{2}).$$

$$16 + 47 + 21 + 19 = 103.$$

$$\frac{3}{4} + \frac{2}{3} + \frac{7}{8} + \frac{7}{8} + \frac{1}{2} = \frac{486}{1782} + \frac{396}{1782} + \frac{918}{1782} + \frac{693}{1782} + \frac{891}{1782} = \frac{3384}{1782} = \frac{376}{198} = \frac{188}{99} = 1\frac{89}{99}.$$

$$103 + 1\frac{88}{99} = 104\frac{89}{99}.$$

(16)

$$17\frac{1}{2} + 43\frac{3}{4} + 168\frac{1}{4} + 207\frac{8}{21} + 506\frac{125}{26} = 17 + 43 + 168 + 207 + 506 + (\frac{1}{2} + \frac{3}{4} + \frac{1}{4} + \frac{8}{21} + \frac{125}{26}).$$

$$17 + 43 + 168 + 207 + 506 = 941.$$

$$\frac{1}{2} + \frac{3}{4} + \frac{1}{4} + \frac{8}{21} + \frac{125}{26} = \frac{63}{126} + \frac{54}{126} + \frac{56}{126} + \frac{148}{126} + \frac{125}{126} = \frac{346}{126} = \frac{173}{63} = 2\frac{47}{63}.$$

$$941 + 2\frac{47}{63} = 943\frac{47}{63}.$$

(17)

$$6\frac{3}{4} + 11\frac{1}{4} + \frac{9}{8} + 16\frac{7}{8} + \frac{1}{2} + \frac{5}{21} + 17\frac{1}{12} = 6 + 11 + 16 + 17 + (\frac{3}{4} + \frac{1}{4} + \frac{9}{8} + \frac{7}{8} + \frac{1}{2} + \frac{5}{21} + \frac{1}{12}).$$

$$6 + 11 + 16 + 17 = 50.$$

$$\frac{3}{4} + \frac{1}{4} + \frac{9}{8} + \frac{7}{8} + \frac{1}{2} + \frac{5}{21} + \frac{1}{12} = \frac{352}{336} + \frac{84}{336} + \frac{54}{336} + \frac{56}{336} + \frac{147}{336} + \frac{68}{336} + \frac{30}{336} = \frac{1201}{336} = 3\frac{193}{336}.$$

$$50 + 3\frac{193}{336} = 53\frac{193}{336}.$$

(18)

$$\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + 68\frac{1}{8} = 68 + (\frac{1}{8} + \frac{1}{4} + \frac{7}{8} + \frac{1}{8}).$$

$$\frac{1}{8} + \frac{2}{8} + \frac{7}{8} + \frac{1}{8} = \frac{36}{80} + \frac{120}{80} + \frac{140}{80} + \frac{15}{80} = \frac{341}{80} = 1\frac{61}{80}.$$

$$68 + 1\frac{61}{80} = 69\frac{61}{80}.$$

(19)

$$173\frac{3}{4} + 8\frac{5}{8} + 91\frac{1}{3} = 173 + 8 + 91 + (\frac{1}{4} + \frac{5}{8} + \frac{1}{3}).$$

$$173 + 8 + 91 = 272.$$

$$\frac{1}{4} + \frac{5}{8} + \frac{1}{3} = \frac{91}{364} + \frac{260}{364} + \frac{308}{364} = \frac{659}{364} = 1\frac{295}{364}.$$

$$272 + 1\frac{295}{364} = 273\frac{295}{364}.$$

(20)

$$1\frac{1}{6} + 2\frac{2}{3} + 3\frac{3}{4} + 4\frac{3}{8} = 1 + 2 + 3 + 4 + (\frac{1}{6} + \frac{2}{3} + \frac{3}{4} + \frac{3}{8}).$$

$$1 + 2 + 3 + 4 = 10.$$

$$\frac{1}{6} + \frac{2}{3} + \frac{3}{4} + \frac{3}{8} = \frac{6750}{7200} + \frac{6000}{7200} + \frac{6012}{7200} + \frac{6060}{7200} = \frac{27522}{7200} = \frac{1529}{400} = 3\frac{329}{400}.$$

$$10 + 3\frac{329}{400} = 13\frac{329}{400}.$$

(21)

$$\frac{1}{8} + \frac{3}{8} + \frac{4}{8} + \frac{5}{8} + \frac{7}{8} + \frac{3}{8} + \frac{1}{2} + \frac{5}{6} = \frac{6}{8} + \frac{12}{8} + \frac{4}{8} + \frac{10}{8} + \frac{21}{8} + \frac{40}{8} = \frac{149}{8} = 3\frac{5}{8}.$$

(22)

$$7 + 11\frac{1}{2} + 18 + 26\frac{3}{4} + 79\frac{1}{4} = 7 + 11 + 18 + 26 + 79 + (\frac{1}{2} + \frac{3}{4} + \frac{1}{4}).$$

$$7 + 11 + 18 + 26 + 79 = 141.$$

$$\frac{1}{2} + \frac{3}{4} + \frac{1}{4} = \frac{77}{154} + \frac{66}{154} + \frac{56}{154} = \frac{199}{154} = 1\frac{45}{154}.$$

$$141 + 1\frac{45}{154} = 142\frac{45}{154}.$$

(23)

$$\frac{1}{5} \text{ of } \frac{3}{4} \text{ of } \frac{2}{3} = \frac{18}{80} = \frac{3}{10}. \quad \frac{2}{3} + 7\frac{2}{3} + 3\frac{2}{3} = 10 + (\frac{2}{3} + \frac{2}{3} + \frac{2}{3}).$$

$$\frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{110}{165} + \frac{30}{165} + \frac{90}{165} = \frac{230}{165} = 1\frac{74}{165}.$$

$$10 + 1\frac{74}{165} = 11\frac{74}{165}.$$

(24)

$$\frac{4\frac{1}{2}}{\frac{7}{18}} = \frac{\frac{13}{3}}{\frac{7}{18}} = \frac{13 \times 18}{3 \times 7} = \frac{78}{7} = 11\frac{1}{7}.$$

$$\frac{1}{2} \text{ of } \frac{36}{11} \text{ of } \frac{4}{15} \text{ of } \frac{1}{4} = \frac{1 \times 36 \times 4 \times 11}{2 \times 11 \times 15 \times 4} = \frac{18}{15} = 1\frac{3}{5}.$$

$$\frac{20\frac{3}{4}}{\frac{7}{11}} = \frac{\frac{83}{4}}{\frac{7}{11}} = \frac{83 \times 11}{4 \times 7} = \frac{11}{4} = 2\frac{3}{4}.$$

$$11\frac{1}{7} + 1\frac{3}{5} + 2\frac{3}{4} = 11 + 1 + 2 + (\frac{1}{7} + \frac{3}{5} + \frac{3}{4}) = 14 + (\frac{1}{7} + \frac{3}{5} + \frac{3}{4}).$$

$$\frac{1}{7} + \frac{3}{5} + \frac{3}{4} = \frac{60}{420} + \frac{84}{420} + \frac{315}{420} = \frac{459}{420} = \frac{153}{140} = 1\frac{13}{140}.$$

$$14 + 1\frac{13}{140} = 15\frac{13}{140}.$$

(25)

$$3\frac{5}{8} + 11\frac{1}{6} + 14\frac{33}{48} = 3 + 11 + 14 + (\frac{5}{8} + \frac{1}{6} + \frac{33}{48}) = 28 + (\frac{5}{8} + \frac{1}{6} + \frac{33}{48}).$$

$$\frac{5}{8} + \frac{1}{6} + \frac{33}{48} = \frac{30}{48} + \frac{8}{48} + \frac{33}{48} = \frac{71}{48} = 1\frac{23}{48}. \quad 28 + 1\frac{23}{48} = 29\frac{23}{48}.$$

(26)

$$\frac{1}{2} \text{ of } \frac{3}{4} = \frac{3}{8}, \quad \frac{2}{3} \text{ of } \frac{6}{7} = \frac{4}{7}, \quad \frac{3}{8} \text{ of } \frac{7}{9} = \frac{7}{12}, \quad \frac{2}{9} \text{ of } \frac{27}{20} = \frac{3}{10}, \quad \frac{9}{2} \text{ of } \frac{1}{2} \text{ of } \frac{1}{2} \text{ of } \frac{1}{2} \text{ of } \frac{1}{2} = \frac{3}{80}.$$

$$\frac{3}{8} + \frac{4}{7} + \frac{7}{12} + \frac{3}{10} + \frac{3}{80} = \frac{630}{1680} + \frac{960}{1680} + \frac{784}{1680} + \frac{504}{1680} + \frac{63}{1680} = \frac{2941}{1680} = 1\frac{361}{1680}.$$

(27)

$$41\frac{1}{2} + 105\frac{2}{3} + 300\frac{3}{4} + 241\frac{3}{5} + 472\frac{1}{4} = 41 + 105 + 300 + 241 + 472 + (\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{3}{5} + \frac{1}{4}).$$

$$41 + 105 + 300 + 241 + 472 = 1159.$$

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{3}{5} + \frac{1}{4} = \frac{90}{180} + \frac{120}{180} + \frac{135}{180} + \frac{108}{180} + \frac{45}{180} = \frac{498}{180} = \frac{83}{30} = 2\frac{29}{30}.$$

$$1159 + 2\frac{29}{30} = 1161\frac{29}{30}.$$

(28)

$$92\frac{5}{14} + 37\frac{8}{19} + 7\frac{1}{6} = 92 + 37 + 7 + (\frac{5}{14} + \frac{8}{19} + \frac{1}{6}) = 136 + (\frac{5}{14} + \frac{8}{19} + \frac{1}{6}).$$

$$\frac{5}{14} + \frac{8}{19} + \frac{1}{6} = \frac{285}{798} + \frac{336}{798} + \frac{133}{798} = \frac{754}{798} = \frac{377}{399} = 1\frac{355}{399}.$$

$$136 + 1\frac{355}{399} = 137\frac{355}{399}.$$

(29)

$$\frac{10\frac{3}{5}}{2\frac{2}{5}} = \frac{5\frac{2}{5}}{1\frac{2}{5}} = \frac{53 \times 5}{5 \times 12} = \frac{53}{12} = 4\frac{5}{12}. \quad \frac{2}{3} \text{ of } \frac{7}{8} = \frac{7}{12}.$$

$$21\frac{1}{2} + 35\frac{1}{3} + 4\frac{5}{12} + \frac{7}{12} = 21 + 35 + 5 + (\frac{1}{2} + \frac{1}{3}) = 61\frac{5}{6}.$$

(30)

$$\frac{1}{4} \text{ of } \frac{1}{3} = \frac{1}{12} = 10\frac{1}{12}. \quad \frac{1}{5} \text{ of } \frac{3}{8} \text{ of } \frac{1}{8} = \frac{2541}{160} = 15\frac{141}{160}.$$

$$\frac{1}{16} = 6\frac{1}{16}. \quad \frac{1}{3} \text{ of } \frac{1}{16} \text{ of } \frac{1}{8} \text{ of } \frac{1}{7} = \frac{119}{63} = 1\frac{5}{6}.$$

$$10\frac{1}{12} + 6\frac{1}{16} + 15\frac{141}{160} + 1\frac{5}{6} = 10 + 6 + 15 + 1 + (\frac{1}{12} + \frac{1}{16} + \frac{141}{160} + \frac{5}{6}).$$

$$\frac{1}{12} + \frac{1}{16} + \frac{141}{160} + \frac{5}{6} = \frac{840}{10080} + \frac{9450}{10080} + \frac{8880}{10080} + \frac{8960}{10080} = \frac{28133}{10080} = 2\frac{1139}{1440}.$$

$$32 + 2\frac{1139}{1440} = 34\frac{1139}{1440}.$$

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(34)

oz. dr. scr. grs.

$$\frac{1}{11} \text{ of a lb.} = 4 \quad 2 \quad 2 \quad 14\frac{6}{11}$$

$$\frac{2}{7} \text{ of an oz.} = 3 \quad 1 \quad 5\frac{1}{7}$$

$$\frac{1}{11} \text{ of a dr.} = 1 \quad 1\frac{9}{11}$$

$$\frac{5}{6} \text{ of a scr.} = 16\frac{2}{3}$$

$$4 \quad 6 \quad 2 \quad 18\frac{172}{31}$$

(35)

qr. na. in.

$$\frac{3}{8} \text{ of a yard} = 2 \quad 1 \quad 1\frac{7}{80}$$

$$\frac{1}{4} \text{ of an Eng. ell} = 2 \quad 1\frac{1}{4}$$

$$\frac{6}{7} \text{ of a qr.} = 3 \quad 0\frac{2}{7}$$

$$3 \quad 3 \quad 1\frac{39}{40}$$

(36)

in.

$$\frac{1}{7} \text{ of a yd.} = 5\frac{1}{7}$$

$$\frac{1}{7} \text{ of a ft.} = 1\frac{1}{7}$$

$$\frac{1}{7} \text{ of an in.} = \frac{1}{7}$$

$$7$$

(37)

fur. per. yds. ft. in.

$$\frac{7}{11} \text{ of a mile} = 5 \quad 3 \quad 3 \quad 1 \quad 6$$

$$\frac{1}{13} \text{ of a fur.} = 12 \quad 1 \quad 2 \quad 0\frac{1}{13}$$

$$\frac{9}{22} \text{ of a yd.} = 1 \quad 2\frac{8}{11}$$

$$5 \quad 16 \quad 0 \quad 0 \quad 3\frac{23}{43}$$

(38)

day hrs. min.

$$\frac{1}{4} \text{ of a week} = 1 \quad 18 \quad 0$$

$$\frac{1}{3} \text{ of a day} = 8 \quad 0$$

$$\frac{1}{6} \text{ of an hour} = 12$$

$$2 \quad 2 \quad 12$$

(39)

s. d.

$$\frac{1}{7} \text{ of a £} = 2 \quad 10\frac{2}{7}$$

$$\frac{2}{9} \text{ of a s.} = 2\frac{2}{9}$$

$$\frac{5}{12} \text{ d.} = 1\frac{5}{12}$$

$$3 \quad 1\frac{31}{4}$$

(40)

	£	s.	d.
$\frac{5}{8}$ of 21s.	=	13	1 $\frac{1}{2}$
$\frac{5}{8}$ of 5s.	=	3	1 $\frac{1}{2}$
$\frac{5}{8}$ of £3 12s. 6d.	=	2	5 3 $\frac{3}{4}$
$\frac{7}{13}$ of a £	=	10	9 $\frac{3}{13}$
$\frac{1}{52}$ d.	=		$\frac{1}{52}$
		<hr/>	<hr/>
		3	12 4 $\frac{1}{2}$

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(7)

$$\frac{3}{4} - \frac{7}{20} = \frac{15}{20} - \frac{7}{20} = \frac{8}{20} = \frac{2}{5}.$$

(8)

$$\begin{aligned} \frac{7}{17} \text{ of } \frac{3}{14} \text{ of } \frac{96}{11} &= \frac{3 \times 48}{17 \times 11} = \frac{144}{187}. \quad \frac{905}{1496} + \frac{144}{187} = \frac{905}{1496} + \frac{1152}{1496} = \\ \frac{2057}{1496} &= 1\frac{561}{1496} = 1\frac{3}{8}. \\ \frac{8\frac{1}{2}}{6\frac{1}{11}} &= \frac{\frac{17}{2}}{\frac{70}{11}} = \frac{35 \times 11}{4 \times 70} = \frac{11}{4 \times 2} = \frac{11}{8} = 1\frac{3}{8}. \\ 1\frac{3}{8} - 1\frac{3}{8} &= 0. \end{aligned}$$

(9)

$$\begin{aligned} 982\frac{17}{87} - 29\frac{19}{20} &= 982\frac{340}{1740} - 29\frac{653}{1740} = 981 + 1\frac{340}{1740} - \\ 29\frac{653}{1740} &= 981\frac{2080}{1740} - 29\frac{653}{1740} = 952\frac{427}{1740}. \end{aligned}$$

(10)

$$\begin{aligned} 69\frac{1}{51} - 18\frac{86}{46} &= 69\frac{146}{446} - 18\frac{386}{446} = 68 + 1\frac{146}{446} - 18\frac{386}{446} = \\ 68\frac{7592}{446} - 18\frac{386}{446} &= 50\frac{3206}{446} = 50\frac{1603}{223}. \end{aligned}$$

(11)

$$100\frac{1}{2} - 9\frac{5}{8} = 100\frac{4}{8} - 9\frac{5}{8} = 99 + 1\frac{4}{8} - 9\frac{5}{8} = 99\frac{12}{8} - 9\frac{5}{8} = 90\frac{7}{8}.$$

(12)

$$\begin{aligned} \frac{1}{2} \text{ of } \frac{37}{4} &= \frac{37}{8} = 4\frac{5}{8}. \quad 6\frac{1}{4} - 4\frac{5}{8} = 6\frac{2}{4} - 4\frac{5}{8} = 5 + 1\frac{2}{8} - 4\frac{5}{8} = \\ 5\frac{10}{8} - 4\frac{5}{8} &= 1\frac{5}{8}. \end{aligned}$$

(13)

$$\begin{aligned} 611\frac{43}{91} - 610\frac{98}{99} &= 611\frac{8557}{38009} - 610\frac{37818}{38009} = 610 + 1\frac{8557}{38009} - \\ 610\frac{37818}{38009} &= 610\frac{8556}{38009} - 610\frac{37818}{38009} = \frac{8745}{38009}. \end{aligned}$$

(14)

$$\frac{5}{9} \text{ of } \frac{2}{1} = \frac{10}{9}. \quad \frac{1}{8} + \frac{1}{9} = \frac{9}{45} + \frac{5}{45} = \frac{14}{45}. \quad \frac{6}{7} \text{ of } \frac{14}{45} = \frac{4}{5}.$$

$$\frac{10}{9} - \frac{4}{5} = \frac{50}{45} - \frac{36}{45} = \frac{14}{45}.$$

(15)

(16)

oz. dr.	$24\frac{1}{4} - 21\frac{1}{21} = 24\frac{7}{168} - 21\frac{8}{168} =$
$\frac{2}{3}$ of a lb. = 10 10 $\frac{2}{3}$	$23 + 1\frac{7}{168} - 21\frac{8}{168} = 23\frac{75}{168} =$
$\frac{8}{9}$ of a dr. = 8	$21\frac{8}{168} = 21\frac{67}{168}.$
<hr/>	
10 9 $\frac{7}{9}$	

(17)

	fur.	per.	yds.	ft.	in.
$\frac{2}{9}$ of a mile =	1	31	0	1	10
$\frac{7}{11}$ of a fur. =		25	2	1	6
	<hr/>				
	1	5	3	1	10

(18)

$$\frac{2}{9} \text{ of } \frac{135}{4} = \frac{15}{2} = 7\frac{1}{2}. \quad \frac{1}{16} \text{ of } \frac{57}{2} = \frac{57}{32} = 1\frac{25}{32}.$$

$$7\frac{1}{2} - 1\frac{25}{32} = 7\frac{16}{32} - 1\frac{25}{32} = 6 + 1\frac{16}{32} - 1\frac{25}{32} = 6\frac{16}{32} - 1\frac{25}{32} = 5\frac{33}{32}.$$

(19)

$$\frac{1}{2} \text{ of } \frac{3}{7} \text{ of } \frac{2}{9} \text{ of } \frac{33}{4} \text{ of } \frac{\frac{52}{6}}{\frac{33}{6}} = \frac{1 \times 3 \times 2 \times 33 \times 62 \times 5}{2 \times 7 \times 9 \times 4 \times 33 \times 6} = \frac{155}{252}.$$

$$12\frac{319}{1764} + \frac{155}{252} = 12\frac{319}{1764} + \frac{1085}{1764} = 12\frac{1404}{1764} = 12\frac{351}{441} = 12\frac{39}{49}.$$

$$17\frac{9}{11} = \frac{196}{11} = \frac{196 \times 33}{11 \times 56} = \frac{21}{2} = 10\frac{1}{2}.$$

$$12\frac{39}{49} - 10\frac{1}{2} = 12\frac{78}{98} - 10\frac{49}{98} = 2\frac{29}{98}.$$

(20)

$$3\frac{1}{2} + 8\frac{1}{9} + 5\frac{1}{5} + 6\frac{1}{2} = 3 + 8 + 5 + 6 + (\frac{1}{2} + \frac{1}{9} + \frac{1}{5} + \frac{1}{2}) =$$

$$22 + (\frac{1}{2} + \frac{1}{9} + \frac{1}{5} + \frac{1}{2}).$$

$$\frac{1}{2} + \frac{1}{9} + \frac{1}{5} + \frac{1}{2} = \frac{15}{180} + \frac{20}{180} + \frac{36}{180} + \frac{90}{180} = \frac{161}{180}.$$

$$22 + \frac{161}{180} = 22\frac{161}{180}.$$

$$3\frac{1}{10} + 2\frac{5}{6} + 16\frac{1}{4} = 3 + 2 + 16 + (\frac{1}{10} + \frac{5}{6} + \frac{1}{4}) = 21 +$$

$$(\frac{1}{10} + \frac{5}{6} + \frac{1}{4}).$$

$$\frac{1}{10} + \frac{5}{6} + \frac{1}{4} = \frac{18}{180} + \frac{150}{180} + \frac{45}{180} = \frac{213}{180} = \frac{71}{60}.$$

$$21 + \frac{71}{60} = 21\frac{71}{60} = 22\frac{11}{60}.$$

$$22\frac{161}{180} - 22\frac{11}{60} = 22\frac{161}{180} - 22\frac{33}{180} = \frac{128}{180} = \frac{32}{45}.$$

(21)

		r.	per.	yds.	ft.	in.
$\frac{1}{11}$ of an acre	=	1	18	5	4	72
$\frac{1}{9}$ of a per.	=			13	4	
		<hr/>				
		1	17	22	2	108

(22)

$$16\frac{1}{7} - 9\frac{1}{19} = 16\frac{19}{133} - 9\frac{98}{133} = 15 + 1\frac{19}{133} - 9\frac{98}{133} = 15\frac{152}{133} - 9\frac{98}{133} = 6\frac{54}{133}.$$

$$169\frac{17}{100} - 83\frac{17}{25} = 169\frac{221}{1300} - 83\frac{850}{1300} = 168 + 1\frac{221}{1300} - 83\frac{850}{1300} = 168\frac{1521}{1300} - 83\frac{850}{1300} = 85\frac{671}{1300}.$$

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(6)

$$\frac{7}{12} \text{ of } \frac{5}{6} = \frac{7 \times 5}{12 \times 6} = \frac{35}{72}.$$

(7)

$$\frac{5}{8} \times \frac{4}{5} = \frac{1}{2}.$$

(8)

$$\frac{4}{15} \times \frac{5}{24} = \frac{1}{18}.$$

(9)

$$\frac{7}{8} \times \frac{5}{6} \times \frac{7}{16} = \frac{245}{768}.$$

$$\frac{14}{1} \times \frac{241}{16} \times \frac{32}{9} = \frac{14 \times 241 \times 2}{9} = \frac{6748}{9} = 749\frac{7}{9}.$$

(10)

(11)

$$\frac{3}{10} \times \frac{7}{4} \times \frac{9}{11} \times \frac{11}{12} = \frac{3 \times 7 \times 9}{2 \times 4 \times 4} = \frac{189}{32} = 5\frac{29}{32}.$$

(12)

$$\frac{4}{5} \times \frac{6}{11} \times \frac{9}{17} \times \frac{182}{200} \times \frac{5}{9} = \frac{3 \times 182}{11 \times 17 \times 25} = \frac{546}{4675}.$$

(13)

$$\frac{3}{6} \times \frac{11}{8} \times \frac{3}{6} \times \frac{3}{21} \times \frac{8}{5} \times \frac{5}{1} = \frac{3 \times 3 \times 3}{2} = \frac{27}{2} = 13\frac{1}{2}.$$

(14)

$$\frac{2}{6} \times \frac{8}{5} \times \frac{6}{11} \times \frac{4}{19} \times \frac{11}{209} = \frac{2 \times 6 \times 4}{5} = \frac{48}{5} = 9\frac{3}{5}.$$

(15)

$$\frac{18}{2} \times \frac{80}{7} \times \frac{180}{11} \times \frac{2}{18} \times \frac{7}{80} \times \frac{1}{90} = \frac{2}{11}.$$

(16)

$$\frac{4}{7} \times \frac{3}{11} \times \frac{9}{16} \times \frac{7}{1} \times \frac{8}{7} \times \frac{8}{18} \times \frac{13}{91} \times \frac{167}{24} \times \frac{3 \times 9 \times 167}{4} = \frac{1509}{4} = 1127\frac{1}{4}.$$

(17)

$$\frac{1}{7} \times \frac{8}{7} \times \frac{64}{9} \times \frac{19}{4} \times \frac{3}{27} \times \frac{9}{8} = \frac{1}{7 \times 8} \times \frac{8 \times 2}{7 \times 19} \times \frac{64 \times 9}{9 \times 8} \times \frac{19 \times 14}{101 \times 4} \times \frac{1}{9} \times \frac{9}{8} = \frac{1}{7 \times 101} = \frac{1}{707}.$$

(18)

$$\frac{1}{4} \times \frac{2}{8} \times \frac{2}{7} \times \frac{19}{1} = \frac{2 \times 2 \times 19}{7} = \frac{76}{7} = 10\frac{6}{7}.$$

(19)

$$\frac{9}{10} \times \frac{7}{1} \times \frac{11}{15} \times \frac{\overset{32}{\cancel{64}}}{11} = \frac{9 \times 7 \times 32}{5} = \frac{2016}{5} = 403\frac{1}{5}.$$

(20)

$$\frac{27}{4} \times \frac{\overset{7}{\cancel{8}}}{2} \times \frac{4}{5} \times \frac{4}{\cancel{7}} = \frac{27}{10} = 2\frac{7}{10}.$$

(21)

$$\frac{11}{3} \times \frac{\overset{13}{\cancel{39}}}{8} \times \frac{15}{1} = \frac{11 \times 13 \times 15}{8} = \frac{2145}{8} = 268\frac{1}{8}.$$

(22)

$$\frac{1}{3} \times \frac{\overset{7}{\cancel{35}}}{4} \times \frac{\overset{3}{\cancel{6}}}{19} \times \frac{19}{2} \times \frac{\overset{9}{\cancel{94}}}{11} \times \frac{\overset{2}{\cancel{16}}}{17} \times \frac{49}{8} \times \frac{\overset{2}{\cancel{4}}}{5} \times \frac{27}{31} \times \frac{\overset{31}{\cancel{31}}}{2} \times \frac{191}{\cancel{188}_2} =$$

$$\frac{7 \times 49 \times 27 \times 191}{2 \times 11 \times 17} = \frac{1768851}{374} = 4729\frac{205}{374}.$$

(23)

$$\frac{27}{37\frac{1}{2}} \times \frac{87\frac{2}{3}}{98\frac{1}{8}} \times \frac{\frac{7}{8}}{2\frac{1}{2}} \times \frac{81\frac{5}{11}}{128} = \frac{\frac{27}{1}}{\frac{182}{5}} \times \frac{\frac{185}{9}}{\frac{785}{8}} \times \frac{\frac{7}{8}}{\frac{7}{3}} \times \frac{\frac{896}{11}}{\frac{128}{1}} =$$

$$\frac{\overset{9}{\cancel{27}} \times 5}{\frac{189}{21}_3} \times \frac{\overset{7}{\cancel{875}} \times 8}{9 \times \overset{8}{\cancel{785}}} \times \frac{\overset{7}{\cancel{7}} \times 3}{8 \times 7} \times \frac{\overset{7}{\cancel{896}} \times 1}{11 \times \overset{7}{\cancel{128}}} = \frac{5}{3 \times 11} = \frac{5}{33}.$$

(24)

$$\text{\$ } \frac{\overset{5}{\cancel{55}}}{11} \times \frac{1}{7} \times \frac{3}{5} \times \frac{17}{19} = \frac{3 \times 17}{11 \times 7} = \text{\$ } \frac{17}{11}.$$

(25)

$$\frac{75\frac{3}{8}}{6\frac{1}{11}} \times \frac{\frac{3}{7} \text{ of } 8\frac{1}{4} \times \frac{1}{16} \text{ of } 28}{\frac{2}{11} \text{ of } 6\frac{3}{8} \times \frac{1}{17} \text{ of } 24} \times \frac{7\frac{1}{5}}{15} \times \frac{\frac{3}{4}}{\frac{5}{7}} \times 14\frac{3}{7} \times \frac{100}{121} \times$$

$$\frac{4}{5\frac{1}{3}} \times \frac{\frac{5}{7}}{9} =$$

$$\frac{60\frac{3}{8}}{\frac{67}{11}} \times \frac{\frac{3}{7} \times \frac{33}{4} \times \frac{1}{16} \times \frac{28}{1}}{\frac{2}{11} \times \frac{51}{8} \times \frac{1}{17} \times \frac{24}{1}} \times \frac{36}{\frac{15}{1}} \times \frac{\frac{3}{4}}{\frac{5}{7}} \times \frac{101}{7} \times \frac{100}{121} \times$$

$$\frac{\frac{4}{1}}{16\frac{2}{3}} \times \frac{\frac{5}{7}}{9} =$$

$$\frac{9}{608 \times 11} \times \frac{11 \times 4}{3 \times 33 \times 28 \times 11 \times 8 \times 17} \times \frac{9}{36} \times \frac{3 \times 7}{4 \times 5} \times$$

$$\frac{8 \times 67}{8 \times 67} \times \frac{7 \times 4 \times 15 \times 2 \times 51 \times 24}{5 \times 3 \times 6 \times 2}$$

$$\frac{101}{7} \times \frac{\frac{4}{20}}{121} \times \frac{\frac{2}{4 \times 3}}{16} \times \frac{5}{7 \times 9} =$$

$$\frac{11 \times 9 \times 101}{5 \times 7 \times 16} = \frac{9999}{560} = 17\frac{479}{560}.$$

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(28)

$$\frac{13}{36} \text{ of 4 days, 5 hours, } = \frac{180 \text{ d. } 23 \text{ h.}}{36} = 5 \text{ d. } 0 \text{ h. } 38 \text{ min. } 20 \text{ sec.}$$

(29)

$$\frac{13}{42} \text{ of } £29 = \frac{£29 \times 13}{42} \times \frac{£377}{42} = £8 \text{ } 19\text{s. } 6\frac{1}{2}\text{d.}$$

(30)

$$\frac{7}{9} \text{ of } 186 \text{ a. } 3 \text{ r.} = \frac{186 \text{ a. } 3 \text{ r.} \times 7}{9} = \frac{1307 \text{ a. } 1 \text{ r.}}{9} = 145 \text{ a. } 1 \text{ r.}$$

(31)

$$\frac{1}{4} \text{ of } \frac{2}{7} \text{ of } \frac{1}{30} \text{ of } \frac{1}{2} \text{ of } 24 \text{ h. } 30 \text{ m.} = \frac{1}{15} \text{ of } 24 \text{ h. } 30 \text{ m.} = 1 \text{ h. } 38 \text{ m.}$$

(32)

$$\begin{aligned} \frac{7}{9} \text{ of } \frac{4}{9} \text{ of } \frac{21}{40} \text{ of } \frac{7}{9} \text{ of } 33 \text{ bu. } 2 \text{ p. } 1 \text{ ga.} &= \frac{7}{90} \text{ of } 33 \text{ bu. } 2 \text{ p. } 1 \text{ ga.} = \\ 33 \text{ bu. } 2 \text{ p. } 1 \text{ ga.} \times 7 &= 235 \text{ b. } 1 \text{ p. } 1 \text{ g.} \\ \frac{\quad}{90} &= \frac{\quad}{90} = 2 \text{ b. } 2 \text{ p. } 0 \text{ g. } 3 \text{ q. } 1 \frac{1}{4} \text{ p.} \end{aligned}$$

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(5)

$$\frac{1}{2} \text{ of } \frac{3}{5} \div \frac{2}{3} \text{ of } \frac{35}{4} = \frac{1}{2} \times \frac{3}{5} \times \frac{4}{3} \times \frac{4}{35} = \frac{2 \times 4}{5 \times 35} = \frac{8}{175}.$$

(6)

$$\frac{15}{22} \div \frac{9}{5} \div \frac{5}{11} = \frac{15}{22} \times \frac{5}{9} \times \frac{11}{5} = \frac{5}{2 \times 3} = \frac{5}{6}.$$

(7)

$$82 \frac{1}{17} \div 26 \frac{5}{11} = \frac{155}{17} \times \frac{41}{119} = \frac{155 \times 41}{17 \times 119} = \frac{6355}{2023} = 3 \frac{286}{2023}.$$

(8)

$$2 \frac{1}{2} \div \frac{2}{3} + \frac{5}{8} = \frac{5}{2} \div \frac{11}{8} = \frac{5}{2} \times \frac{8}{11} = \frac{5 \times 4}{11} = \frac{20}{11} = 1 \frac{9}{11}.$$

(9)

$$1\frac{1}{2} \div \frac{1}{7} \text{ of } 2\frac{3}{4} \text{ of } 16 \text{ of } \frac{3^5}{4} \text{ of } \frac{1}{70} = \frac{7}{\frac{4}{2}} \times \frac{7}{1} \times \frac{4}{11} \times \frac{1}{\frac{16}{4}} \times \frac{4}{35} \times$$

$$\frac{\frac{2}{70}}{1} = \frac{7 \times 7}{2 \times 11} = \frac{49}{22} = 2\frac{5}{22}.$$

(10)

$$2\frac{1}{2} \div (\frac{5}{9} \div \frac{6}{32} \text{ of } 9) = \frac{7}{3} \div (\frac{5}{9} \text{ of } \frac{3^2}{6} \text{ of } \frac{1}{9}) = \frac{7}{3} \times \frac{9}{5} \times \frac{\frac{3}{6}}{\frac{32}{16}} \times$$

$$\frac{\frac{3}{9}}{1} = \frac{7 \times 9 \times 3 \times 3}{5 \times 16} = \frac{567}{80} = 7\frac{7}{80}.$$

(11)

$$48\frac{1}{2} \div \frac{2}{9} + \frac{2}{3} \text{ of } 6 = \frac{97}{2} \div \frac{2}{9} + \frac{2}{3} = \frac{97}{2} \div \frac{82}{36} = \frac{97}{2} \times \frac{\frac{18}{36}}{\frac{82}{89}} =$$

$$\frac{97 \times 18}{89} = \frac{1746}{89} = 19\frac{55}{89}.$$

(12)

$$6\frac{1}{2} \div \frac{2}{6} \text{ of } \frac{9}{10} + \frac{8}{17} = \frac{13}{2} \div \frac{27}{30} + \frac{8}{17} = \frac{13}{2} \div \frac{859}{330} = \frac{13}{2} \times$$

$$\frac{425}{859} = \frac{13 \times 425}{859} = \frac{5525}{859} = 6\frac{371}{859}.$$

(13)

$$\frac{2}{2} \times \frac{10}{3} \div \frac{9}{4} \times \frac{2^5}{4} = \frac{9}{2} \times \frac{\frac{5}{10}}{3} \times \frac{4}{9} \times \frac{4}{\frac{25}{5}} = \frac{4 \times 4}{3 \times 5} = \frac{16}{15} = 1\frac{1}{15}.$$

(14)

$$\frac{\frac{57}{9}}{\frac{35}{3}} \div \frac{\frac{7}{3}}{\frac{33}{8}} = \frac{67 \times 3}{9 \times 35} \div \frac{3 \times 8}{7 \times 33} = \frac{67 \times 3}{3 \times 35} \times \frac{7 \times 33}{3 \times 8} =$$

$$\frac{67 \times 11}{3 \times 5 \times 8} = \frac{737}{120} = 6\frac{17}{20}.$$

(15)

$$\frac{5}{9} \text{ of } \frac{80}{11} \div \frac{4}{11} \text{ of } \frac{132}{7} = \frac{5}{9} \times \frac{10}{11} \times \frac{11}{4} \times \frac{7}{132} = \frac{5 \times 10 \times 7}{9 \times 61} = \frac{350}{549}.$$

(16)

$$\frac{15}{8} \text{ of } \frac{10}{13} \text{ of } \frac{3}{4} \text{ of } \frac{7}{5} \div \frac{5}{6} \text{ of } \frac{3}{28} \text{ of } \frac{3}{4} \text{ of } \frac{5}{1} = \frac{45}{28} \times \frac{10}{13} \times \frac{3}{4} \times \frac{7}{5} \times \frac{6}{5} \times$$

$$\frac{28}{3} \times \frac{4}{3} \times \frac{1}{5} = \frac{3 \times 2 \times 7 \times 6}{13 \times 5} = \frac{252}{65} = 3\frac{57}{65}.$$

(17)

$$\frac{\frac{7}{4}}{\frac{3}{2}} \div \frac{\frac{7}{3}}{\frac{9}{4}} = \frac{7 \times 2}{4 \times 9} \div \frac{7 \times 4}{3 \times 9} = \frac{7 \times 2}{4 \times 9} \times \frac{3 \times 9}{7 \times 4} = \frac{3}{2 \times 4} = \frac{3}{8}.$$

(18)

$$\frac{\frac{3}{25}}{\frac{35}{2}} \div \frac{\frac{21}{35}}{\frac{3}{25}} = \frac{3}{25} \div \frac{21 \times 2}{5 \times 35} = \frac{3}{25} \times \frac{5 \times 35}{21 \times 2} = \frac{1}{2}.$$

(19)

$$\frac{113}{8} \times \frac{1}{9} \div \frac{3}{7} \times \frac{107}{13} \times \frac{13}{7} = \frac{113}{8} \times \frac{1}{9} \times \frac{7}{3} \times \frac{13}{107} \times$$

$$\frac{17}{2 \times 136} = \frac{113 \times 2 \times 17}{9 \times 3 \times 107} = \frac{3842}{2889} = 1\frac{953}{2889}.$$

(20)

$$\begin{aligned}
 & \frac{31}{2} \times \frac{\frac{2}{7}}{\frac{7}{9}} \times \frac{\frac{7}{1}}{\frac{3}{2}} \times \frac{\frac{7}{10}}{\frac{3}{1}} \div \frac{\frac{41}{9}}{\frac{7}{1}} \times \frac{\frac{3}{1}}{\frac{19}{4}} \times \frac{\frac{7}{8}}{\frac{7}{2}} \times \frac{\frac{11}{4}}{\frac{4}{7}} = \\
 & \frac{31}{2} \times \frac{2 \times 9}{7 \times 7} \times \frac{7 \times 3}{2} \times \frac{7}{10 \times 3} \div \frac{41}{9 \times 7} \times \frac{3 \times 4}{19} \times \frac{7 \times 2}{8 \times 7} \times \frac{11 \times 7}{4 \times 4} = \\
 & \frac{31}{2} \times \frac{\frac{3}{2 \times 7}}{\frac{7 \times 7}{2}} \times \frac{7 \times 3}{2} \times \frac{7}{10 \times 3} \times \frac{9 \times 7}{41} \times \frac{19}{3 \times 4} \times \frac{\frac{4}{7 \times 2}}{\frac{8 \times 7}{2}} \times \frac{\frac{2}{4 \times 4}}{11 \times 7} = \\
 & \frac{31 \times 3 \times 9 \times 19 \times 4}{5 \times 41 \times 11} = \frac{63612}{2255} = 28\frac{472}{2255}.
 \end{aligned}$$

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(22)

$$\begin{aligned}
 \frac{19}{\frac{5}{3}} &= \frac{19 \times 3}{11 \times 5} = \frac{57}{55}. \quad £8 \text{ 14s. } 6\frac{3}{4}\text{d.} \div \frac{57}{55} = £8 \text{ 14s. } 6\frac{3}{4}\text{d.} \times \frac{55}{57} = \\
 & \frac{£8 \text{ 14s. } 6\frac{3}{4}\text{d.} \times 55}{57} = £8 \text{ 8s. } 5\frac{1}{4}\text{d.}
 \end{aligned}$$

(23)

$$\begin{aligned}
 \frac{23}{8} \times \frac{20}{11} &= \frac{115}{22}. \quad 1 \text{ m. } 5 \text{ fur. } 91 \text{ yds. } 2 \text{ ft.} \div \frac{115}{22} = 1 \text{ m. } \\
 5 \text{ fur. } 91 \text{ yds. } 2 \text{ ft.} \times \frac{22}{115} &= \\
 1 \text{ m. } 5 \text{ fur. } 91 \text{ yds. } 2 \text{ ft.} \times 22 &= \\
 \frac{115}{115} &= 2 \text{ fur. } 124 \text{ yds. } 2 \text{ ft.}
 \end{aligned}$$

(24)

$$\begin{aligned}
 3 \text{ a. } 3 \text{ r. } 3 \text{ per.} \div \frac{2}{3} &= 3 \text{ a. } 3 \text{ r. } 3 \text{ p.} \times \frac{5}{3} = \frac{3 \text{ a. } 3 \text{ r. } 3 \text{ p.} \times 5}{3} = \\
 6 \text{ a. } 1 \text{ r. } 5 \text{ per.} &
 \end{aligned}$$

(25)

$$\begin{aligned}
 £7 \text{ 16s. } 2\text{d.} \div \frac{4}{9} &= £7 \text{ 16s. } 2\text{d.} \times \frac{9}{4} = \frac{£7 \text{ 16s. } 2\text{d.} \times 9}{4} = \\
 £17 \text{ 11s. } 4\frac{1}{2}\text{d.} &
 \end{aligned}$$

Page 178.

(28)

$$\begin{array}{r} 12\frac{1}{4} \\ \hline 7 \\ \hline 3\frac{1}{4} \\ \hline 9 \\ \hline 3 \\ \hline 7 \\ \hline 5 \\ \hline 4\frac{1}{2} \end{array} = \begin{array}{r} 4\frac{9}{4} \\ \hline 7 \\ \hline 1\frac{3}{4} \\ \hline 9 \\ \hline 3 \\ \hline 7 \\ \hline 5 \\ \hline 9\frac{1}{2} \end{array} = \begin{array}{r} 7 \\ \hline 4 \\ \hline 1\frac{3}{4} \\ \hline 2\frac{7}{4} \\ \hline 7 \\ \hline 5 \\ \hline 9\frac{1}{2} \end{array} = \begin{array}{r} 7 \\ \hline 13 \\ \hline 2\frac{43}{14} \\ \hline 5 \\ \hline 9\frac{1}{2} \end{array} = \begin{array}{r} 7 \\ \hline 13 \\ \hline 2\frac{43}{70} \\ \hline 9\frac{1}{2} \end{array} = \begin{array}{r} 7 \\ \hline 13 \\ \hline 2\frac{7}{35} \end{array} = \frac{7 \times 35}{13 \times 27}$$

$$\begin{array}{r} \frac{2}{3} \text{ of } 32 \\ \hline 7 \\ \hline 9\frac{1}{3} \\ \hline 3\frac{1}{2} \\ \hline \frac{7}{8} \end{array} = \begin{array}{r} 6\frac{4}{3} \\ \hline 7 \\ \hline 2\frac{8}{3} \\ \hline 7\frac{1}{2} \\ \hline \frac{7}{8} \end{array} = \begin{array}{r} 6\frac{4}{7} \\ \hline 8 \\ \hline 7\frac{1}{8} \end{array} = \begin{array}{r} 6\frac{4}{7} \\ \hline 2\frac{1}{2} \end{array} = 3.$$

$$\frac{7 \times 35}{13 \times 27} \times \frac{3}{1} = \frac{735}{351} = \frac{245}{117} = 2\frac{11}{117}.$$

(29)

$$\begin{array}{r} \frac{1}{7} \\ \hline 6\frac{1}{2} \\ \hline 9\frac{1}{2} \\ \hline 3 \\ \hline \frac{1}{2} \end{array} = \begin{array}{r} \frac{1}{7} \\ \hline 1\frac{3}{2} \\ \hline 1\frac{9}{2} \\ \hline 3 \\ \hline \frac{1}{2} \end{array} = \begin{array}{r} \frac{1}{21} \\ \hline 1\frac{3}{2} \\ \hline 1\frac{9}{2} \\ \hline 6 \\ \hline 1 \end{array} = \begin{array}{r} 2 \\ \hline 21 \times 13 \\ \hline 1\frac{9}{2} \end{array} = \frac{8}{7 \times 13 \times 19}$$

$$\frac{\frac{5}{6}}{\frac{7}{1}} = \frac{5}{42} \cdot \frac{8}{7 \times 13 \times 19} \div \frac{5}{42} = \frac{8}{7 \times 13 \times 19} \times \frac{42}{5} = 1\frac{8}{35}.$$

(30)

$$\frac{12\frac{1}{2}}{5\frac{1}{4}} = \frac{2\frac{5}{2}}{2\frac{1}{4}} = \frac{50}{21} = \frac{220}{63}.$$

$$\frac{3\frac{3}{4}}{5\frac{1}{2}} = \frac{1\frac{5}{4}}{1\frac{1}{2}} = \frac{15}{22} = \frac{220}{63}.$$

$$\frac{2\frac{1}{4}}{5} = \frac{\frac{9}{4}}{5} = \frac{9}{20}$$

$$\frac{4\frac{1}{2}}{3\frac{3}{4}} = \frac{\frac{9}{2}}{1\frac{5}{4}} = \frac{\frac{9}{2}}{1\frac{5}{4}} = \frac{10}{80} = \frac{8}{9}.$$

$$\frac{16\frac{2}{3}}{\frac{1}{2}} = \frac{50}{\frac{1}{2}} = \frac{100}{3}$$

$$\frac{220}{63} \div \frac{8}{9} = \frac{220}{63} \times \frac{9}{8} = \frac{55}{14} = 3\frac{3}{4}.$$

Page 180.

(1)

$$\frac{800}{2000} = \frac{2}{5}, \quad \frac{420}{2000} = \frac{21}{100}, \quad \frac{100}{2000} = \frac{1}{20}, \quad \frac{160}{2000} = \frac{2}{25}.$$

$$\frac{35}{2000} = \frac{7}{400}.$$

(2)

$$\frac{2}{5} \text{ of } \frac{5}{4} \text{ of } \frac{8}{5} \text{ of } \frac{1}{4} \text{ of } \frac{5}{2} \text{ of } \frac{4}{5} = \frac{2}{5} \times \frac{5}{4} \times \frac{8}{5} \times \frac{1}{4} \times \frac{5}{2} \times \frac{4}{5} = \frac{2}{5}.$$

(3)

$$6\frac{7}{8} \times 65\frac{3}{4} \text{ cts.} = \frac{55}{8} \times \frac{263}{4} \text{ cts.} = \frac{14465}{32} \text{ cts.} = \$4.52\frac{1}{32}.$$

(4)

$$\frac{3}{8} + \frac{1}{17} = \frac{51}{136} + \frac{8}{136} = \frac{59}{136}.$$

(5)

$$\frac{1}{3} + \frac{1}{10} + \frac{1}{8} + \frac{1}{6} = \frac{40}{120} + \frac{12}{120} + \frac{15}{120} + \frac{20}{120} = \frac{87}{120} = \frac{29}{40}.$$

$$1 \text{ or } \frac{40}{40} - \frac{29}{40} = \frac{11}{40}.$$

(6)

$$\frac{5\frac{1}{2} - 2\frac{1}{2}}{3\frac{1}{2} + \frac{9}{20}} \text{ of } \frac{4\frac{1}{2} + 5\frac{1}{20}}{4\frac{1}{20}} \text{ of } \frac{2\frac{3}{4} + 1\frac{1}{2}}{7\frac{1}{24} - 2\frac{1}{4}} = \frac{5\frac{3}{40} - 2\frac{5}{40}}{3\frac{1}{20} + \frac{9}{20}} \text{ of } \frac{4\frac{2}{5} + 5\frac{3}{5}}{\frac{8}{5}} \text{ of } \frac{2\frac{9}{15} + 1\frac{1}{15}}{7\frac{1}{24} - 2\frac{6}{24}} = \frac{3\frac{2}{40}}{4\frac{4}{40}} \text{ of } \frac{10\frac{1}{30}}{\frac{8}{20}} \text{ of } \frac{4\frac{1}{15}}{5\frac{1}{24}} = \frac{1\frac{4}{40}}{\frac{8}{20}} \text{ of } \frac{5\frac{1}{30}}{\frac{8}{20}} \text{ of } \frac{6\frac{1}{15}}{\frac{1}{24}} =$$

$$\frac{7}{8} \times \frac{2 \times 57}{5 \times \frac{9}{3}} \times \frac{8 \times 64}{5 \times \frac{133}{19}} = \frac{2 \times 64}{5 \times 3 \times 5} = \frac{128}{75} = 1\frac{53}{75}$$

(7)

$$1670\frac{7}{13} \times 12\frac{1}{4} \text{ cts.} = 2\frac{17}{13} \times 5\frac{1}{4} \text{ cts.} = 110\frac{75}{52} \text{ cts.} = \$212.99\frac{1}{2}.$$

(8)

$\frac{3}{4}$ of the longer = $\frac{3}{4}$ of the shorter; therefore $\frac{1}{4}$ of the longer = $\frac{1}{4}$ of $\frac{3}{4}$ = $\frac{3}{16}$ of the shorter.

Hence the longer = $\frac{3}{4} \times 3 = \frac{9}{4}$ of the shorter.

The whole tree = longer + shorter = $\frac{9}{4} + \frac{8}{8}$ of shorter = $1\frac{1}{4}$ of the shorter.

If 136 ft. = $\frac{1}{8}$ of the shorter, $\frac{1}{17}$ of 136 = 8 = $\frac{1}{8}$ of the shorter.

Hence shorter = $8 \times 8 = 64$ ft.; and longer = $136 - 64 = 72$ ft.

(9)

$$97\frac{1}{2} + 127\frac{2}{5} + 500\frac{3}{8} + 333\frac{1}{2} = 97\frac{30}{120} + 127\frac{48}{120} + 500\frac{15}{120} + 333\frac{40}{120} = 1057\frac{163}{120} = 1058\frac{43}{120}.$$

$$\$1000 + \$1375\frac{1}{2} + \$6831 + \$4013\frac{3}{8} = \$1000 + \$1375\frac{8}{16} + \$6831 + \$4013\frac{3}{8} = \$13219\frac{1}{8} = \$13219.68\frac{1}{2}.$$

(10)

$$12\frac{5}{6} + \frac{8}{15} = 13\frac{11}{30}. \quad 8\frac{3}{4} + 1\frac{1}{10} = 9\frac{17}{20}. \quad 13\frac{11}{30} - 9\frac{17}{20} = 3\frac{31}{60} = 2\frac{11}{60}.$$

$$7\frac{5}{12} - 6\frac{1}{2} = \frac{1}{2}. \quad 2\frac{11}{60} \times \frac{9}{2} \times \frac{1}{2} = 2\frac{321}{160} = 14\frac{81}{160}.$$

$$\frac{2}{3} \div \frac{12}{7} = \frac{2}{3} \times \frac{7}{12} = \frac{7}{18}. \quad \frac{5}{8} \div \frac{35}{11} = \frac{5}{8} \times \frac{11}{35} = \frac{11}{56}. \quad \frac{7}{18} - \frac{11}{56} = \frac{97}{504}.$$

(11)

$$19\frac{7}{8} \times \$6\frac{3}{4} = \frac{159}{8} \times \$\frac{27}{4} = \$\frac{4293}{32} = \$134.15\frac{5}{8}.$$

(12)

$$376\frac{1}{8} \times \$75\frac{3}{8} = \frac{6779}{18} \times \$\frac{603}{8} = \frac{4087737}{144} = \$28387.06\frac{1}{4}.$$

(13)

$$147\frac{2}{3} + 320\frac{1}{5} = 147\frac{10}{15} + 320\frac{3}{15} = 467\frac{13}{15}. \quad 467\frac{13}{15} - 156\frac{1}{4} =$$

$$467\frac{52}{60} - 156\frac{15}{60} = 311\frac{37}{60}.$$

(14)

$$\frac{7 \text{ (} 1\frac{1}{2} \text{ of } \frac{3}{4} \text{)}}{\frac{1}{6} \left(\frac{3}{3\frac{1}{2}} \text{ of } 7 \right)} \div 7\frac{7}{8} = \frac{\frac{7}{1} \times \frac{3}{2} \times \frac{4}{4}}{\frac{1}{6} \times \frac{3}{2} \times \frac{7}{2}} \div \frac{63}{8} = \frac{\frac{7 \times 3 \times 3}{1 \times 2 \times 4}}{\frac{1}{6} \times \frac{6}{7} \times \frac{7}{2}} \times \frac{8}{63} =$$

$$\frac{7 \times 3 \times 3}{1 \times 2 \times 4} \times \frac{\frac{2}{8}}{\frac{63}{8}} = 1. \quad \frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}}{\frac{1}{2\frac{1}{2}} + \frac{1}{3\frac{1}{4}} + \frac{1}{4\frac{1}{2}}} = \frac{\frac{6}{12} + \frac{4}{12} + \frac{3}{12}}{\frac{1}{\frac{5}{2}} + \frac{1}{\frac{13}{4}} + \frac{1}{\frac{9}{2}}} =$$

$$\frac{\frac{13}{12}}{\frac{2}{5} + \frac{4}{13} + \frac{2}{9}} = \frac{\frac{13}{12}}{\frac{234}{585} + \frac{180}{585} + \frac{130}{585}} = \frac{\frac{13}{12}}{\frac{544}{585}} = \frac{2535}{2176} = 1\frac{359}{176}.$$

(15)

$$17\frac{1}{7} \div 7\frac{1}{7} = \frac{123}{7} \div \frac{53}{7} = \frac{123}{53} \times \frac{7}{53} = \frac{123}{53} = 2\frac{17}{53}.$$

(16)

$$3\frac{3}{4} + 4\frac{1}{2} + 4\frac{1}{5} = 3\frac{40}{60} + 4\frac{12}{60} + 4\frac{12}{60} = 13\frac{64}{60} = 7\frac{23}{60}.$$

$$7\frac{6}{7} - 5\frac{5}{6} = 7\frac{36}{42} - 5\frac{35}{42} = 2\frac{1}{42} = \frac{85}{42}.$$

$$94\frac{1}{8} + 93\frac{1}{9} = 94\frac{9}{72} + 93\frac{8}{72} = 187\frac{17}{72} = \frac{13481}{72}.$$

$$\frac{793}{60} \times \frac{85}{42} \div \frac{13481}{72} = \frac{793}{60} \times \frac{17}{85} \times \frac{72}{13481} = \frac{1}{7}.$$

(17)

$$2\frac{3}{4} + \frac{1}{5} + 4 = 2\frac{10}{20} + \frac{4}{20} + 4 = 7\frac{14}{20} = \frac{113}{10}.$$

$$2 \div \frac{113}{10} = 2 \times \frac{10}{113} = \frac{20}{113}. \quad 1\frac{2}{3} - \frac{7}{9} = \frac{15}{9} - \frac{7}{9} = \frac{8}{9}.$$

$$\frac{15}{56} + \frac{8}{9} = \frac{583}{504}. \quad 5\frac{1}{8} - 4\frac{1}{2} = 4\frac{12}{24} - 4\frac{12}{24} = \frac{7}{24}.$$

$$\frac{583}{504} = \frac{7}{10} = \frac{583}{720}.$$

(18)

$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}. \quad 1\frac{1}{2} + 2\frac{3}{4} = 4\frac{1}{2} = \frac{9}{2}. \quad 2\frac{1}{4} - 1\frac{1}{2} = \frac{1}{4} = \frac{1}{4}.$$

$$3\frac{1}{10} - \frac{3}{7} = 2\frac{47}{70} = \frac{187}{70}.$$

$$\frac{5}{6} \times \frac{49}{12} \times \frac{2}{7} \times \frac{187}{70} = \frac{187}{3 \times 12} = \frac{187}{36} = 5\frac{7}{36}.$$

$$1\frac{3}{4} \div 2\frac{1}{2} = \frac{7}{4} \times \frac{2}{5} = \frac{7}{10}. \quad 5\frac{1}{2} \div 3\frac{1}{8} = \frac{11}{2} \times \frac{8}{25} = \frac{44}{25} = 1\frac{19}{25}.$$

$$\frac{7}{10} + 1\frac{19}{25} = 2\frac{23}{50}.$$

(19)

$$1 - (\frac{1}{3} + \frac{1}{4}) = \frac{1}{6}. \quad \frac{2}{3} \text{ of } \frac{1}{2} = \frac{1}{3}. \quad \frac{1}{3} - \frac{1}{5} = \frac{2}{15}. \quad \frac{1}{6} + \frac{2}{15} = \frac{9}{30}.$$

$$\frac{1}{3} - \frac{9}{30} = \frac{1}{30}. \quad \frac{1}{30} \text{ of } \$40000 = \$1333.33\frac{1}{3}.$$

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(13)

$$\begin{array}{r} 1=2)1 \\ \hline .5 \end{array} \quad \begin{array}{r} 3=8)3 \\ \hline .375 \end{array}$$

(14)

$$\begin{array}{r} 9=25)9 \\ \hline .36=\frac{36}{100} \end{array} \quad \begin{array}{r} 4=5)1 \\ \hline .25=\frac{25}{100} \end{array}$$

(15)

75)73 (.9733+

67.5

5.50

5.25

.250

.225

250

225

25

123)574(4.666+

492

82.0

73.8

8.20

7.38

.820

.738

82

34)15 (.44117+

13.6

1.40

1.36

40

34

60

34

260

238

22

(16)

7)6

.857142

12)5

.4166+

9)4

.44444+

(17)

112)17 (.15178571428+

11.2

5.80

5.60

.200

.112

880

784

960

896

640

560

800

800

784

160

112

480

448

320

224

960

896

64

1296)718 (.554012+

648.0

70.00

64.80

5.200

5.184

1600

1296

3040

2592

448

Page 184.

(20)

(21)

(22)

12)1·0 in.

 3)2·083333 ft.

 5½)3·694444 yd.
 2 2

 11) 7·388888

 40)·671717 per.

 ·01679+ fur.

12)17·0 grs.

 2)1·41666666

 20)3·70833333 dwt.

 12)·18541666 oz.

 ·01545138+ lb.

20)7·0 grs.

 3)2·35 scr.

 8)·7833333 dr.

 12)·0979166 oz.

 ·0081597+ lb.

(23)

(24)

(26)

12)9·0 in.

 3)2·75 ft.

 5½)2·91666
 2 2

 11)5·83333

 40)35·53030 per.

4)2·0 na.

 4)3·5 qr.

 ·875 yd.

60)21·0 sec.

 60)55·35 min.

 12)12·9225 hr.

 2)1·076875

 ·5384375 day.

(25)

8)5·88825 fur.

 ·73603+ mile.

13s. 4d. = 160d.
 5s. = 60d.
 $\frac{60}{160} = \frac{3}{8} = \cdot375$

(27)

(28)

$\frac{7}{8}$ of $\frac{1}{2}$ of 6½d. = $\frac{7}{8}$ d. and £½ = 80d.
 $\frac{7}{8}$ d. = $\frac{1}{80}$ of $\frac{7}{8}$ of £½ = $\frac{7}{2240}$ of £½.
 $27 \div 2240 = 0\cdot012053$.

$\frac{7}{8}$ of $\frac{1}{2}$ of 1 mil. = 1267½ in.
 3½)12672
 2 2

 7)25344

 3620·571428+

(29)

$$\frac{1}{3} \text{ of } \frac{2}{5} \text{ of } \frac{1}{4} \text{ lbs.} = \frac{1}{30} \text{ lb.} = 110\frac{1}{10} \text{ drs.} = \frac{1664}{15} \text{ drs.}$$

$$\frac{3}{4} \text{ of an oz.} = 12 \text{ drs.} = \frac{1664}{15} \div 12 = \frac{1664}{180}$$

$$180)1664(9\cdot2444+$$

1620

440

360

800

720

800

720

800

720

80

(30)

2)1·0 pts.

4)1·5 qt.

2)1·375 gal.

4)3·6875 pk.

·921875 bush.

Page 186.

(33)

·3945

24

15780

7890

9·4680 hrs.

60

28·0800 min.

60

4·8000 sec.

(34)

·3965

8

3·1720 fur.

40

6·8800 per.

5½

44000

4400

4·8400 yds.

3

2·5200 ft.

12

6·2400 in.

(35)

·309153

20

6·183060 dwt.

24

732240

366120

4·393440 grs.

(36)

(37)

$$22.75 = 22\frac{75}{100} = 22\frac{3}{4}. \quad 7 \text{ b. } 1 \text{ p. } 1 \text{ g. } 1 \text{ qts.} = 237 \text{ qts.}$$

$$£2 \text{ 2s. } 6\text{d.} \times 22\frac{3}{4} = £48 \text{ 6s. } 10\frac{1}{2}\text{d.} \quad 11.17825 \times 237 = 2649.24525 \text{ qt.} =$$

$$82 \text{ b. } 3 \text{ p. } 0 \text{ g. } 1 \text{ q. } 0.4905 \text{ pts.}$$

(38)

(39)

(40)

$\begin{array}{r} .2057 \\ 12 \\ \hline 2.4684 \text{ oz.} \\ 20 \\ \hline 9.3680 \text{ dwt.} \\ 24 \\ \hline 14720 \\ 7360 \\ \hline 8.8320 \text{ grs.} \end{array}$	$\begin{array}{l} 1 \text{ f. } 36 \text{ p. } 2 \text{ y. } 5 \text{ in.} = 15125 \text{ in.} \\ 15125 \times .176 = 2662 \text{ in.} = \\ 13 \text{ per. } 2 \text{ yds. } 1 \text{ ft. } 4 \text{ in.} \end{array}$	$\begin{array}{r} .625 \\ 3 \\ \hline 1.875 \text{ mil.} \\ 8 \\ \hline 7.000 \text{ fur.} \end{array}$
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(41)

(42)

(43)

$\begin{array}{r} .015625 \\ 4 \\ \hline .062500 \text{ pk.} \\ 2 \\ \hline .125000 \text{ gal.} \\ 4 \\ \hline .500000 \text{ qt.} \\ 2 \\ \hline 1.000000 \text{ pt.} \end{array}$	$\begin{array}{r} .9378 \\ 4 \\ \hline 3.7512 \text{ r.} \\ 40 \\ \hline 30.0480 \text{ per.} \\ 30\frac{1}{4} \\ \hline 14400 \\ 120 \\ \hline 1.4520 \text{ yd.} \\ 9 \\ \hline 4.0680 \text{ ft.} \\ 144 \\ \hline 2720 \\ 2720 \\ 680 \\ \hline 9.7920 \text{ in.} = 9\frac{89}{125} \text{ in.} \end{array}$	$\begin{array}{l} 1 \text{ sq. yd. } 3 \text{ ft. } 72 \text{ in.} = 1800 \text{ in.} \\ .2775 \times 1800 = 499.5 \text{ in.} = \\ 3 \text{ ft. } 67\frac{1}{2} \text{ in.} \end{array}$
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(54)

$$\begin{aligned}\cdot\dot{8} &= \frac{8}{9}. \\ \cdot\ddot{05} &= \frac{5}{99}. \\ \cdot\dot{342} &= \frac{342}{999} = \frac{38}{111}. \\ \cdot\dot{7004} &= \frac{7004}{9999}. \\ \cdot\dot{002003} &= \frac{2003}{999999}.\end{aligned}$$

(55)

$$\begin{aligned}\cdot\dot{19} &= \frac{19}{99}. \\ \cdot\dot{1067} &= \frac{1067}{9999} = \frac{97}{909}. \\ \cdot\dot{11115} &= \frac{11115}{99999} = \frac{1235}{11111}. \\ \cdot\dot{704103} &= \frac{704103}{999999} = \frac{334701}{333333}.\end{aligned}$$

(56)

$$\begin{aligned}\cdot\dot{102} &= \frac{102}{999} = \frac{34}{333}. \\ \cdot\ddot{0013} &= \frac{13}{9999}. \\ \cdot\dot{00007103} &= \frac{7103}{99999999}. \\ \cdot\dot{01020304} &= \frac{1020304}{99999999}. \\ \cdot\dot{987654321} &= \frac{987654321}{999999999} = \frac{109739369}{111111111}.\end{aligned}$$

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(58)

$\begin{array}{r} \cdot\ddot{8325} \\ 83 \\ \hline \end{array}$	$\begin{array}{r} \cdot\dot{147658} \\ 147 \\ \hline \end{array}$	$\begin{array}{r} \cdot\dot{4320075} \\ 432 \\ \hline \end{array}$
$\frac{8212}{9900} = \frac{1121}{4950}$	$\frac{147511}{999000}$	$\frac{4319643}{9999000} = \frac{1439881}{3333000}.$

(59)

$\begin{array}{r} 875\cdot\ddot{4965} \\ 49 \\ \hline \end{array}$	$\begin{array}{r} 301\cdot\dot{82756} \\ 82 \\ \hline \end{array}$
$875\frac{4916}{9900} = 875\frac{1229}{2475}$	$301\frac{82674}{999000} = 301\frac{9186}{11100} = 301\frac{531}{1830}.$

(60)

$$\begin{array}{l} \cdot 083 \\ 8 \\ \hline \frac{75}{900} = \frac{1}{12} \end{array} \quad \begin{array}{l} \cdot \cdot \\ \frac{714285}{9999990} = \frac{79365}{1111110} = \frac{7215}{101010} = \frac{1443}{20202} = \\ \frac{481}{6734} = \frac{1}{14} \end{array}$$

$$\begin{array}{l} \cdot \cdot \cdot \\ \cdot 123456 \\ 123 \\ \hline \frac{1233333}{999000} = \frac{41111}{333000} \end{array}$$

(61)

$$\begin{array}{l} \cdot \\ \cdot 7034 \\ 703 \\ \hline \frac{6331}{9000} \end{array} \quad \begin{array}{l} \cdot \cdot \\ \cdot 96432 \\ 96 \\ \hline \frac{96336}{99900} \end{array} = \frac{10704}{11100} = \frac{3568}{3700} = \frac{892}{925}$$

$$\begin{array}{l} \cdot \cdot \\ \cdot 00207 \\ 2 \\ \hline \frac{205}{99000} = \frac{41}{19800} \end{array} \quad \begin{array}{l} \cdot \cdot \cdot \\ \cdot 143271 \\ 1432 \\ \hline \frac{141839}{990000} \end{array}$$

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(63)

Dissimilar.		Similar.		Similar and Coterminous.
$\cdot 9$	=	$\cdot 99999$	=	$\cdot 9999999999$
$6 \cdot 327$	=	$6 \cdot 327272$	=	$6 \cdot 3272727272$
$19 \cdot 43$	=	$19 \cdot 43000$	=	$19 \cdot 4300000000$
$27 \cdot 0278$	=	$27 \cdot 027878$	=	$27 \cdot 0278787878$
$\cdot 0347123$	=	$\cdot 0347123$	=	$\cdot 0347123123$
				2 carried.
				<hr/>
		Sum,	=	$53 \cdot 8198638274$

(64)		
Dissimilar.	Similar.	Similar and Coterminous.
$7.\ddot{4}2\ddot{7} =$	$7.427\ddot{2}7 =$	$7.427\ddot{2}727272727\ddot{2}7$
$9.1234 =$	$9.123423 =$	$9.123423423423423\ddot{4}$
$17.2987643 =$	$17.2987643 =$	$17.298764376437643\ddot{3}$
$18.\ddot{6}7 =$	$18.676\ddot{7}6 =$	$18.676\ddot{7}67676767676\ddot{6}$
2 carried.		
Sum, = 52.526228203901471		

(65)		
Dissimilar.	Similar.	Similar and Coterminous.
$4.\ddot{9}5 =$	$4.959\ddot{5}95 =$	$4.9595959595\ddot{5}$
$7.164 =$	$7.1641641 =$	$7.1641641641\ddot{1}$
$4.7123 =$	$4.7123123 =$	$4.7123123123\ddot{3}$
$.97317 =$	$.97317 =$	$.9731777777\ddot{7}$
2 carried.		
Sum, = 17.8092502138		

(66)		
Dissimilar.	Similar.	Similar and Coterminous.
$1.5 =$	$1.5000 =$	$1.500000000\ddot{0}$
$99.083 =$	$99.0830 =$	$99.083000000\ddot{0}$
$.162 =$	$.162162 =$	$.162162162\ddot{2}$
$.814 =$	$.814814 =$	$.814814814\ddot{4}$
$2.93 =$	$2.93939 =$	$2.939393939\ddot{9}$
$3.769230 =$	$3.769230769 =$	$3.769230769\ddot{9}$
$97.26 =$	$97.2666 =$	$97.266666666\ddot{6}$
$134.09 =$	$134.09090 =$	$134.090909090\ddot{0}$
3 carried.		
Sum, = 339.626177443		

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(68)

Dissimilar.		Similar.		Similar and Coterminous.
729·3427̇	=	729·342742̇	=	729·342742̇
93·126	=	93·1260̇	=	93·126000̇
				<hr/>
				636·216742̇

(69)

Dissimilar.		Similar.		Similar and Coterminous.
1·437291̇	=	1·43729137̇	=	1·4372913729137̇
·00713̇	=	·00713̇	=	·0071313131313̇
				<hr/>
				1·4301600597824̇

(70)

Dissimilar.		Similar.		Similar and Coterminous.
1·12754̇	=	1·12754̇	=	1·12754754754754̇
·47384̇	=	·473847̇	=	·47384738473847̇
				<hr/>
				·65370016280907̇

(71)

Dissimilar.		Similar.		Similar and Coterminous.
42·18763̇	=	42·1876333̇	=	42·1876333333̇
17·0000008432̇	=	17·0000008432̇	=	17·0000008432̇
				<hr/>
				25·1876324900̇

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(74)

$$2.\dot{9} = 2\frac{2}{3} = 3. \quad 7.25 \times 3 = 21.75.$$

(75)

$$\cdot 29\dot{7} = \frac{297}{999} = \frac{11}{37} \text{ and } 7.72 = 7\frac{72}{100} = 7\frac{18}{25} = \frac{193}{25}.$$

$$\frac{11}{37} \times \frac{193}{25} = \frac{2123}{925} = 2.29513.$$

(76)

$$\cdot 8\dot{1}8 = \frac{810}{990} = \frac{9}{11} \text{ and } \cdot 77 = \frac{77}{100}. \quad \frac{9}{11} \times \frac{77}{100} = \frac{63}{100} = .63.$$

(77)

$$1.\dot{7}35 = \frac{1738}{990} = \frac{1364}{495} = \frac{852}{495} \text{ and } \cdot 47053 = \frac{42348}{90000} = \frac{3528}{7500}.$$

$$\frac{852}{495} \times \frac{3528}{7500} = \frac{3031411}{37125} = .81654168350.$$

(78)

$$4.\dot{7}22 = \frac{4650}{990} = 4\frac{13}{18} = \frac{85}{18} \text{ and } \cdot 198 = \frac{198}{999} = \frac{22}{111}.$$

$$\frac{85}{18} \times \frac{22}{111} = \frac{935}{999} = .935.$$

(80)

$$\cdot 082 = \frac{82}{999} \text{ and } \cdot 123 = \frac{123}{999} = \frac{41}{333}.$$

$$\frac{82}{999} \div \frac{41}{333} = \frac{82}{999} \times \frac{333}{41} = \frac{3}{3} = .6.$$

(81)

$$389.\dot{1}85 = 389\frac{185}{999} = \frac{388796}{999} \text{ and } 15.\dot{7} = 15\frac{7}{9} = \frac{142}{9}.$$

$$\frac{388796}{999} \div \frac{142}{9} = \frac{388796}{999} \times \frac{9}{142} = \frac{2738}{111} = 24.\dot{6}.$$

(82)

$$\cdot 81654168350 = \frac{816540886696}{999999900000} = \frac{10206760837}{12499987500}.$$

$$\cdot 47053 = \frac{42348}{90000} = \frac{10587}{22500}.$$

$$\frac{10206760837}{12499987500} \div \frac{10587}{22500} = \frac{10206760837}{12499987500} \times \frac{22500}{10587} = \frac{10206760837}{6881660785} =$$

1.735.

(83)

$$\cdot\ddot{45} = \frac{45}{99} = \frac{5}{11} \text{ and } \cdot\dot{118881} = \frac{118881}{999999} = \frac{17}{143}.$$

$$\frac{5}{11} \div \frac{17}{143} = \frac{5}{11} \times \frac{143}{17} = \frac{65}{17} = 3.\dot{8}235294117647058.$$

MISCELLANEOUS EXERCISES.

(84)

$$\frac{1}{2} \text{ of } \frac{3}{7} \text{ of } \frac{4}{15} \text{ of } 14 = \frac{1}{2} \times \frac{3}{7} \times \frac{4}{15} \times 14 = \frac{4}{5} = .8.$$

(85)

$$\cdot\dot{67} = \frac{67}{90} \text{ and } 2.\dot{13} = 2\frac{13}{99} = \frac{211}{99}.$$

$$\frac{67}{90} \times \frac{211}{99} = \frac{12871}{8910} = 1.4445566778 +$$

(86)

wk.

$$\cdot 678125 = 4 \text{ days } 17 \text{ hours } 55 \text{ minutes } 30 \text{ seconds.}$$

7

(87)

$$4.746875 \text{ days.}$$

24

$$\cdot\dot{92437}$$

92

$$2987500$$

$$1493750$$

$$\cdot\dot{92347} = \frac{92347}{99900} = \frac{18469}{19980}.$$

$$17.925000 \text{ hours.}$$

60

$$55.500000 \text{ minutes.}$$

60

$$30.000000 \text{ seconds.}$$

(88)

Dissimilar.		Similar.		Similar and Coterminous.
-------------	--	----------	--	--------------------------

$$67.\dot{2}3\dot{4} = 67.23434\dot{3}4 = 67.234343434\dot{3}4$$

$$98.\dot{7}1\dot{3} = 98.71371\dot{3}71 = 98.71371371371\dot{3}71$$

$$91.0347123\dot{4} = 91.0347123\dot{4} = \underline{91.0347123423\dot{4}}$$

$$\text{Sum,} = 256.9827694903\dot{9}$$

Dissimilar.		Similar and Coterminous.
-------------	--	--------------------------

$$256.9827694903\dot{9} = 256.9827694903\dot{9}$$

$$100.123456789\dot{ } = \underline{100.1234567894\dot{5}}$$

$$\text{Difference} = 156.8593127009\dot{4}$$

(89)

$$12) 9 \text{ in.}$$

$$3) 2.75 \text{ ft.}$$

$$5\frac{1}{2}) 2.916 \text{ yds.}$$

$$2 \quad \quad 2$$

$$11) 5.833$$

$$40) 36.5303 \text{ rds.}$$

$$8) 5.913257 \text{ fur.}$$

$$.739157196 \text{ miles.}$$

(90)

$$17.\dot{4}28571 \text{ sq. ft.} = 17\frac{128571}{999999} \text{ sq. ft.} = 17\frac{7}{9} \text{ sq. ft.} = 17 \text{ sq. ft. } 61\frac{1}{3} \text{ in.}$$

$$100.\dot{8} \text{ sq. in.} = \underline{100\frac{8}{9}}$$

$$\text{Difference,} = 16 \text{ sq. ft. } 104\frac{5}{9} \text{ in.}$$

(91)

$$\begin{array}{r} \cdot 91789772 \\ 917897 \end{array}$$

$$\cdot 91789772 \text{ of } 2 \text{ a.} = \frac{90871875}{99000000} \times \frac{2}{1} \text{ a.} = \frac{90871875}{49500000} = \frac{3231}{1760} = 1 \text{ a. } 3 \text{ r. } 13 \text{ per. } 22 \text{ yds.}$$

(92)

$$\begin{array}{r} 11 \cdot 287 \\ 2 \end{array}$$

$$11 \cdot 287 = 11 \frac{287}{990} = 11 \frac{19}{66}. \quad 1 \cdot 0428571 = 1 \frac{428571}{9999990} = 1 \frac{3}{70}.$$

(93)

$$47 \cdot 345 = \frac{47345}{1000} \text{ and } 1 \cdot 76 = \frac{176}{99} = \frac{175}{99}.$$

$$\frac{47345}{1000} \div \frac{175}{99} = \frac{47345}{1000} \times \frac{99}{175} = \frac{937431}{35000} = 26 \cdot 7837428571.$$

(94)

Dissimilar.		Similar.		Similar and Coterminous.
85·62	=	85·626	=	85·62626
13·76432	=	13·76432	=	13·76432
		Difference,	=	71·86193

(95)

$$\begin{array}{l} \cdot 734 \text{ of a lb.} = 11 \cdot 744 \text{ oz.} \\ \cdot 198 \text{ of an oz.} = \cdot 198 \text{ oz.} \end{array}$$

$$\text{Difference,} = 11 \cdot 546 \text{ oz.}$$

(96)

$$2 \text{ ft. } 5\frac{1}{2} \text{ in.} = 29\frac{1}{2} \text{ in.} = \frac{88}{3} \text{ in.}$$

$$27 \cdot 3 \text{ ft.} = 27\frac{1}{2} \text{ ft.} = 328 \text{ in.}$$

$$20 \cdot 16 \text{ ft.} = 20\frac{1}{6} \text{ ft.} = 242 \text{ in.}$$

$$328 \times 242 \div \frac{88}{3} = \frac{328}{1} \times \frac{242}{1} \times \frac{3}{88} = 2706 \text{ in.} = 75\frac{1}{6} \text{ yds.}$$

(97)

$$3 \cdot 145 = 3 \frac{145}{990} = 3 \frac{8}{55} = \frac{173}{55} \text{ and } 4 \cdot 297 = 4 \frac{297}{999} = 4 \frac{11}{37} = \frac{159}{37}.$$

$$\frac{173}{55} \times \frac{159}{37} = \frac{27507}{2035} = 13 \cdot 5169533.$$

(98)

$\frac{3}{40}$.	Here	$40 = 2^3 \times 5$.	Therefore the equivalent decimal	will contain 3 places.
$\frac{7}{24}$.	"	$24 = 2^3 \times 3$.	" " "	3 "
$\frac{8}{15}$.	"	$15 = 5 \times 3$.	" " "	1 "
$\frac{11}{144}$.	"	$144 = 2^4 \times 3^2$.	" " "	4 "
$\frac{6}{90}$.	"	$90 = 2 \times 5 \times 3^2$.	" " "	1 "
$\frac{119}{3584}$.	"	$3584 = 2^9 \times 7$.	" " "	9 "

(99)

$$81\frac{1}{3} = 81.\dot{6} \text{ and } 328\frac{2}{3} = 328.\ddot{2}\ddot{3}.$$

Dissimilar.

Similar.

Similar and Coterminous.

$$81.\dot{6} = 81.\dot{6}\dot{6}\dot{6} = 81.\dot{6}\dot{6}\dot{6}\dot{6}\dot{6}\dot{6}\dot{6}\dot{6}$$

$$61.12\dot{6} = 61.12\dot{6} = 61.12\dot{6}\dot{6}\dot{6}\dot{6}\dot{6}\dot{6}\dot{6}\dot{6}$$

$$328.\ddot{2}\ddot{3} = 328.\ddot{2}\ddot{3}\ddot{2}\ddot{3} = 328.\ddot{2}\ddot{3}\ddot{2}\ddot{3}\ddot{2}\ddot{3}\ddot{2}\ddot{3}$$

$$5.\dot{6}24 = 5.\dot{6}24\dot{6}2 = 5.\dot{6}24\dot{6}24\dot{6}2$$

2 carried.

$$\text{Sum,} = 476.\dot{6}502811\dot{9}$$

(100)

$$\left(\frac{4.\dot{4} - 2.\dot{8}\dot{3}}{1.\dot{6} + 2.\dot{6}2\dot{9}} \times \frac{6.8 \times 3}{2.25} \right) + \frac{2.8 \times 2.\ddot{2}\ddot{7}}{1.1\dot{3}\dot{6}}$$

$$= \left(\frac{1.\dot{6}\dot{1}}{4.\dot{2}9\dot{6}} \times \frac{20.4}{2.25} \right) + \frac{2\frac{4}{5} \times 2\frac{7}{9}}{1\frac{135}{90}}$$

$$= \left(\frac{1\frac{55}{90}}{4\frac{296}{99}} \times \frac{20\frac{2}{5}}{2\frac{1}{4}} \right) + \frac{2\frac{4}{5} \times 2\frac{7}{11}}{1\frac{3}{2}}$$

$$= \left(\frac{1\frac{11}{18}}{1\frac{296}{99}} \times \frac{10\frac{2}{5}}{\frac{9}{4}} \right) + \frac{1\frac{4}{5} \times 2\frac{5}{11}}{\frac{25}{22}}$$

$$= \left(\frac{2\frac{9}{18}}{4\frac{292}{99}} \times \frac{3\frac{4}{5}}{\frac{3}{4}} \right) + \frac{1\frac{4}{5} \times \frac{5}{11}}{\frac{25}{22}}$$

$$= \left(\frac{\frac{1}{2}}{1\frac{148}{111}} \times \frac{3\frac{4}{5}}{\frac{3}{4}} \right) + \frac{7\frac{0}{11}}{2\frac{5}{22}} = \left(\frac{1\frac{11}{96}}{2\frac{96}{96}} \times \frac{1\frac{36}{15}}{1\frac{5}{15}} \right) + \frac{2\frac{8}{6}}{2\frac{8}{6}}$$

$$= \left(\frac{3}{8} \times \frac{1\frac{36}{15}}{1\frac{5}{15}} \right) + \frac{2\frac{8}{6}}{2\frac{8}{6}} = \frac{1\frac{7}{5}}{1\frac{7}{5}} + \frac{2\frac{8}{6}}{2\frac{8}{6}} = \frac{4\frac{5}{6}}{4\frac{5}{6}} = 9.$$

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(1)

V	V	V
9)4312131	3)4312131	8)4312131
<u> </u>	<u> </u>	<u> </u>
9)224322..8	3)1234023..2	8)242343..7
<u> </u>	<u> </u>	<u> </u>
9)12043..5	3)224322..2	8)14022..2
<u> </u>	<u> </u>	<u> </u>
9)344..7	3)41240..2	8)1032..1
<u> </u>	<u> </u>	<u> </u>
9)21..0	3)12043..1	8)32..6
<u> </u>	<u> </u>	<u> </u>
1..2	3)2144..1	2..1
	<u> </u>	
	3)344..2	
	<u> </u>	
	3)113..0	
	<u> </u>	
	3)21..0	
	<u> </u>	
	3)3..2	
	<u> </u>	
	1..0	

V	IX	III	VIII
4312131 =	120758 =	10200211222 =	216127
5	9	3	8
<u> </u>	<u> </u>	<u> </u>	<u> </u>
23	11	3	17
5	9	3	8
<u> </u>	<u> </u>	<u> </u>	<u> </u>
116	99	11	142
5	9	3	8
<u> </u>	<u> </u>	<u> </u>	<u> </u>
582	898	33	1137
5	9	3	8
<u> </u>	<u> </u>	<u> </u>	<u> </u>
2911	8087	99	9098
5	9	3	8
<u> </u>	<u> </u>	<u> </u>	<u> </u>
14558	72791 dec.	299	72791 dec.
5		3	
<u> </u>		<u> </u>	
72791 decimals.		72791 dec.	

lbs.	oz.	dr.	scr.	lbs.	oz.	dr.	scr.	grs.
9	7	7	2)	97	3	4	17
12					12			
<hr/>					<hr/>			
115					1167			
8					8			
<hr/>					<hr/>			
927					9340			
3					3			
<hr/>					<hr/>			
2783					28021			
20					20			
<hr/>					<hr/>			
55660)				560437	(10	³⁸³⁷ ₅₅₆₆₀	
					55660			
					<hr/>			
					3837			

(6)

$$15 \text{ yds.} = 540 \text{ in. and } 7 \text{ ft.} = 84 \text{ in.}$$

$$6 \text{ ft.} = 72 \text{ in. and } 4 \text{ ft.} = 48 \text{ in.}$$

$$(540 \times 84 \times 13) - (72 \times 48 \times 13) = 589680 - 44928 = 544752.$$

$$544752 \div 108 = 5044.$$

(7)

9 ft.	6'	4''	7'''			
11	7	9	11			
		8	8	10'''	2''''	5'''''
	7	1	9	5	3	
5	6	8	8	1		
104	10	2	5			
111	0	9	7	4	5	5

(8)

$$\frac{4\frac{2}{7} + \frac{8}{9} - \frac{7}{12}}{\frac{2}{3} \text{ of } \frac{8}{13} + \frac{1}{6} \text{ of } \frac{5}{9}} = \frac{4\frac{7}{12} - \frac{7}{12}}{\frac{6}{13} + \frac{5}{54}} = \frac{\frac{1157}{252}}{\frac{389}{702}} = \frac{\frac{1157}{14}}{\frac{389}{39}} = \frac{45123}{5446} = 8\frac{1555}{5446}.$$

(9)

(10)

pts.	77	77..42..27..21..33..14..7..11..63..30
2)782436	27	6..27.. 3.. 3.. 2
4)391218..0 pt.	10	2
2)97804..2 qt.		10

$$77 \times 27 \times 10 = 20790 = 1 \text{ c. m.}$$

$$4)43902..0 \text{ gal.}$$

$$12225..2 \text{ pks.}$$

$$12225 \text{ bush. } 2 \text{ pks. } 0 \text{ gal. } 2 \text{ qts.}$$

(11)

XII	IX
28e4)36t87942(1375t·12	3762814
28e4	9
<hr/>	<hr/>
9e47	34
82t0	9
<hr/>	<hr/>
18679	312
17274	9
<hr/>	<hr/>
14054	2810
11888	9
<hr/>	<hr/>
23882	25298
23554	9
<hr/>	<hr/>
32t·0	227683
28e·4	9
<hr/>	<hr/>
5t·80	2049151
55·t8	
<hr/>	
4·94	

(12)

$$150528 = 2^{10} \times 3 \times 7^2.$$

$$10 + 1 = 11$$

$$1 + 1 = 2$$

$$2 + 1 = 3$$

$$11 \times 3 \times 2 = 66.$$

(13)

$$2 \text{ wks. } 2 \text{ dys.} = 16 \text{ dys.}$$

$$\cdot 1234625$$

$$16$$

$$7407750$$

$$1234625$$

$$1 \cdot 9754000 \text{ dys.}$$

$$24$$

$$39016000$$

$$19508000$$

$$23 \cdot 4096000 \text{ hrs.}$$

$$60$$

$$24 \cdot 5760000 \text{ min.}$$

$$60$$

$$34 \cdot 5600000 = 34\frac{1}{2} \text{ sec.}$$

(14)

$$728\frac{1}{2} = 8\frac{1}{2} + 2 \times 10 + 7 \times 10 \times 10.$$

$$\text{lbs. oz. dr.}$$

$$\text{lbs. oz. dr.}$$

$$27 \ 4 \ 3 \times 8\frac{1}{2} = 231 \ 11 \ 9\frac{1}{2}$$

$$10$$

$$272 \ 9 \ 14 \times 2 = 545 \ 3 \ 12$$

$$10$$

$$2726 \ 2 \ 12 \times 7 = 19083 \ 3 \ 4$$

$$19860 \ 2 \ 9\frac{1}{2}$$

(15)

$$\begin{aligned} \text{£}16 \text{ 3s. } 8\frac{1}{2}\text{d.} &= \$64.74\frac{7}{12} \text{ and } \text{£}67 \text{ 17s. } 7\frac{1}{2}\text{d.} = \$271.52\frac{1}{2}. \\ \$98.17 + \$42.29 + \$64.74\frac{7}{12} + \$97.19 + \$127.87\frac{1}{2} &= \$430.27\frac{1}{2}. \\ \$430.27\frac{1}{2} - \$271.52\frac{1}{2} &= \$158.75. \end{aligned}$$

(16)

$$\begin{aligned} \cdot\dot{8} &= \frac{8}{9}. & \cdot\ddot{76} &= \frac{76}{99}. & \cdot\ddot{9123} &= \frac{9123-91}{9900} = \frac{9032}{9900} = \frac{2258}{2475}. \\ \cdot 003327 &= \frac{3327-3}{999000} = \frac{3324}{999000} = \frac{377}{83250}. \end{aligned}$$

(18)

$$\begin{aligned} & \frac{[\{(2\frac{1}{3} \times .5 \text{ of } 1\frac{1}{7}) + 9\frac{1}{2}\frac{1}{7} + \cdot 09 + \frac{23}{31}\} - 11\frac{6}{17}] \div (\frac{1}{61} \text{ of } \cdot 16)}{[(\cdot 7632763 \times 11) \times \frac{1}{8} \text{ of } \frac{101}{106}] \times (\frac{1}{2} \text{ of } \cdot 2 \text{ of } \cdot 3 \text{ of } \cdot 25 \text{ of } 96) \div \cdot 2} \\ & \qquad \qquad \qquad \frac{1}{4} \text{ of } \cdot 6732467 \div \frac{1}{9} \\ & \frac{[\{(\frac{7}{3} \times \frac{1}{2} \times \frac{1}{7}) + 9\frac{1}{2}\frac{1}{7} + \frac{1}{11} + \frac{23}{31}\} - 11\frac{6}{17}] \div (\frac{1}{61} \text{ of } \frac{1}{6})}{(\frac{7632}{9999} \times \frac{11}{1} \times \frac{1}{8} \times \frac{101}{106} \times \frac{1}{2} \times \frac{1}{6} \times \frac{1}{3} \times \frac{1}{4} \times \frac{96}{1}) \div \frac{2}{9}} \\ & \qquad \qquad \qquad \frac{1}{4} \times \frac{67324}{99999} \div \frac{1}{9} \\ & \frac{(2 + 9\frac{1}{2}\frac{1}{7} + \frac{1}{11} + \frac{23}{31}) - 11\frac{6}{17} \div \frac{1}{61} \times \frac{1}{6}}{\frac{7632}{9999} \times \frac{11}{1} \times \frac{1}{8} \times \frac{101}{106} \times \frac{1}{2} \times \frac{1}{6} \times \frac{1}{3} \times \frac{1}{4} \times \frac{96}{1} \times \frac{9}{2}} \\ & \qquad \qquad \qquad \frac{1}{4} \times \frac{67324}{99999} \times \frac{9}{1} \\ & \frac{(12 - 11\frac{6}{17}) \div \frac{1}{61} \times \frac{1}{6}}{\frac{18}{6}} = \frac{\frac{11}{7} \times \frac{51}{11} \times \frac{6}{1}}{\frac{18}{6}} = \frac{\frac{18}{6}}{\frac{18}{6}} = \frac{\frac{5}{1}}{\frac{16831}{11111}} = \\ & \frac{16831}{11111} = 3\frac{5062}{16831}. \end{aligned}$$

(19)

8 children will have 8 children's shares.

One woman will have 3 children's shares \therefore 6 women will have
 $6 \times 3 = 18$ children's shares.

One man will have 6 children's shares \therefore 4 men will have
 $4 \times 6 = 24$ children's shares.

4 men, 6 women, and 8 child. will therefore have 50 child. shares.

$\pounds 550$ 3s. $1\frac{1}{2}$ d. $\div 50 = \pounds 11$ 0s. $0\frac{3}{4}$ d. = child's share.

$\pounds 11$ 0s. $0\frac{3}{4}$ d. $\times 3 = \pounds 33$ 0s. $2\frac{1}{4}$ d. = woman's share.

$\pounds 33$ 0s. $2\frac{1}{4}$ d. $\times 2 = \pounds 66$ 0s. $4\frac{1}{2}$ d. = man's share.

(20)

$$16\frac{7}{11} + 19\frac{4}{5} + 23\frac{7}{8} + 129\frac{6}{7} = 16 + 19 + 23 + 129 + \\ (\frac{7}{11} + \frac{4}{5} + \frac{7}{8} + \frac{6}{7}) = 187 + 3\frac{519}{3080} = 190\frac{519}{3080}.$$

(21)

$$8100 = 2^2 \times 3^4 \times 5^2.$$

1..3..9..27..81

1..2..4

1..3..9..27..81..2..6..18..54..162..4..12..36..108..324

1..5..25

1..3..9..27..81..2..6..18..54..162..4..12..36..108..324..

5..15..45..135..405..10..30..90..270..810..20..60..180..

540..1620..25..75..225..675..2025..50..150..450..1350..

4050..100..30..900..2700..8100.

Therefore the divisors of 8100 are 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 25, 27, 30, 36, 45, 50, 54, 60, 75, 81, 90, 100, 108, 135, 150, 162, 180, 225, 270, 300, 324, 405, 450, 540, 675, 810, 900, 1350, 1620, 2025, 2700, 4050, 8100.

(22)

$$\begin{array}{r} 2691 \overline{)11817(4} \\ 10764 \end{array}$$

$$\begin{array}{r} 1053 \overline{)2691(2} \\ 2106 \end{array}$$

$$\begin{array}{r} 585 \overline{)1053(1} \\ 585 \end{array}$$

$$\begin{array}{r} 468 \overline{)585(1} \\ 468 \end{array}$$

$$\begin{array}{r} 117 \overline{)468(4} \\ 468 \end{array}$$

9828 is divisible by 117 \therefore 117 is the G. C. M.

(23)

$$\begin{array}{r} \text{sec.} \\ 60 \overline{)2551443} \end{array}$$

$$60 \overline{)42524 \dots 3}$$

$$24 \overline{)708 \dots 44}$$

$$29 \dots 12$$

$$29 \text{ d., } 12 \text{ h., } 44 \text{ m., } 3 \text{ sec.}$$

$$\begin{array}{r} \text{sec.} \\ 60 \overline{)31556928} \end{array}$$

$$60 \overline{)525948 \dots 48}$$

$$24 \overline{)8765 \dots 48}$$

$$365 \dots 5$$

$$365 \text{ d., } 5 \text{ h., } 48 \text{ m., } 48 \text{ sec.}$$

(24)

$$14 \text{ ft. } 11 \text{ in.} = 179 \text{ in.}$$

$$38 \text{ miles} = 2407680 \text{ in.}$$

$$2407680 \div 179 = 13450 \frac{1}{7} \frac{9}{9}.$$

(25)

$$11 \text{ ft.} \times 13 \text{ ft.} \times 15 \text{ ft.} = 2145 \text{ cub. ft.}$$

$$\text{One cubic foot weighs } 62 \frac{1}{2} \text{ lbs. } 2145 \times 62 \frac{1}{2} = 134062 \frac{1}{2} = \text{weight of } 2145 \text{ cub. ft.}$$

$$\text{One gallon weighs } 10 \text{ lbs. } 134062 \frac{1}{2} \div 10 = 13406 \frac{1}{4} = \text{gals. in } 134062 \frac{1}{2} \text{ lbs.}$$

(26)

$$£73 \times 400 = \$292.00$$

$$17\text{s.} \times 20 = 3.40$$

$$11 \frac{1}{4} \text{ d.} = 47 \text{ far.} \times 5 \div 12 = .19 \frac{7}{12}$$

$$£73 \quad 17\text{s.} \quad 11 \frac{1}{4} \text{ d.} = \$295.59 \frac{7}{12}$$

(27)

$$93 \frac{4}{11} - 76 \frac{1}{2} = 92 \frac{1}{11} - 76 \frac{1}{2} = 16 \frac{1}{2} \frac{5}{11} = \frac{4206}{253}$$

$$\frac{4206}{253} \div \frac{17}{3} = \frac{4206}{253} \times \frac{3}{17} = \frac{4206}{17} = 247 \frac{7}{17}.$$

(28)

$$\frac{5\frac{5}{8} \div \frac{2}{3}}{1\frac{1}{6} \text{ of } \frac{5}{9} \div 10\frac{1}{3}} \times \frac{\frac{2}{5} \text{ of } \frac{1\frac{1}{2} \text{ of } 4\frac{1}{9}}{13\frac{7}{8} \text{ of } 5\frac{1}{3}}}{\frac{45 \times 3}{16}} = \frac{\frac{45}{8} \times \frac{3}{2}}{\frac{6}{5} \times \frac{5}{9} \times \frac{3}{31}} \times \frac{\frac{2}{5}}{\frac{111}{8} \times \frac{16}{3}} =$$

$$\frac{\frac{45 \times 3}{16}}{\frac{2}{1} \times \frac{1}{31}} \times \frac{\frac{2}{5}}{\frac{37}{2} \times 3} =$$

$$\frac{9}{45 \times 3 \times 31} \times \frac{8}{5} \times \frac{87}{2 \times 8 \times 87 \times 2} = \frac{3 \times 9 \times 31}{16 \times 2 \times 2 \times 2} =$$

$$\frac{837}{128} = 6\frac{69}{128}.$$

(29)

$$\begin{array}{r} \text{XI} \\ 5)91342 \\ \hline 5)19074..4 \\ \hline 5)4015..1 \\ \hline 5)891..0 \\ \hline 5)184..3 \\ \hline 5)39..3 \\ \hline 5)8..2 \\ \hline 1..3 \end{array}$$

$$\begin{array}{r} \text{XI} \\ 12)91342 \\ \hline 12)834..9 \\ \hline 12)773..1 \\ \hline 12)70..3 \\ \hline 6..5 \end{array}$$

$$\begin{array}{r} \text{XI} \\ 2)91342 \\ \hline 2)46176..1 \\ \hline 2)23093..0 \\ \hline 2)11541..1 \\ \hline 2)6246..0 \\ \hline 2)3153..0 \\ \hline 2)1627..0 \\ \hline 2)869..0 \\ \hline 2)434..1 \\ \hline 2)217..1 \\ \hline 2)109..0 \\ \hline 2)54..0 \\ \hline 24..1 \\ \hline 2)15..0 \\ \hline 2)8..0 \\ \hline 2)4..0 \\ \hline 2)2..0 \\ \hline 1..0 \end{array}$$

(29 continued.)

XI	V	XII	II	
91342	13233014	65319	100000100110000101	
11	5	12	2	
<hr/>	<hr/>	<hr/>	<hr/>	
100	8	77	2	260
11	5	12	2	2
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
1103	42	927	4	521
11	5	12	2	2
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
12137	213	11125	8	1043
11	5	12	2	2
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
133509 dec.	1068	133509 dec.	16	2086
	5		2	2
	<hr/>		<hr/>	<hr/>
	5340		32	4172
	5		2	2
	<hr/>		<hr/>	<hr/>
	26701		65	8344
	5		2	2
	<hr/>		<hr/>	<hr/>
	133509 dec.		130	16688
			2	2
			<hr/>	<hr/>
			260	33377
				2
				<hr/>
				66754
				2
				<hr/>
				133509 dec.

(30)	(31)	(32)
2)7680 = $2^9 \times 3 \times 5$	m. f. p. y. ft. in.	
<u> </u>	72 3 7 2 1 7	
2)3840	8	$\$.47 \times 97 = \$45.59.$
<u> </u>	<u> </u>	
2)1920	579 fur.	(33)
<u> </u>	40	
2)960	<u> </u>	$(73 \times 4 \times 11) \div 128 = 25\frac{3}{2}.$
<u> </u>	23167 per.	
2)480	5½	$\$.3 \cdot 62\frac{1}{2} \times 25\frac{3}{2} = \$90.96\frac{3}{4}.$
<u> </u>	<u> </u>	
2)240	115837	
<u> </u>	11583½	
2)120	<u> </u>	
<u> </u>	127420½ yds.	
2)60	3	
<u> </u>	<u> </u>	
2)30	382262½ ft.	
<u> </u>	12	
3)15	<u> </u>	
<u> </u>	4587157 in.	
5	12.	
	<u> </u>	
	55045884 lines	

(34)

$$93 \cdot 723 = 93\frac{716}{990} = \frac{92786}{990} \text{ and } 29 \cdot 4173 = 29\frac{4169}{9990} = \frac{293879}{9990}.$$

$$\frac{92786}{990} \div \frac{293879}{9990} = \frac{92786}{990} \times \frac{9990}{293879} = \frac{92786 \times 111}{11 \times 293879} =$$

$$\frac{10299246}{3232669} = 3.185988 +$$

(35)

One bushel of oats weighs 34 lbs. ∴ in 73429 lbs. there are
 $73429 \div 34 = 2159\frac{3}{4}$ bushels.

(36)

In 719630 lbs. of wheat there are $719630 \div 60 = 11993\frac{5}{6}$ bus

$$\$1.80 \times 11993\frac{5}{6} = \$21588.90.$$

Or $\$1.80$ per bushel = 3 cents per lb.

$$719630 \times 3 = 2158890 \text{ cents.} = \$21588.90.$$

(38)

$$\begin{array}{r} 21389)180781(8 \\ 171112 \\ \hline \end{array}$$

$$(37) \quad \begin{array}{r} 9669)21389(2 \\ 19338 \\ \hline \end{array}$$

$$\$72.14 + \$93.76 = \$165.90$$

$$\$165.90 \times 9.47 = \$1571.0730$$

$$\$1571.0730 \div 11 = \$142.8248+$$

$$\begin{array}{r} 2051)9669(4 \\ 8204 \\ \hline 1465)2051(1 \\ 1465 \\ \hline 586)1465(2 \\ 1172 \\ \hline 293)586(2 \\ 586 \\ \hline \end{array}$$

Last divisor 293 = G. C. M.

(39)

$$\frac{7}{11}, \frac{4}{5}, \frac{9}{7}, \frac{8}{33}, \frac{11}{14}, \frac{7}{10}, \frac{1}{2}.$$

The least common multiple of 11, 5, 7, 33, 14, 10 and 2 is 2310.

The multiplier for both terms of the first fraction is $\frac{2310}{11} = 210$; for the second, $\frac{2310}{5} = 462$; for the third, $\frac{2310}{7} = 330$; for the fourth, $\frac{2310}{33} = 70$; for the fifth, $\frac{2310}{14} = 165$; for the sixth, $\frac{2310}{10} = 231$; for the seventh, $\frac{2310}{2} = 1155$.

Multiplying by these numbers, we obtain $\frac{1470}{2310}$, $\frac{1848}{2310}$, $\frac{2970}{2310}$, $\frac{560}{2310}$, $\frac{1815}{2310}$, $\frac{1617}{2310}$, and $\frac{1155}{2310}$ for the required fraction.

(40)

$$\begin{aligned} \$ \cdot 11 \times 17 &= \$1.87. \quad \$ \cdot 37\frac{1}{2} \times 19 = \$7.12\frac{1}{2}. \quad \$2.17 \times 14\frac{1}{2} = \\ & \$31.46\frac{1}{2}. \quad \$ \cdot 27 \times 67 = \$18.09. \quad \$1.37\frac{1}{2} \times 15 = \$20.62\frac{1}{2}. \\ \$1.87 + \$7.12\frac{1}{2} + \$31.46\frac{1}{2} + \$4.75 + \$11.50 + \$18.09 + \\ & \$20.62\frac{1}{2} + \$7.93 = \$103.35\frac{1}{2}. \end{aligned}$$

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(17)

$$\begin{array}{r} \text{Baskets.} \\ 11 : 87 :: \$13.42 : \frac{\overset{1.22}{\$13.42} \times 87}{11} = \$106.14 \text{ Ans.} \end{array}$$

(18)

$$\begin{array}{r} \text{Cords.} \\ 28 : 25 :: \$266 : \frac{\overset{19}{\$266} \times 25}{28} = \$237.50 \text{ Ans.} \end{array}$$

(19)

$$\begin{array}{r} \$29.20 : \$83.60 :: 16 : \frac{\overset{4}{\text{days } 16} \times 83.60}{29.20} = 45\frac{5}{7} \text{ days. Ans.} \end{array}$$

(20)

$$\begin{array}{r} \text{Bags.} \\ 16 : 156 :: \$12.80 : \frac{\overset{.8}{\$12.80} \times 156}{16} = \$124.80 \text{ Ans} \end{array}$$

(21)

$$\begin{array}{r} \text{Feet.} \quad \text{ft.} \\ 5 : 112 :: 7 : \frac{7 \times 112}{5} = 156\frac{4}{5} \text{ ft. Ans.} \end{array}$$

(22)

$$\begin{array}{r} \text{Cows.} \quad \text{days.} \\ 55 : 27 :: 99 : \frac{\overset{9}{99} \times 27}{55} = 48\frac{3}{5} \text{ days. Ans.} \end{array}$$

(23)

$$\begin{array}{r} \text{Acres.} \quad \text{bus.} \\ 5 : 48 :: 9 : \frac{9 \times 48}{5} = 86\frac{2}{5} \text{ bush. Ans.} \end{array}$$

(24)

$$\begin{array}{r} 11 \\ \text{Perches. days. } 2 \times 803 \\ 73 : 803 :: 2 : \frac{\quad}{73} = 22 \text{ days. Ans.} \end{array}$$

(25)

$$\begin{array}{r} 141 \\ \text{Pails. lbs. } 100 \times 1128 \\ 176 : 1128 :: 100 : \frac{\quad}{176} = 640\frac{10}{11} \text{ lbs. Ans.} \end{array}$$

(26)

$$\begin{array}{r} .58 \quad 155 \\ \$20.88 \times 465 \\ 108 : 465 :: \$20.88 : \frac{\quad}{108} = \$89.90 \text{ Ans.} \end{array}$$

(27)

$$\begin{array}{r} 9 \quad 639 \\ \$ \quad \$ \quad \text{brls. } 72 \times 1278 \\ 16 : 1278 :: 72 : \frac{\quad}{16} = 5751 \text{ barrels. Ans.} \end{array}$$

(28)

$$\begin{array}{r} 15 \\ \text{Men. Acres } 165 \times 3 \\ 11 : 3 :: 165 : \frac{\quad}{11} = 45 \text{ acres. Ans.} \end{array}$$

(29)

$$\begin{array}{r} 125 \\ \text{Barrels. loaves } 250 \times 67 \\ 4 : 67 :: 250 : \frac{\quad}{4} = 4187\frac{1}{2} \text{ loaves. Ans.} \end{array}$$

(30)

$$\begin{array}{r} \text{Bushels. brls. } 16 \times 38 \\ 190 : 38 :: 16 : \frac{\quad}{190} = 3\frac{1}{5} \text{ barrels. Ans.} \end{array}$$

(31)

$$\begin{array}{l} \text{Days.} \quad \text{men} \quad 90 \times 12 \\ 15 : 12 :: 90 : \frac{1080}{15} = 72 \text{ men. Ans.} \end{array}$$

(32)

$$\begin{array}{l} \text{D'. work. brls.} \quad 2 \times 279 \\ 17 : 279 :: 2 : \frac{558}{17} = 32\frac{14}{17} \text{ barrels. Ans.} \end{array}$$

(33)

$$\begin{array}{l} \text{Hours.} \quad \text{miles.} \\ 1 : 24 :: 27 : 27 \times 24 = 648 \text{ miles. Ans.} \end{array}$$

(34)

$$\begin{array}{l} \text{Cows.} \quad \text{lbs.} \quad 30 \times 23 \\ 7 : 23 :: 30 : \frac{690}{7} = 98\frac{4}{7} \text{ lbs. Ans.} \end{array}$$

(37)

$$\begin{array}{l} 36 : 21 :: \$9750 : \frac{375}{1} \times \frac{7}{28} \times \frac{16}{3} = \$42000 \text{ Ans.} \end{array}$$

(38)

$$\begin{array}{l} \text{Yard.} \quad \text{s.} \quad 5 \quad 1 \quad 8 \\ \frac{7}{8} : \frac{1}{4} :: \frac{5}{6} : \frac{1}{6} \times \frac{1}{4} \times \frac{8}{7} = \frac{5}{21} = 2\frac{2}{7} \text{d. Ans.} \end{array}$$

(39)

$$\begin{array}{l} \text{Tons.} \\ \frac{7}{9} : 8\frac{1}{2} :: \$7.49 : \frac{\$7.49 \times 8\frac{1}{2}}{9} = \frac{1.07}{1} \times \frac{25}{3} \times \frac{3}{7} = \$80.25 \text{ Ans.} \end{array}$$

(40)

$$\begin{array}{l} \text{Yards.} \\ 5\frac{1}{2} : \frac{4}{7} :: \$28.42 : \frac{.14}{1} \times \frac{4}{7} \times \frac{5}{28} = \$2.80 \text{ Ans.} \end{array}$$

(41)

$$\text{Dollar. bag } \frac{4}{25} : \frac{7}{20} :: \frac{4}{5} : \frac{7}{20} \times \frac{5}{25} \times \frac{25}{12} = \frac{7}{12} \text{ of a bag. Ans.}$$

(42)

$$\begin{array}{c} \$ \quad \$ \quad \$ \\ 100 : 472\frac{1}{2} :: 98\frac{7}{8} : \frac{98\frac{7}{8} \times 472\frac{1}{2}}{100} = \frac{98 \cdot 875 \times 472 \cdot 44}{100} = \$467 \cdot 12\frac{1}{2} \text{ Ans.} \end{array}$$

(43)

$$\begin{array}{c} \text{Tons. days} \\ 17\frac{2}{3} : 11\frac{1}{7} :: 107\frac{3}{11} : \frac{107\frac{3}{11} \times 11\frac{1}{7}}{17\frac{2}{3}} = \frac{1180}{11} \times \frac{198}{17} \times \frac{5}{88} = 70\frac{1}{8} \text{ dys. Ans.} \end{array}$$

(44)

$$\begin{array}{c} \text{Tons. cords.} \\ 15\frac{7}{13} : 11\frac{9}{26} :: 22\frac{1}{3} : \frac{22\frac{1}{3} \times 11\frac{9}{26}}{15\frac{7}{13}} = \frac{202}{9} \times \frac{295}{26} \times \frac{18}{202} = 16\frac{7}{18} \text{ cords. Ans.} \end{array}$$

(45)

$$\begin{array}{c} \text{Yds. yds.} \\ \frac{1}{2} \text{ of } \frac{2}{3} \text{ of } 3\frac{1}{2} : \frac{2}{3} \text{ of } \frac{1}{2} \text{ of } \frac{5}{6} :: \frac{2}{7} \text{ of } \frac{3}{11} \text{ of } 4\frac{3}{4} : \frac{\frac{2}{7} \text{ of } \frac{3}{11} \text{ of } 4\frac{3}{4} \times \frac{2}{3} \text{ of } \frac{1}{2} \text{ of } \frac{5}{6}}{\frac{1}{2} \text{ of } \frac{2}{3} \text{ of } 3\frac{1}{2}} = \\ \frac{4}{11} \times \frac{15}{896} = \$\frac{15}{224} \text{ Ans.} \end{array}$$

(47)

$$\begin{array}{c} 37 \text{ sq. yds. 4 ft. 120 in.} = 48648 \text{ in., and } 9 \text{ sq. yds. 2 ft.} = 11952 \text{ in.} \\ \text{Inches.} \\ 11952 : 48648 :: \$3 \cdot 50 : \frac{3 \cdot 50 \times 48648}{11952} = \$14 \cdot 245\frac{1}{2} \text{ Ans.} \end{array}$$

(48)

$$12 \text{ lbs. } 10 \text{ oz.} = 154 \text{ oz.}$$

Ounces.

$$1 : 154 :: \$1.25 : 1.25 \times 154 = \$192.50 \text{ Ans.}$$

(49)

$$10 \text{ yds.} = 40 \text{ qrs., and } 3 \text{ yds. } 2 \text{ qrs.} = 14 \text{ qrs.}$$

$$\begin{array}{r} \text{Quarters.} \quad .17 \quad 7 \\ 40 : 14 :: \$3.40 : \frac{\$3.40 \times 14}{20} = \$1.19 \text{ Ans.} \end{array}$$

(50)

$$15 \text{ lbs. } 12 \text{ dwt. } 16 \text{ grs.} = 7504 \text{ grs., and } 13 \text{ oz. } 14 \text{ grs.} = 6254 \text{ grs.}$$

$$\begin{array}{r} \text{Grains.} \quad .95 \quad 3127 \\ 7504 : 6254 :: \$3.80 : \frac{\$3.80 \times 6254}{938} = \$3.167+ \text{ Ans.} \end{array}$$

(51)

$$3 \text{ lbs. } 1 \text{ oz. } 11 \text{ dwt.} = 751 \text{ dwt. and } 12 \text{ lbs. } 6 \text{ oz. } 4 \text{ dwt.} = 3004 \text{ dwt.}$$

$$\begin{array}{r} \text{Dwt.} \quad \$ \quad 150 \\ 3004 : 751 :: 600 : \frac{600 \times 751}{4} = \$150 \text{ Ans.} \end{array}$$

(52)

$$\begin{array}{r} \text{Barrels.} \quad \text{h. m. s.} \quad 2 \text{ h. } 46 \text{ m. } 30 \text{ s.} \times \frac{4}{24} \\ 54 : 24 :: 2 \text{ } 46 \text{ } 30 : \frac{2 \text{ } 46 \text{ } 30 \times 4}{9} = 1 \text{ hr. } 14 \text{ min. Ans.} \end{array}$$

(53)

73 yds. 3 qrs. 2 na. 1 in. = 2660½ in. 3 Fl. e. 2 qrs. 1 na. = 101½ in.
And £4 17s. 8½d. = 1172½d.

$$\begin{array}{l} \text{Inches.} \quad \text{d.} \quad 1172\frac{1}{2} \times 2660\frac{1}{2} \\ 101\frac{1}{2} : 2660\frac{1}{2} :: 1172\frac{1}{2} : \frac{1172\frac{1}{2} \times 2660\frac{1}{2}}{101\frac{1}{2}} = \frac{521}{4} \times \frac{5321}{2} \times \frac{4}{405} = \\ 277\frac{2241}{90} \text{ d.} = £128 \text{ 6s. } 10\frac{61}{90} \text{ d. Ans.} \end{array}$$

(54)

8½ lbs. = 136½ oz.

$$\begin{array}{l} \text{Ounces.} \quad \text{s.} \quad 287 \quad 205 \quad 3 \\ 4\frac{5}{9} : 136\frac{1}{2} :: 8\frac{31}{2} : \frac{287}{82} \times \frac{205}{8} \times \frac{3}{41} = \frac{4505}{16} = £13 \text{ 9s. } 0\frac{1}{2} \text{ d. Ans.} \end{array}$$

(55)

$$\begin{array}{l} \text{Pages.} \quad 52 \\ 327 : 400 :: 156 : \frac{156 \times 400}{327} = 190\frac{90}{109}, \text{ i. e. on the } 191^{\text{st}} \text{ p. Ans.} \end{array}$$

(56)

46 a., 3 r., 14 p. = 7294 p., and 35 a., 2 r., 10 p. = 5690 p.

$$\begin{array}{l} \text{Perches.} \quad £ \quad 100 \times 5690 \\ 7494 : 5690 :: 100 : \frac{5690}{3747} = £75 \text{ 18s. } 6\frac{166}{747} \text{ d. Ans.} \end{array}$$

(57)

$$\begin{array}{l} \text{Days.} \quad \text{miles.} \quad 12 \times 68 \\ 48 : 68 :: 12 : \frac{17}{4} = 17 \text{ miles per day. Ans.} \end{array}$$

(58)

$$\begin{array}{rcll} \text{Shillings.} & \text{lbs.} & 113 & \\ 21\frac{1}{2} : 32\frac{2}{7} :: 16\frac{1}{7} : \frac{113}{7} \times \frac{226}{7} \times \frac{3}{6\frac{1}{2}} = \frac{38307}{1568} = 24\frac{675}{1568} \text{ lbs. Ans.} \end{array}$$

(59)

$$17493 \times 1000 \times 5 \text{ cub. ft.} = 87465000 \text{ cub. ft.}$$

$$192724 \times 1000 \times 4 \text{ cub. ft.} = 770896000 \text{ cub. ft.}$$

$$87465000 + 770896000 = 858361000 \text{ cub. ft.}$$

$$\begin{array}{rcll} \text{Cubic feet.} & \text{ton.} & 858361000 & \\ 9000 : 858361000 :: 1 : \frac{858361000}{9000} = 95373\frac{4}{9} \text{ tons. Ans.} \end{array}$$

(60)

$$50000 \times 9000 = 450000000 = \text{cub. ft. of gas in 50000 tons of coal.}$$

$$\begin{array}{rcll} \text{Cubic feet.} & \text{hour.} & & \text{Ans.} \\ 4 : 450000000 :: 1 : \frac{450000000}{4} = 112500000 \text{ h.} = 12842 \text{ y. } 170 \text{ d.} \end{array}$$

(61)

$$\text{lbs. lbs. lbs. lb. lb.}$$

$$4 + 3 + 2 + 1 + \frac{1}{2} = 10\frac{1}{2} \text{ lbs.}$$

$$\begin{array}{rcll} \text{lbs.} & 11270 & & \\ 10\frac{1}{2} : 11270 :: 1 : \frac{11270}{10\frac{1}{2}} = 1073, \text{ and } 3\frac{1}{2} \text{ lbs. remaining. Ans.} \end{array}$$

(62)

$$180 \text{ miles} = 180 \times 1760 = 316800 \text{ yards.}$$

$$\text{Yards. day.}$$

$$100 : 316800 :: 1 : \frac{316800 \times 1}{100} = 3168 \text{ dys. or about } 8\frac{2}{3} \text{ yrs. Ans.}$$

Page 216.

(4)

$$\left. \begin{array}{l} 120 : 90 \text{ bush.} \\ 6 : 14 \text{ horses.} \end{array} \right\} :: 56 \text{ days} : \frac{7 \quad 15}{56 \times 90 \times 14} = 7 \times 14 = 98 \text{ days.}$$

$$\frac{120 \times 6}{8}$$

(5)

$$\left. \begin{array}{l} 28 : 32 \text{ ft. high.} \\ 8 : 15 \text{ days.} \end{array} \right\} :: 63 \text{ men} : \frac{9 \quad 4}{63 \times 32 \times 15} = 9 \times 15 = 135 \text{ men.}$$

$$\frac{8 \times 28}{7}$$

(6)

$$\left. \begin{array}{l} 3 : 45 \text{ length.} \\ 1\frac{1}{4} : 1 \text{ width.} \end{array} \right\} :: 1 \text{ lb.} : \frac{45}{3 \times 1\frac{1}{4}} = \frac{45}{\frac{15}{4}} = \frac{3}{15} \times 4 = 3 \times 4 = 12 \text{ lbs.}$$

$$\frac{45 \times 4}{15}$$

(7)

$$\left. \begin{array}{l} 10 : 100 \text{ length.} \\ 1\frac{1}{2} : 1\frac{1}{2} \text{ width.} \end{array} \right\} :: 3 \text{ lbs.} : \frac{3 \times 1\frac{1}{2} \times 100}{1\frac{1}{2} \times 10} = 2 \times 1\frac{1}{2} \times 10 = 25 \text{ lbs.}$$

(8)

$$\left. \begin{array}{l} 44 : 132 \text{ tons.} \\ 18 : 5 \text{ days.} \end{array} \right\} :: 12 \text{ horses} : \frac{2 \quad 3}{12 \times 5 \times 132} = 2 \times 5 = 10 \text{ horses.}$$

$$\frac{44 \times 18}{6}$$

(9)

$$\left. \begin{array}{l} 4 : 14 \text{ men.} \\ 7 : 10 \text{ days} \end{array} \right\} :: 27s. : \frac{2 \quad 5}{27 \times 14 \times 10} = 27 \times 5 = 135s. = £6 \text{ } 15s.}$$

$$\frac{4 \times 7}{2}$$

(10)

$$\begin{array}{l}
 3:5 \text{ masters.} \\
 8:10 \text{ apprentices.} \\
 5:8 \text{ weeks.} \\
 6:5\frac{1}{2} \text{ days per wk.}
 \end{array}
 \left. \vphantom{\begin{array}{l} 3:5 \\ 8:10 \\ 5:8 \\ 6:5\frac{1}{2} \end{array}} \right\} :: \$144 : \frac{\overset{8}{24} \overset{2}{144} \times 5\frac{1}{2} \times 8 \times \overset{2}{10} \times 5}{3 \times 8 \times 5 \times 6} = \$440.$$

(11)

$$\begin{array}{l}
 6 : 18 \text{ s.mak.} \\
 4 : 5 \text{ weeks.}
 \end{array}
 \left. \vphantom{\begin{array}{l} 6 : 18 \\ 4 : 5 \end{array}} \right\} :: 36 \text{ pairs of men's shoes} : \frac{\overset{9}{36} \times \overset{3}{18} \times 5}{6 \times 4} =$$

135 pairs men's and the women's = $\frac{3}{4} = \frac{3}{4}$ of 135 = 90 pairs.

(12)

$$\begin{array}{l}
 9 : 18 \text{ feet high.} \\
 4 : 6 \text{ days.}
 \end{array}
 \left. \vphantom{\begin{array}{l} 9 : 18 \\ 4 : 6 \end{array}} \right\} :: 12 \text{ men} : \frac{\overset{3}{12} \times \overset{2}{18} \times 6}{9 \times 4} = 3 \times 2 \times 6 = 36 \text{ men.}$$

(13)

$$\begin{array}{l}
 130 : 390 \text{ miles.} \\
 7 : 14 \text{ hours.}
 \end{array}
 \left. \vphantom{\begin{array}{l} 130 : 390 \\ 7 : 14 \end{array}} \right\} :: 3 \text{ days} : \frac{\overset{2}{3} \times \overset{3}{14} \times 390}{130 \times 7} = 3 \times 2 \times 3 = 18 \text{ days.}$$

(14)

$$\begin{array}{l}
 10 : 60 \text{ oz.} \\
 22\frac{1}{2} : 30 \text{ d.}
 \end{array}
 \left. \vphantom{\begin{array}{l} 10 : 60 \\ 22\frac{1}{2} : 30 \end{array}} \right\} :: 1 \text{ d.} : \frac{60 \times 30}{10 \times 22\frac{1}{2}} = \frac{\overset{4}{60}}{\overset{3}{10}} \times \frac{\overset{3}{30}}{\overset{1}{1}} \times \frac{\overset{1}{10}}{\overset{2}{45}} \times \frac{\overset{2}{2}}{\overset{15}{15}} = 4 \times 2 = 8 \text{ d.}$$

(15)

$$\begin{array}{l}
 10 : 5 \text{ compositors} \\
 7 : 14 \text{ hours.} \\
 20 : 40 \text{ sheets.} \\
 24 : 16 \text{ pages.} \\
 50 : 60 \text{ lines.} \\
 40 : 50 \text{ letters.}
 \end{array}
 \left. \vphantom{\begin{array}{l} 10 : 5 \\ 7 : 14 \\ 20 : 40 \\ 24 : 16 \\ 50 : 60 \\ 40 : 50 \end{array}} \right\} :: 16 \text{ days} : \frac{\overset{2}{16} \times \overset{2}{5} \times \overset{2}{14} \times \overset{3}{40} \times \overset{3}{16} \times \overset{3}{60} \times \overset{3}{50}}{\underset{5}{10} \times \underset{5}{7} \times \underset{5}{20} \times \underset{8}{24} \times \underset{8}{50} \times \underset{8}{40}} =$$

$2 \times 16 = 32 \text{ days.}$

(16)

$$\begin{array}{l}
 336 : 240 \text{ men.} \\
 5 : 9 \text{ days.} \\
 10 : 12 \text{ hours.} \\
 6 : 5 \text{ degrees.} \\
 5 : 3 \text{ yards wide} \\
 3 : 2 \text{ yards deep}
 \end{array}
 \left. \vphantom{\begin{array}{l} 336 : 240 \text{ men.} \\ 5 : 9 \text{ days.} \\ 10 : 12 \text{ hours.} \\ 6 : 5 \text{ degrees.} \\ 5 : 3 \text{ yards wide} \\ 3 : 2 \text{ yards deep} \end{array}} \right\} :: 70 \text{ yards} : \frac{\overset{7}{70} \times \overset{5}{240} \times 9 \times \overset{2}{12} \times 5 \times 3 \times 2}{\underset{48}{336 \times 5 \times 10 \times 6 \times 5 \times 3}} =$$

$$9 \times 2 \times 2 = 36 \text{ yards.}$$

(17)

$$\begin{array}{l}
 6 : 12 \text{ horses.} \\
 4 : 9 \text{ months.}
 \end{array}
 \left. \vphantom{\begin{array}{l} 6 : 12 \text{ horses.} \\ 4 : 9 \text{ months.} \end{array}} \right\} :: 16 \text{ acres} : \frac{\overset{4}{16} \times \overset{2}{12} \times 9}{\underset{6 \times 4}{}} = 4 \times 2 \times 9 = 72 \text{ acres.}$$

(18)

$$\begin{array}{l}
 25 : 139 \text{ persons} \\
 1 : 7 \text{ years.}
 \end{array}
 \left. \vphantom{\begin{array}{l} 25 : 139 \text{ persons} \\ 1 : 7 \text{ years.} \end{array}} \right\} :: 300 \text{ bus.} : \frac{\overset{12}{300} \times 139 \times 7}{\underset{25}{}} = 11676 \text{ bushels.}$$

(19)

$$\begin{array}{l}
 48 : 32 \text{ men.} \\
 36 : 864 \text{ feet long.} \\
 8 : 5 \text{ feet high.} \\
 4 : 3 \text{ feet wide.}
 \end{array}
 \left. \vphantom{\begin{array}{l} 48 : 32 \text{ men.} \\ 36 : 864 \text{ feet long.} \\ 8 : 5 \text{ feet high.} \\ 4 : 3 \text{ feet wide.} \end{array}} \right\} :: 4 \text{ days} : \frac{\overset{3}{4} \times \overset{2}{32} \times \overset{108}{864} \times 5 \times 3}{\underset{16}{48 \times 36 \times 8 \times 4}} = 30 \text{ days.}$$

(20)

$$\begin{array}{l}
 679 : 22407 \text{ sold's.} \\
 336 : 112 \text{ days.}
 \end{array}
 \left. \vphantom{\begin{array}{l} 679 : 22407 \text{ sold's.} \\ 336 : 112 \text{ days.} \end{array}} \right\} :: 702 \text{ bushels} : \frac{\overset{234}{702} \times \overset{33}{22407} \times 112}{\underset{3}{679 \times 336}} =$$

$$234 \times 33 = 7722 \text{ bushels.}$$

(21)

$$\begin{array}{l}
 13 : 494 \text{ suits.} \\
 19 : 27 \text{ days.}
 \end{array}
 \left. \vphantom{\begin{array}{l} 13 : 494 \text{ suits.} \\ 19 : 27 \text{ days.} \end{array}} \right\} :: 12 \text{ tailors} : \frac{\overset{2}{12} \times \overset{26}{494} \times 27}{\underset{19 \times 19}{}} = 648 \text{ tailors.}$$

(22)

$$\left. \begin{array}{l} 17:40 \text{ head of cattle} \\ 30:51 \text{ days.} \end{array} \right\} :: 5 \text{ a. } 2 \text{ r. } 10 \text{ p.} : \frac{5 \text{ a. } 2 \text{ r. } 10 \text{ p.} \times 40 \times 51}{17 \times 30} =$$

$$5 \text{ a. } 2 \text{ r. } 10 \text{ p.} \times 4 = 22 \text{ a. } 1 \text{ r.}$$

(23)

$$\left. \begin{array}{l} 20 : 100 \text{ ft. long} \\ 6 : 4 \text{ feet wide.} \end{array} \right\} :: 180 \text{ bricks} : \frac{30 \times 5 \times 180 \times 100 \times 4}{20 \times 6} =$$

$$30 \times 5 \times 4 = 600 \text{ bricks.}$$

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(4)

$$\left. \begin{array}{ll} 17 \text{ cords} & = 116 \text{ lbs.} \\ 87 \text{ lbs.} & = 23 \text{ barrels} \\ 19 \text{ barrels} & = 34 \text{ days' work} \\ 92 \text{ days' work} & = 57 \text{ baskets peaches} \\ 31 \text{ baskets peaches} & = 24 \text{ dollars} \\ 12 \text{ dollars} & = 2 \text{ tons} \\ 35 \text{ tons} & = x \text{ cords} \end{array} \right\} =$$

$$\frac{17 \times 87 \times 19 \times 92 \times 31 \times 12 \times 35}{116 \times 23 \times 34 \times 57 \times 24 \times 2} = \frac{31 \times 35}{4 \times 2} = \frac{1085}{8} = 135\frac{5}{8}.$$

(5)

$$\left. \begin{array}{ll} 6 \text{ lbs. tea} & = 29 \text{ lbs. sugar} \\ 17 \text{ lbs. sugar} & = 1 \text{ bushel} \\ 27 \text{ bushels} & = 4 \text{ tons} \\ 34 \text{ tons} & = 15 \text{ cows} \\ 29 \text{ cows} & = 1160 \text{ dollars} \\ 20 \text{ dollars} & = x \text{ lbs. tea.} \end{array} \right\} =$$

$$\frac{6 \times 17 \times 27 \times 34 \times 29 \times 20}{29 \times 1 \times 4 \times 15 \times 1160} = \frac{17 \times 17 \times 27}{5 \times 58} = \frac{7803}{290} = 26\frac{23}{290}$$

(6)

11 bush. barley	=	21 bush. potatoes	}
19 " potatoes	=	29 " oats	
115 " oats	=	44 " wheat	
14½ " wheat	=	38 " peas	
60 " peas	=	55 " rye	
75 " rye	=	11½ " clover sd.	
36 " clover sd.	=	x " barley	}

$$\frac{11 \times 19 \times 115 \times 14\frac{1}{2} \times 60 \times 75 \times 36}{21 \times 29 \times 44 \times 38 \times 55 \times 11\frac{1}{2}} = \frac{5 \times 75 \times 18}{7 \times 11} = \frac{6750}{77} = 87\frac{51}{77}.$$

(7)

16 baskets pears	=	29 turkeys	}
17 turkeys	=	7 days' work	
7½ days' work	=	187 loaves	
3½ loaves	=	4 lbs. veal	
1 lb. veal	=	11 cents	
792 cents	=	63 lbs. sugar	
x lbs. sugar	=	21 baskets pears	}

$$\frac{29 \times 7 \times 187 \times 4 \times 11 \times 63 \times 21}{16 \times 17 \times 7\frac{1}{2} \times 3\frac{1}{2} \times 1 \times 792} = \frac{11 \times 7 \times 21}{4} = \frac{1617}{4} = 404\frac{1}{4}.$$

(8)

$$\left. \begin{array}{l} 7 A = 11 B \\ 5 B = 8 C \\ 15 C = 21 D \\ 11 D = 5 E \\ 42 E = x A \end{array} \right\} = \frac{7 \times 5 \times 15 \times 11 \times 42}{11 \times 8 \times 21 \times 5} = \frac{7 \times 15}{4} = \frac{105}{4} = 26\frac{1}{4}.$$

(9)

$$\begin{array}{rcl}
 7 \text{ barrels flour} & = & 23 \text{ cords} \\
 6 \text{ cords} & = & 11 \text{ cwt.} \\
 46 \text{ cwt.} & = & £28 \\
 £77 & = & 9 \text{ sheep} \\
 5 \text{ sheep} & = & 8 \text{ tons} \\
 9 \text{ tons} & = & x \text{ barrels flour}
 \end{array}
 \left. \vphantom{\begin{array}{rcl} 7 \text{ barrels flour} & = & 23 \text{ cords} \\ 6 \text{ cords} & = & 11 \text{ cwt.} \\ 46 \text{ cwt.} & = & £28 \\ £77 & = & 9 \text{ sheep} \\ 5 \text{ sheep} & = & 8 \text{ tons} \\ 9 \text{ tons} & = & x \text{ barrels flour} \end{array}} \right\} =$$

$$\begin{array}{r}
 \begin{array}{ccccccc}
 & 3 & & 2 & & 7 & \\
 7 & \times & 6 & \times & 46 & \times & 77 & \times & 5 & \times & 9 \\
 \hline
 23 & \times & 11 & \times & 28 & \times & 9 & \times & 8
 \end{array} & = & \frac{3 \times 7 \times 5}{8} = \frac{105}{8} = 13\frac{1}{8}.
 \end{array}$$

(10)

$$\begin{array}{rcl}
 15 \text{ N. England} & = & 20 \text{ New York} \\
 24 \text{ New York} & = & 22\frac{1}{2} \text{ N. Jersey} \\
 30 \text{ New Jersey} & = & 20 \text{ Canada} \\
 4807\frac{1}{2} \text{ Canada} & = & x \text{ N. England}
 \end{array}
 \left. \vphantom{\begin{array}{rcl} 15 \text{ N. England} & = & 20 \text{ New York} \\ 24 \text{ New York} & = & 22\frac{1}{2} \text{ N. Jersey} \\ 30 \text{ New Jersey} & = & 20 \text{ Canada} \\ 4807\frac{1}{2} \text{ Canada} & = & x \text{ N. England} \end{array}} \right\} =$$

$$\begin{array}{r}
 \begin{array}{ccccccc}
 & 2 & & 6 & & 3 & & 961\frac{1}{2} \\
 15 & \times & 24 & \times & 30 & \times & 4807\frac{1}{2} \\
 \hline
 20 & \times & 22\frac{1}{2} & \times & 20
 \end{array} & = & 961\frac{1}{2} \times 6 = 5769 \text{ s.} = £288 \text{ 9s.}
 \end{array}$$

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(1)

$$\begin{array}{ccccccc}
 \frac{7}{8} & \times & \frac{17}{11} & \times & \frac{23}{29} & \times & \frac{\frac{11}{319}}{\frac{119}{17}} & \times & \frac{\frac{2}{16}}{\frac{69}{3}} & = & \frac{2}{3} & = & 2 : 3.
 \end{array}$$

(2)

$$\begin{array}{rcl}
 £119 \times 400 & = & \$476.00 \\
 16\text{s.} \times 20 & = & 3.20 \\
 6\frac{1}{2}\text{d.} = 26 \text{ far.} \times 5 \div 12 & = & .10\frac{5}{6} \\
 \hline
 £119 \text{ 16s. } 6\frac{1}{2}\text{d.} & = & \$479.30\frac{5}{6}
 \end{array}$$

(4)

$$\left. \begin{array}{l} 9 : 13 = 9 \div 13 = .692 \\ 21 : 27 = 21 \div 27 = .777 \\ 7 : 10 = 7 \div 10 = .7 \\ 11 : 15 = 11 \div 15 = .733 \end{array} \right\} \text{Hence } 21 : 27 \text{ is the greatest,} \\ \text{and } 9 : 13 \text{ the least.}$$

(5)

Dissimilar.		Similar.		Similar and Coterminous.
76·23478	=	76·234784	=	76·234784784784784
19·1342291	=	19·1342291	=	19·134229122912291
				<u>57·100555661872493</u>
		Difference,	=	57·100555661872493

(6)

71324t undenary = 1146287 denary, 23421 quinary = 1736 denary, and t4e7 duodenary = 17995 denary.

1146287 × 1736 = 1989954232 ÷ 17995 = 110583 $\frac{13147}{17995}$.
 110583 $\frac{13147}{17995}$ denary * = 53ee3 $\frac{7137}{147}$ duodenary, 12014313 $\frac{11042}{1033440}$ quinary, and 760t0 $\frac{2272}{12571}$ undenary.

(7)

$$\left. \begin{array}{l} 5.63 : 7.9 \text{ cubic inches.} \\ 1 : 1.220 \text{ spec. grav.} \end{array} \right\} :: 3.254 \text{ oz.} : \frac{3.254 \times 7.9 \times 1.220}{5.63} =$$

$$\frac{31.362052}{5.63} = 5.57052 \text{ oz. Ans.}$$

* To reduce the fractional part, reduce both numerator and denominator separately.

(8)

$$\begin{array}{r} \text{yds. qrs. na. in.} \quad \text{yds. qrs. na. in.} \\ 17) 63 \quad 3 \quad 2 \quad 1 \quad (\quad 3 \quad 3 \quad 0 \quad 0 \frac{1}{4} \end{array} \quad (9)$$

51

—
12

4

—
51

51

—
0

4

—
2

2½

$$5\frac{1}{2} = \frac{11}{2} \div 17 = \frac{11}{34}.$$

$$\begin{aligned} &\cdot 916325 \text{ of an acre} = \cdot 916325 \times 4840 = \\ &4421 \cdot 945 \text{ sq. yds.} \\ &4421 \cdot 945 \times \$ \cdot 67 = \$ 2962 \cdot 70 + \end{aligned}$$

(10)

$$\begin{aligned} &\frac{1}{2} \text{ of } \frac{2}{3} \text{ of } \frac{7}{8} \text{ of } 20 \text{ bush.} \times \cdot 5 \times \cdot 6 \times \frac{7}{8} = \\ &\frac{1}{2} \times \frac{2}{3} \times \frac{7}{8} \times 20 \times \frac{1}{2} \times \frac{3}{4} \times \frac{7}{8} = \\ &4\frac{1}{2} \text{ bush.} = 1 \text{ bush. } 2 \text{ pks. } 0 \text{ gal. } 1 \text{ qt.} \end{aligned}$$

(12)

$$\text{Whole amount of increase} = 2571437 - 1842265 = 729172.$$

$$1842265 : 100 :: 729172 : \frac{729172 \times 100}{1842265} = 39 \text{ per cent.}$$

(13)

$$\frac{1}{2} \text{ of } \frac{2}{3} \text{ of } \frac{18}{29} - \frac{1}{2} \text{ of } \frac{2}{3} \text{ of } \frac{5}{7} = \frac{6}{29} - \frac{5}{84} = \frac{359}{2436}.$$

(14)

$$100 : 7 :: 11 : \frac{11 \times 7}{100} = \frac{77}{100}. \quad 11 - \frac{77}{100} = 10\frac{23}{100}.$$

(15)

$$79 \times 16 \times £ \cdot 00163 = £ 2 \cdot 06032 = £ 2 \text{ ls. } 2\frac{2}{3}\frac{2}{3}\text{d.}$$

(16)

$$\left. \begin{array}{l} 4 : 3 \text{ men} \\ 10 : 12 \text{ hours} \\ 20 : 35 \text{ acres} \end{array} \right\} :: 2\frac{1}{2} \text{ days} : \frac{\overset{3}{2}\overset{7}{2} \times 3 \times \overset{3}{1}\overset{5}{2} \times \overset{3}{2}\overset{5}{2}}{\underset{4}{4} \times \overset{3}{1}\overset{5}{2} \times \overset{3}{2}\overset{5}{2}} = \frac{63}{16} = 3\frac{1}{2} \text{ days.}$$

(17)

$$(\frac{4}{5} \text{ of } \frac{9}{11} \times .02 \times .456) \div (\frac{16}{17} \text{ of } \frac{2}{3} \text{ of } \frac{1}{8} \text{ of } 51) =$$

$$\frac{\frac{2}{4} \times \frac{3}{9} \times \frac{1}{11} \times \frac{2}{25} \times \frac{38}{37}}{\frac{16}{5} \times \frac{2}{11} \times \frac{1}{25} \times \frac{1}{37}} = \frac{2 \times 38}{5 \times 11 \times 25 \times 37} = \frac{76}{80375}.$$

(18)

$$\frac{2}{1} \times \frac{4}{7} \times \frac{13}{5} \times \frac{7}{2} \times \frac{5}{1} = 4 \times 13 = 52.$$

(19)

$$\left. \begin{array}{l} 50 \text{ barrels} = 125 \text{ yards} \\ 80 \text{ yards} = 6 \text{ bales} \\ 13 \text{ bales} = 3\frac{1}{2} \text{ hogsheads} \\ x \text{ hogsheads} = 1000 \text{ barrels} \end{array} \right\} =$$

$$\frac{50 \times 3 \times 125}{125 \times 6 \times 3\frac{1}{2} \times 1000} = \frac{125 \times 3 \times 3\frac{1}{2}}{50 \times 80 \times 13} = 50\frac{25}{2}.$$

(20)

$$\frac{7347}{100} \times \frac{63}{10000} \times \frac{3330}{57391} = \frac{7347}{100} \times \frac{63}{10000} \div \frac{57391}{3330} = \frac{154132713}{5739100000} = .026856599989+$$

(21)

$$2 \text{ roods } 7 \text{ per. } 4 \text{ yds. } 3 \text{ ft. } 117 \text{ in.} = 3416481 \text{ in. and } 7 \text{ acres} = 43908480 \text{ inches.}$$

$$3416481 \div 43908480 = .0778+$$

(22)

 $\frac{2}{7}$ of $\frac{1}{5}$ of $\frac{1}{3}$ of 70 miles = $\frac{1}{3}$ miles = 5.33333+ miles.

 $\cdot 73$ of 11 fur. = 8.03 fur. = 1.00375 mile.

 $5.33333 - 1.00375 = 4.32958$ miles.

(23)

274312 nonary = 167195 denary, 1101011010 = 858 denary, and
 .5555 septenary = 2000 denary.

 $167195 - 858 = 166337 \times 2000 = 332674000.$
 332674000 denary = 764876837 nonary,

 $= 10011110101000011001111010000$ binary,

 $= 11146453021$ septenary.

(24)

 $275 \mid 275 \dots 18 \dots 190 \dots 209 \dots 225$
 $38 \mid 4 \dots 18 \dots 38 \dots 19 \dots 9$
 $18 \mid 2 \dots 9 \dots 9$
 $275 \times 38 \times 18 = 188100 = 1. \text{ c. m.}$

(25)

10 : 6 weeks	}	men	60 :	$\frac{6 \times 2914 \times 2}{10 \times 6 \times 11 \times 2400 \times 18 \times 11}$	=
6 : 5 days					
11 : 10 hours					
2400 : 8742 feet long					
18 : 20 feet wide					
11 : 8 feet high				$\frac{5 \times 2914 \times 2}{11 \times 3 \times 11}$	

$$\frac{5 \times 2914 \times 2}{11 \times 3 \times 11} = \frac{29140}{363} = 80\frac{100}{363}.$$

(26)

$172000 = 2^5 \times 3^3 \times 43$. Increasing each exponent by 1 and multiplying them together we obtain $6 \times 4 \times 2 = 48$.

(27)

$$42\dot{\cdot}7 = 42\dot{7} = \frac{385}{9} \text{ and } 9\dot{\cdot}7123 = 9\frac{7116}{9990} = 9\frac{1186}{1665} = \frac{16171}{1665}.$$

$$\frac{385}{9} \times \frac{16171}{1665} = \frac{6225835}{14985} = 415\dot{\cdot}471137804.$$

(28)

$$100 : 27 :: \$73\cdot42 : \frac{73\cdot42 \times 27}{100} = \$19\cdot8234.$$

$$\$73\cdot42 - \$19\cdot8234 = \$53\cdot5966.$$

(29)

$$6300 = 2^2 \times 3^2 \times 5^2 \times 7.$$

1..5..25

1..2.. 4

1..5..25..2..10..50..4..20..100

1..3.. 9

1..5..25..2..10..50..4..20..100..3..15..75..6..30..150..

12..60..300..9..45..225..18..90..450..36..180..900

1..7

1..5..25..2..10..50..4..20..100..3..15..75..6..30..150..

12..60..300..9..45..225..18..90..450..36..180..900 ..7 ..

35..175..14..70..350..28..140..700..21..105 ..525 ..42..

210..1050..84..420..2100..63..315..1575..126..630 ..3150

..252..1260..6300.

Therefore the divisors of 6300 are 1, 2, 3, 4, 5, 6, 7, 9, 10, 12, 14, 15, 18, 20, 21, 25, 28, 30, 35, 36, 42, 45, 50, 60, 63, 70, 75, 84, 90, 100, 105, 126, 140, 150, 175, 180, 210, 225, 252, 300, 315, 350, 420, 450, 525, 630, 700, 900, 1050, 1260, 1575, 2100, 3150, 6300.

(30)

$$\frac{2}{7} \text{ of } \frac{3}{8} \text{ of } 3\frac{1}{2} \text{ lbs.} = \frac{3}{8} \text{ lbs.}, \quad \frac{9}{7} \text{ of } \frac{3}{8} \text{ of } \frac{2}{7} \text{ of } \frac{1}{2} \text{ of } \$1 = \$\frac{3}{7},$$

$$\text{and } \frac{3}{8} \text{ of } \frac{7}{9} \text{ of } \frac{6}{10} \text{ of } \frac{2}{50} \text{ of } 90 \text{ lbs.} = \frac{1}{200} \text{ lbs.}$$

$$\begin{array}{r} \text{lbs.} \\ \frac{3}{8} : \frac{1323}{200} :: \frac{2}{7} : \frac{\frac{2}{7} \times \frac{1323}{200}}{\frac{3}{8}} = \frac{2}{7} \times \frac{1323}{200} \times \frac{8}{3} = \frac{1323}{25} = \$5\cdot04. \end{array}$$

(31)

7 men will have 7 men's shares.

One woman has $\frac{3}{11}$ of a man's share; \therefore 2 women will have $2 \times \frac{3}{11} = \frac{6}{11}$ of a man's share.

One child has $\frac{2}{7}$ of $\frac{3}{11} = \frac{6}{77}$ of a man's share; \therefore 11 children will have $11 \times \frac{6}{77} = \frac{6}{7}$ of a man's share.

7 men, 2 women and 11 children will have $7 + \frac{6}{11} + \frac{6}{7} = 8\frac{31}{77}$ men's shares.

$\$2739 \cdot 18 \div 8\frac{31}{77} = \$325 \cdot 99\frac{133}{44} =$ a man's share.

$\frac{3}{11}$ of $\$325 \cdot 99\frac{133}{44} = \$88 \cdot 90\frac{448}{447} =$ a woman's share.

$\frac{2}{7}$ of $\$88 \cdot 90\frac{448}{447} = \$25 \cdot 40\frac{128}{447} =$ a child's share.

(33)

yds. ft. in.

$$\begin{array}{rcl}
 \frac{4}{9} \text{ of } 6\frac{1}{2} \text{ yds.} & = \frac{26}{9} \text{ yds.} & = 2 \ 2 \ 8 \\
 \frac{2}{6} \text{ of } \frac{4}{7} \text{ of } 8\frac{3}{4} \text{ ft.} & = 1 \ 0 \ 0 & \\
 \frac{2}{7} \text{ of } \frac{3}{11} \text{ of } 7\frac{7}{10} \text{ in.} & = & \frac{3}{8} \\
 \hline
 \text{Sum} & = 3 \ 2 \ 8\frac{3}{8} &
 \end{array}
 \left. \begin{array}{l}
 2 \ 28 : 7 \ 2 \\
 4 : 11 \\
 2 \ 6 : 5 \\
 13 : 11\frac{1}{2}
 \end{array} \right\} = 104 : 5.$$

(34)

(35)

23 bush. 2 pks. 1 gal. 1 qt. 1 pt. = 1515 pts.

$1515 \times 9000 \times \frac{1}{3} = 4545000 \text{ in.} = 71 \text{ miles } 5 \text{ fur. } 34 \text{ per. } 3 \text{ yds.}$

(36)

$$\frac{4158}{10395} = \frac{462}{1155} = \frac{66}{165} = \frac{22}{55} = \frac{2}{5}.$$

(37)

VIII.

$\frac{1}{2}, \frac{2}{3}, \frac{1}{5}, \frac{2}{7}$. Here the common denominator is $2 \times 3 \times 5 \times 7 = 322$. The numerators of the fractions are, for the first, $1 \times 3 \times 5 \times 7 = 151$; for the second, $2 \times 2 \times 5 \times 7 = 214$; for the third, $4 \times 2 \times 3 \times 7 = 250$; for the fourth, $2 \times 2 \times 3 \times 5 = 74$; and the equivalent fractions are, $\frac{151}{322}, \frac{214}{322}, \frac{250}{322}$ and $\frac{74}{322}$, which when added together $= \frac{731}{322} = 2\frac{65}{322}$, the numbers all through being in the octenary scale.

(38)

$$\left. \begin{array}{l} 17 \text{ sheep} = 6 \text{ cows} \\ 26 \text{ cows} = 27\frac{1}{2} \text{ acres} \\ 12 \text{ acres} = 13 \text{ horses} \\ 11 \text{ horses} = 28 \text{ goats} \\ x \text{ goats} = 68 \text{ sheep} \end{array} \right\} = \frac{3}{6} \times \frac{2\frac{1}{2}}{27\frac{1}{2}} \times \frac{13}{12} \times \frac{28}{11} \times \frac{17}{68} = 2\frac{1}{2} \times 28 = 70 \text{ goats.}$$

(39)

$$\left. \begin{array}{l} 27:54 \text{ days} \\ 24:18 \text{ cel.} \\ 36:48 \text{ ft. l.} \\ 21:28 \text{ ft. w.} \\ 10:9 \text{ ft. d.} \\ 3:5 \text{ hrs.} \end{array} \right\} :: 50 \text{ men} : \frac{50 \times 54 \times 18 \times 48 \times 28 \times 9 \times 5}{27 \times 24 \times 36 \times 21 \times 10 \times 3} = 200 \text{ men.}$$

Page 226.

(7)

$$\$.35 \times 92647 = \$32426.45.$$

(8)

	£	s.	d.	
4d. $\frac{1}{3}$	4746	17	0	= cost of 94937 pails at 1s.
1d. $\frac{1}{4}$	1582	5	8	= " " " at 4d.
	395	11	5	= " " " at 1d.
<hr/>				
	£6724	14	1	= cost of 94937 pails at 1s. 5d.

(9)

(10)

$$\$.07\frac{1}{2} \times 95974 = \$7197.90 \quad \$28.80 \times 62 = \$1785.60.$$

(11)

(12)

$$\$.32\frac{1}{2} \times 2310 = \$750.75. \quad \$.37\frac{1}{2} \times 2117 = \$793.87\frac{1}{2}.$$

(13)

	£	s.	d.	
6d. $\frac{1}{2}$	375	6	0	= price of 7506 pairs at 1s.
3d. $\frac{1}{2}$	187	13	0	= " " at 6d.
$\frac{3}{4}$ d. $\frac{1}{4}$	93	16	6	= " " at 3d.
	23	9	1 $\frac{1}{2}$	= " " at $\frac{3}{4}$ d.
<hr/>				
	£680	4	7 $\frac{1}{2}$	= price of 7506 pairs at 1s. 9 $\frac{1}{2}$ d.

$$\begin{array}{l} (14) \\ \$ \cdot 17\frac{1}{2} \times 1217 = \$212 \cdot 97\frac{1}{2}. \end{array} \quad \begin{array}{l} (15) \\ \$3 \cdot 07\frac{1}{2} \times 2103 = \$6466 \cdot 72\frac{1}{2}. \end{array}$$

(16)

10s.	$\frac{1}{2}$	2096							
		<u>3</u>							
		6288	0	0	=	cost of 2096 oz. at £3.			
5s.	$\frac{1}{2}$	1048	0	0	=	"	"	at 0 10s.	
2s. 6d.	$\frac{1}{2}$	524	0	0	=	"	"	at 0 5s.	
1s. 3d.	$\frac{1}{2}$	262	0	0	=	"	"	at 0 2s. 6d.	
1½d.	$\frac{1}{10}$	131	0	0	=	"	"	at 0 1s. 3d.	
		13	2	0	=	"	"	at 0 0 1½d.	
		£8266	2	0	=	"	"	at £3 18s. 10½d.	

(17)

10 dwt.	$\frac{1}{2}$	\$1.55							
		<u>6</u>							
		\$9.30	=	cost of 6 oz.					
5 dwt.	$\frac{1}{2}$	·77½	=	"	10 dwt.				
2 dwt. 12 grs.	$\frac{1}{2}$	·38¾	=	"	5 dwt.				
1 dwt. 6 grs.	$\frac{1}{2}$	·19¾	=	"	2 dwt. 12 grs.				
2 grs.	$\frac{1}{16}$	·09½	=	"	1 dwt. 6 grs.				
		·00¾	=	"	2 grs.				
		\$10.75¾	=	cost of 6 oz. 18 dwt. 20 grs.					

(18)

10s.	$\frac{1}{2}$	£98	0	0	=	cost of 98 yards at £1.			
5s.	$\frac{1}{2}$	49	0	0	=	"	"	0 10s.	
		24	10	0	=	"	"	0 5s.	
		£171	10	0	=	cost of 98 yards at £1 15s.			
2 qrs.	$\frac{1}{2}$	£1	15						
1 qr.	$\frac{1}{2}$	17	6	=	cost of 2 qrs.				
1 na.	$\frac{1}{4}$	8	9	=	"	1 qr.			
		2	2½	=	"	1 na.			

£1 8 5¼ = cost of 3 qrs. 1 na.

Then £171 10 0 = cost of 98 yards at £1 15s.

1 8 5¼ = cost of 3 qrs. 1 na. at £1 15s. per yard.

£172 18 5¼ = cost of 98 yds. 3 qrs. 1 na. at £1 15s per yd.

(19)

1s.	$\frac{1}{20}$	344							
		4							
		<hr/>							
		£1376	0	0	=	rent of 344 acres at £4			
1d.	$\frac{1}{12}$	17	4	0	=	"	"	at 0 1s.	
		1	8	8	=	"	"	at 0 0 1d.	
		<hr/>							
		£1394	12	8	=	rent of 344 acres at £4 1s. 1d.			

2 r.	$\frac{1}{2}$	£4	1	1					
		<hr/>							
1 r.	$\frac{1}{2}$	2	0	6 $\frac{1}{2}$	=	rent of 2 roods.			
10 per.	$\frac{1}{4}$	1	0	3 $\frac{1}{4}$	=	" 1 rood.			
5 per.	$\frac{1}{2}$	5	0 $\frac{1}{2}$	3 $\frac{1}{2}$	=	" 10 perches.			
		2	6 $\frac{1}{2}$	3 $\frac{1}{2}$	=	" 5 perches.			
		<hr/>							
		£3	8	4 $\frac{3}{4}$	=	" 3 roods 15 perches.			

£1394 12 8 = rent of 344 acres at £4 1s. 1d.

3 8 4 $\frac{3}{4}$ = " 3 roods 15 per. at £4 1s. 1d. per ac.£1398 1 0 $\frac{3}{4}$ = " 344 a. 3 r. 15 per. at £4 1s. 1d.

(20)

5 dwt.	$\frac{1}{4}$	5	10						
		5							
		<hr/>							
		£1	9	2	=	price of 5 oz. at 5s. 10d. per oz.			
1 dwt.	$\frac{1}{8}$	1	5 $\frac{1}{2}$	=	" 5 dwt.	"	"		
12 grs.	$\frac{1}{2}$	3 $\frac{1}{2}$	=	" 1 dwt.	"	"			
4 grs.	$\frac{1}{3}$	1 $\frac{1}{2}$	=	" 12 grs.	"	"			
1 gr.	$\frac{1}{4}$	0 $\frac{7}{8}$	=	" 4 grs.	"	"			
		0 $\frac{7}{8}$	=	" 1 gr.	"	"			
		<hr/>							
		£1	11	1 $\frac{23}{8}$	=	" 5 oz. 6 dwt. 17 grs. at			
						5s. 10d. per oz.			

(21)

2 grs.	$\frac{1}{2}$	£1 2 4			
		4			
		<hr/>			
		£4 9 4	=	price of 4 yards at £1 2 4 per yard.	
2 na.	$\frac{1}{4}$	11 2	=	" 2 qrs.	" "
1 na.	$\frac{1}{2}$	2 9 $\frac{1}{2}$	=	" 2 na.	" "
		1 4 $\frac{3}{4}$	=	" 1 na.	" "
		<hr/>			
		£5 4 8 $\frac{1}{4}$	=	price of 4 yds. 2 qrs. 3 na.	" "

(22)

1 rood	$\frac{1}{4}$	£1 16			
		32			
		<hr/>			
		£57 12 0	=	price of 32 acres at £1 16s.	
10 per.	$\frac{1}{4}$	9 0	=	" 1 rood.	"
2 per.	$\frac{1}{8}$	2 3	=	" 10 per.	"
2 per.	$\frac{1}{8}$	5 $\frac{2}{5}$	=	" 2 per.	"
		5 $\frac{2}{5}$	=	" 2 per.	"
		<hr/>			
		£58 4 1 $\frac{1}{5}$	=	price of 32 acres 1 rood 14 per.	

(23)

4 pts.	$\frac{1}{2}$	7 6			
		3			
		<hr/>			
		£1 2 6	=	price of 3 gals. at 7s.6d. per gal.	
1 pt.	$\frac{1}{4}$	3 9	=	" 4 pts.	"
		11 $\frac{1}{4}$	=	" 1 pt.	"
		<hr/>			
		£1 7 2 $\frac{1}{4}$	=	price of 3 gals. 5 pts.	

(24)

$$\$1.67\frac{1}{2} \times 724 = \$1212.70.$$

(25)

$$\$1.93\frac{3}{4} \times 721 = \$1396.93\frac{3}{4}.$$

(26)

10s.	$\frac{1}{2}$	4514							
		2							
		<hr/>							
		£9028	0	0	=	cost of 4514 rods at £2			
6s. 8d.	$\frac{1}{3}$	2257	0	0	=	"	"	at 0 10	
10d.	$\frac{1}{8}$	1504	13	4	=	"	"	at 0 6 8	
1d.	$\frac{1}{10}$	188	1	8	=	"	"	at 0 0 10	
$\frac{1}{2}$ d.	$\frac{1}{2}$	18	16	2	=	"	"	at 0 0 1	
		9	8	1	=	"	"	at 0 0 0 $\frac{1}{2}$	
		<hr/>							
		£13005	19	3	=	"	"	at £2 17 7 $\frac{1}{2}$	

(27)

10s.	$\frac{1}{2}$	3749	7	6					
		3							
		<hr/>							
		£11248	2	6	=	price of 3749 $\frac{3}{8}$ acres at £3			
5s.	$\frac{1}{2}$	1874	13	9	=	"	"	at 0 10	
6d.	$\frac{1}{10}$	937	6	10 $\frac{1}{2}$	=	"	"	at 0 5	
		93	14	8 $\frac{1}{4}$	=	"	"	at 0 0 6	
		<hr/>							
		£14153	17	9 $\frac{3}{4}$	=	price of 3749 $\frac{3}{8}$ acres at £3 15 6			

(28)

4s.	$\frac{1}{5}$	£17	0	0	=	cost of 17 cwt. at £1			
8d.	$\frac{1}{6}$	3	8	0	=	"	"	at 0 4	
1d.	$\frac{1}{8}$	11	4		=	"	"	at 0 0 8	
		1	5		=	"	"	at 0 0 1	
		<hr/>							
		£21	0	9	=	cost of 17 cwt. at £1 4 9			
1 qr.	$\frac{1}{4}$	£1	4	9					
		<hr/>							
16 lbs.	$\frac{1}{7}$	6	2 $\frac{1}{4}$		=	cost of 1 qr.			
1 lb.	$\frac{1}{16}$	3	6 $\frac{3}{4}$		=	" 16 lbs.			
		0	2 $\frac{7}{11\frac{1}{2}}$		=	" 1 lb.			
		<hr/>							
		9	11 $\frac{37}{11\frac{1}{2}}$		=	" 1 qr. 17 lbs.			
£21	0	9			=	cost of 17 cwt. at £1 4s. 9d. per cwt.			
	9	11 $\frac{37}{11\frac{1}{2}}$			=	" 1 qr. 17 lbs.		"	"
<hr/>									
£21	10	8 $\frac{37}{11\frac{1}{2}}$			=	" 17cwt. 1qr. 17lbs.		"	"

(29)

2 qrs.	$\frac{1}{2}$	\$11.55			
		78			
		<hr/>			
		9240			
		8085			
		<hr/>			
		\$900.90	=	cost of 78 cwt. at \$11.55 per cwt.	
1 qr.	$\frac{1}{2}$	5.77 $\frac{1}{2}$	=	" 2 qrs.	" "
7 lbs.	$\frac{1}{4}$	2.88 $\frac{3}{4}$	=	" 1 qr.	" "
4 lbs.	$\frac{1}{8}$.72 $\frac{3}{16}$	=	" 7 lbs.	" "
1 lb.	$\frac{1}{4}$.41 $\frac{1}{4}$	=	" 4 lbs.	" "
		.10 $\frac{5}{16}$	=	" 1 lb.	" "
		<hr/>			
		\$910.80	=	cost of 78 cwt. 3 qrs. 12 lbs.	

(30)

£10 10

20

 £210 0 = price of 20 tons at £10 10s.

19 cwt. 3 qrs. 27 $\frac{1}{2}$ lbs. = 1 ton — $\frac{1}{2}$ lb. The price of 1 ton is £10 10s., and the price of $\frac{1}{2}$ lb. = $\frac{1}{4480}$ of £10 10s. = $\frac{63}{112}$ d. ∴ the price of 19 cwt. 3 qrs. 27 $\frac{1}{2}$ lbs. = £10 10s. — $\frac{63}{112}$ d. = £10 9s. 11 $\frac{49}{112}$ d.

£210 0 0 = price of 20 tons at £10 10s.

10 9 11 $\frac{49}{112}$ = " 19 cwt. 3 qrs. 27 $\frac{1}{2}$ lbs.

 £220 9 11 $\frac{49}{112}$ = price of 20 tons 19 cwt. 3 qrs. 27 $\frac{1}{2}$ lbs. at £10 10s. per ton,

(31)

10 cwt.	$\frac{1}{2}$	\$45.50			
		219			
		<hr/>			
		40950			
		4550			
		9100			
		<hr/>			
		\$9964.50	=	price of 219 tons at \$45.50 per ton.	
5 cwt.	$\frac{1}{2}$	22.75	=	"	10 cwt. " "
1 cwt.	$\frac{1}{5}$	11.37 $\frac{1}{2}$	=	"	5 cwt. " "
2 qrs.	$\frac{1}{2}$	2.27 $\frac{1}{2}$	=	"	1 cwt. " "
1 qr.	$\frac{1}{2}$	1.13 $\frac{3}{4}$	=	"	2 qrs. " "
		.56 $\frac{7}{8}$	=	"	1 qr. " "

\$10002.60 $\frac{5}{8}$ = price of 219 tons 16 cwt. 3 qrs.

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BILLS OF PARCELS.

(No. 2.)

	s.	d.		£	s.	d.
9 pair of worsted stockings, at.....	4	6	per pair	2	0	6
6 pair of silk ditto, at	15	9	"	4	14	6
17 pair of thread ditto, at.....	5	4	"	4	10	8
23 pair of cotton ditto, at.....	4	10	"	5	11	2
14 pair of yarn ditto, at	2	4	"	1	12	8
18 pair of women's silk gloves, at...	4	2	"	3	15	0
19 yards of flannel, at	1	7½	per yard	1	10	10½

Ans. £23 15 4 $\frac{1}{2}$

(No. 3.)

75 $\frac{1}{2}$ lbs. of sugar, at.....	7 $\frac{3}{4}$	cents per lb.	\$5.85 $\frac{1}{8}$
63 lbs. of tea, at.....	93	"	58.59
126 lbs. of butter, at.....	13	"	16.38
35 $\frac{1}{2}$ lbs. of raisins, at	18 $\frac{1}{2}$	"	6.71 $\frac{1}{4}$
17 lbs. of sago, at	15	"	2.55
23 lbs. of rice, at	9	"	2.07
58 $\frac{1}{2}$ lbs. of starch, at	22	"	12.87

Ans. \$105.02 $\frac{3}{8}$

(No. 4.)

198 Sangster's National Arithmetic, at.....	\$0·60	\$118·80
197 Robertson's Philosophy of Grammar, at ...	0·50	98·50
83 Hodgins' Geography, at	1·00	83·00
57 Sangster's Algebraic Formula, at	0·12½	7·12½
217 Strachan's Canadian Penmanship, at.....	0·37½	81·37½
143 Hodgins' Geography of British Provinces, at	0·45	64·35
227 Sangster's First Arithmetic, at.....	0·30	68·10

Ans. \$521·25

(No. 5.)

	s.	d.	£	s.	d.
9½ yards of silk, at	12	9 per yard	6	1	1½
13 yards of flowered ditto, at...15	6	"	10	1	6
11½ yards of lustring, at.....	6	10	4	0	3½
14 yards of brocade, at.....11	3	"	7	17	6
12½ yards of satin, at	10	8	6	10	8
11½ yards of velvet, at	18	0	10	4	9

Ans. £44 15 10

(No. 6.)

14 oz. ipecacuanha, at	0·67	9·38
23 " laudanum, at.....	0·89	20·47
17 " emetic tartar, at.....	1·25	21·25
25 " cantharides, at	2·17	54·25
27 " gum mastic, at	0·61	16·47
56 " gum camphor, at	0·27	15·12

Ans. \$136·94

(No. 7.)

	s.	d.	£	s.	d.
15½ lbs. of currants, at	0	4 per lb.	5	2	
17½ lbs. of Malaga raisins, at.....	0	5½ "	7	10½	
19½ lbs. of sun raisins, at	0	6 "	9	10½	
17 lbs. of rice, at	0	3½ "	4	11½	
8½ lbs. of pepper, at.....	1	6 "	12	9	
3 loaves of sugar, weight 32½ lbs. at.	0	8½ "	1	3	0½
13 oz. of cloves, at.....	0	9 per oz.	9	9	

Ans. £3 13 5

(4)

Dissimilar.		Similar.		Similar and Coterminous.
$73.\dot{7}2\dot{3}$	=	$73.\dot{7}23\dot{7}2\dot{3}$	=	$73.\dot{7}23\dot{7}23\dot{7}2\dot{3}$
$11.\dot{3}4\dot{2}$	=	$11.\dot{3}42\dot{2}$	=	$11.\dot{3}42\dot{2}222\dot{2}$
$16.\dot{7}1\dot{3}$	=	$16.\dot{7}13\dot{0}$	=	$16.\dot{7}1300000\dot{0}$
$19.\dot{0}3\dot{4}$	=	$19.\dot{0}340\dot{3}4$	=	$19.\dot{0}340\dot{3}40\dot{3}4$
$713.\dot{2}134\dot{3}7$	=	$713.\dot{2}134\dot{3}7$	=	$713.\dot{2}134\dot{3}721\dot{3}$
$12.\dot{3}456\dot{7}8$	=	$12.\dot{3}456\dot{7}834\dot{5}$	=	$12.\dot{3}456\dot{7}834\dot{5}$
				2 carried.
		Sum	=	$846.\dot{3}72095\dot{7}6\dot{3}$

(5)

$$\begin{array}{l}
 5:7 = 5 \div 7 = .714 + \\
 9:13 = 9 \div 13 = .692 + \\
 12:17 = 12 \div 17 = .705 + \\
 7:10 = 7 \div 10 = .7
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{Hence } 5:7 \text{ is the greatest,} \\ \text{and } 9:13 \text{ least.} \end{array}$$

$$\frac{5}{7} \times \frac{9}{13} \times \frac{12}{17} \times \frac{7}{10} = \frac{54}{221} = 54:221.$$

(6)

1 acre = 160 rods, and 25 acres 2 roods 35 rods = 4115 rods.

$$\begin{array}{r}
 \text{rods.} \quad \quad \quad 40 \cdot 25 \quad 83 \\
 160 : 4115 : \$80 \cdot 50 : \frac{80 \cdot 50 \times 415}{160} = \$2070 \cdot 3593.
 \end{array}$$

(8)

$$\$3 \cdot 68\frac{1}{2} \times 7439 = \$27431 \cdot 31\frac{1}{2}.$$

(9)

$\frac{135795}{222210}$. The G. C. M. of 135795 and 222210 is 12345; when both terms of the fraction are divided by 12345, it becomes $\frac{11}{18}$.

$\frac{714235}{999999}$. Here 714235 and 999999 have no G. C. M.; \therefore the fraction cannot be reduced.

$\frac{109375}{100000}$. The G. C. M. of 109375 and 100000 is 3125; when both terms of the fraction are divided by 3125, it becomes reduced to $\frac{35}{32}$.

$\frac{20301}{33633}$. The G. C. M. of 20301 and 33633 is 303; when both terms of the fraction are divided by 303, it is reduced to its lowest terms, viz., $\frac{67}{111}$.

(10)

$$\left. \begin{array}{ll} 34\frac{1}{2} \text{ bushels turnips} & = 17 \text{ bushels potatoes} \\ 9 \text{ " potatoes} & = 59\frac{1}{2} \text{ lbs. tea} \\ 6 \text{ lbs. tea} & = 11\frac{1}{2} \text{ stone flour} \\ 13 \text{ stone flour} & = 360 \text{ cents} \\ 38 \text{ cents} & = 12 \text{ loaves} \\ 119 \text{ loaves} & = x \text{ bushels turnips} \end{array} \right\} =$$

$$\frac{3}{34\frac{1}{2}} \times \frac{9}{59\frac{1}{2}} \times \frac{6}{11\frac{1}{2}} \times \frac{13}{360} \times \frac{19}{12} \times \frac{7}{119} = \frac{3 \times 13 \times 19}{8\frac{1}{2} \times 40} = 2\frac{61}{40}.$$

(11)

$$\left. \begin{array}{l} 54 : 27 \text{ men} \\ 11 : 8 \text{ hours} \\ 42 : 77 \text{ floors} \\ 20 : 24 \text{ feet long} \\ 16 : 22 \text{ feet wide} \\ 3 : 5 \text{ coats paint} \end{array} \right\} \therefore 7 \text{ days : } \frac{7 \times 27 \times 8 \times 77 \times 24 \times 22 \times 5}{54 \times 11 \times 42 \times 20 \times 16 \times 3} = \frac{7 \times 11}{2 \times 3} = 12\frac{5}{6} \text{ days.}$$

(13)

$$\begin{array}{r} \text{IX.} \\ 12)72342 \\ \hline \end{array}$$

$$12)5403..2$$

$$12)407..0$$

$$12)30..7$$

$$2..3$$

$$\begin{array}{r} \text{IX.} \\ 6)72342 \\ \hline \end{array}$$

$$6)11806..2$$

$$6)1731..0$$

$$6)264..4$$

$$6)40..4$$

$$6)6..0$$

$$1..0$$

$$\begin{array}{r} \text{IX.} \\ 3)72342 \\ \hline \end{array}$$

$$3)23713..2$$

$$3)7234..0$$

$$3)2371..1$$

$$3)723..1$$

$$3)237..0$$

$$3)72..1$$

$$3)23..2$$

$$3)7..0$$

$$2..1$$

IX.		XII.		VI.		III.	
72342	=	23702	=	1004402	=	2102101102	
9		12		6		3	
<u> </u>		<u> </u>		<u> </u>		<u> </u>	
65		27		6		7	
9		12		6		3	
<u> </u>		<u> </u>		<u> </u>		<u> </u>	
588		331		36		21	
9		12		6		3	
<u> </u>		<u> </u>		<u> </u>		<u> </u>	
5296		3972		220		65	1765
9		12		6		3	3
<u> </u>		<u> </u>		<u> </u>		<u> </u>	<u> </u>
47666		47666		1324		196	5296
				6		3	3
				<u> </u>		<u> </u>	<u> </u>
				7944		588	15888
				6		3	3
				<u> </u>		<u> </u>	<u> </u>
				47666		1765	47666

(14)

II.	II.	IV.	IV.
111111	100000	333333	100000
2	2	4	4
—	—	—	—
3	2	15	4
2	2	4	4
—	—	—	—
7	4	63	16
2	2	4	4
—	—	—	—
15	8	255	64
2	2	4	4
—	—	—	—
31	16	1023	256
2	2	4	4
—	—	—	—
63 Greatest.	32 Least.	4095 Greatest.	1024 Least.

VI.	VI.	VIII.	VIII.
555555	100000	777777	100000
6	6	8	8
—	—	—	—
35	6	63	8
6	6	8	8
—	—	—	—
215	36	511	64
6	6	8	8
—	—	—	—
1295	216	4095	512
6	6	8	8
—	—	—	—
7775	1296	32767	4096
6	6	8	8
—	—	—	—
46655 Greatest.	7776 Least.	262143 Greatest.	32768 Least.

(Continued on next page.)

(14 continued.)

XII	XII
<i>eeeeee</i>	•100000
12	12
<hr/>	<hr/>
143	12
12	12
<hr/>	<hr/>
1727	144
12	12
<hr/>	<hr/>
20735	1728
12	12
<hr/>	<hr/>
248831	20736
12	12
<hr/>	<hr/>
2985983 Greatest.	248832 Least.

(15)

$$1728 = 2^6 \times 3^3.$$

1..2..4..8..16..32..64

1..3..9..27

1..2..4.. 8..16..32..64..3..6..12..24..48..96..192 .. 9 ..
 18..36..72..144..288..576..27..54..108..216..432 .. 864 ..
 1728.

Therefore the divisors of 1728 are 1, 2, 3, 4, 6, 8, 9, 12, 16,
 18, 24, 27, 32, 36, 48, 54, 64, 72, 96, 108, 144, 192, 216, 288, 432,
 576, 864, 1728.

(16)

30		2..4..6..8..10..12..14..16..18..20..22..24..26..28..30
14		2 4 2.. 7.. 8.. 3.. 2..11.. 4..13..14
12		2 4.. 3 11.. 2..13
143		11 13

$$30 \times 14 \times 12 \times 143 = 720720 = 1. \text{ m. c.}$$

(17)

Dissimilar.		Similar.		Similar and Coterminous.
97·91342	=	97·913423	=	97·913423423423423
18·1234567	=	19·1234567	=	18·123456745674567
				<hr/>
		Difference	=	18·789966677748855

(18)

20 ft. 7'	
19 ft. 5 7"	
<hr/>	
1 0 0 1'''	
8 6 11	
391 1	
<hr/>	
400 7 11 1	= 44 sq. yds. + $\frac{4}{9}$ + $\frac{7}{108}$ + $\frac{11}{1296}$ + $\frac{1}{18662} =$
	$44\frac{8053}{18662}$ sq. yds. = 44·517 + sq. yds.
	$\$2\cdot87\frac{1}{2} \times 44\cdot517 = \$127\cdot98 +.$

(19)

916 acres 3 roods 17 per 7 yds. = 4437591 $\frac{1}{4}$ sq. yds., and 43 acres 1 rood 2 per 17 yds. = 209407 $\frac{1}{2}$ sq. yds.
 $4437591\frac{1}{4} \div 209407\frac{1}{2} = 4437591\cdot25 \div 207407\cdot5 = 21\cdot19117+.$

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(14)

(15)

$$\$742\cdot10 \times \cdot05 = \$37\cdot10\frac{1}{2}. \quad \$1000 \times \cdot11 = \$110.$$

(16)

$$\$734\cdot19 \times \cdot10 = \$73\cdot419.$$

(17)

$$\$1624\cdot50 \times \cdot875 = \$1421\cdot4375.$$

(18)

$$\$994.70 \times .125 = \$124.3375.$$

(19)

$$\$777.50 \times .0875 = \$68.03125, \text{ or } \$68.03\frac{1}{2}.$$

(20)

$$\$7135.80 \times .0225 = \$160.5555.$$

(21)

$$2740 \times .20 = 548.$$

(22)

$$\$7490 \times .10 = \$749$$

$$\$7490 \times .17 = \$1273.30$$

$$\$7490 \times .27 = \$2022.30$$

$$\$7490 \times .46 = \$4445.40$$

(23)

$$\$740 \times .045 = \$33.30$$

$$\$1680 \times .025 = \$42.00$$

$$\$42.00 - \$33.30 = \$8.70$$

(24)

$$729 \times .11 = 80.19$$

$$729 - 80.19 = 648.81 = 648\frac{81}{100}.$$

(25)

$$\$763.22 \times 25 = \$190.8050$$

$$\$847.16 \times 16 = 135.5456$$

$$\$1234.17 \times .0625 = 77.135625$$

$$\text{Sum} = \$403.486225$$

(26)

$$\$17429.40 \times .43 = \$7494.64\frac{1}{2}$$

$$\$17429.40 \times .37 = 6448.87\frac{1}{2}$$

$$\$13943.52$$

$$\$17429.40 - \$13943.52 = \$3485.88.$$

(27)

$$68978 \times .36 = 24832.08.$$

(28)

$$29800 \times .17 = 5066$$

$$29800 - 5066 = 24734$$

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(3)

$$\$1000 \times .045 = \$45.$$

(4)

$$\$1678.30 \times .0225 = \$37.76175.$$

(5)

(6)

$$\$7531.19 \times .0375 = \$282.419625, \quad \$508.60 \times .0125 = \$6.3575$$

(7)

(8)

$$\$7863.50 \times .0175 = \$137.61125, \quad \$878.30 \times .025 = \$21.9575$$

(9)

(10)

$$\$7193.16 \times .03125 = \$224.78625, \quad \$6734.10 \times .17 = \$1144.797.$$

(11)

$$\$7.13 \times 718 \div .0425 = \$217.57195.$$

(12)

$$\$1.85 \times 8243 \times .05625 = \$857.7871875.$$

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(13)

(14)

$$\$7893.87 \times .02 = \$157.8774, \quad \$8000 \times .00875 = \$70.$$

(15.)

$$\$8643.22 \times .0125 = \$108.04025.$$

(16.)

$$\$78963.80 \times .00875 = \$690.93325.$$

(17)

$$\$1987.27 \times .0375 = \$74.522625.$$

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(19)

$$\$4000 \div 1.0125 = \$3950.61728 + = \text{sum to be invested.}$$

$$\$4000 - 3950.61728 = \$49.38271 = \text{commission.}$$

(20)

$$\$7500 \div 1.045 = \$7177.03349 = \text{sum to be expended in laces.}$$

$$\$7500 - \$322.96651 = \$322.96651 = \text{commission.}$$

(21)

$$\$8470 \div 1.05 = \$8066.66\frac{2}{3} = \text{sum to be invested.}$$

$$\$8066.66\frac{2}{3} \div \$6.40 = 1260\frac{5}{12}. \text{ Ans.}$$

(22)

$$\$11000 \div 1.00875 = \$10904.584882 = \text{sum to be invested.}$$

(23)

$$\$13000 \div 1.045 = \$12440.1913 + = \text{sum to be invested.}$$

$$\$13000 - \$12440.1913 = \$559.8086 + = \text{commission.}$$

$$\$12440.1913 + \div \$3.63 = 3427.0499 \text{ yds. Ans.}$$

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(4)

(5)

$$\$9000 \div 0.83 = \$10843.373. \quad \$8500 \div 1.11 = \$7657.6576.$$

(6)

$$\$17500 \div 1.0125 = \$17283.951 = \text{amount to be invested.}$$

$$\$17283.951 \div 1.07 = \$16153.22 = \text{stock.}$$

(7)

 $\$20000 \div 1.0175 = \$19656.01965 = \text{amount to be invested.}$ $\$19656.01985 \div 0.97 = \$20263.937 = \text{stock remitted.}$

(8)

 $\$200 \times 100 = \$20000 = \text{par value of 200 shares.}$ $\$1 \text{ stock costs } \$1.055. \$1.055 \times 20000 = \$21100 = \text{cost of stock.}$ $\$21100 \times .00875 = \$184.625 = \text{brokerage.}$ $\$21100 + \$184.625 = \$21284.625 = \text{whole cost.}$

 Page 240.

(2)

 $\$7500 \times .0175 = \$131.25.$

(3)

 $\$8375 \times .0075 = \$62.8125.$

(4)

 $\$6000 \times .01875 = \$112.50.$

(5)

 $\$5000 \times .0117 = \$58.50.$

(6)

 $\$6400 \times .0090 = \$57.60.$

(7)

 $\$4500 \times .0035 = \$15.75.$

(8)

 $\$36000 \times .03 = \$1080.$

(9)

 $\$27000 \times 4.82 \times 4 = \$5205.60.$

(10)

 $\$39000 \times .022 = \$858.$

(11)

 $\$17800 \times .005 = \$89.$

(12)

 $\$12350 \times .01\frac{1}{2} \times 7 = \$1235.$

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(15)

$$\$17000 \div .965 = \$17616.58.$$

(16)

$$\$22750 \div .94 = \$24202.127.$$

(17)

$$\$15000 \div .9775 = \$15345.2685.$$

(18)

$$\$33000 \div .9425 = \$35013.2625.$$

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(3)

$$1347 \times 5 = 6735 \text{ lbs.} = \text{gross weight.}$$

$$6735 \times .06 = 404.1 \text{ lbs. tare.}$$

$$6330.9 \text{ lbs.} = \text{net at } 3\frac{1}{2} \text{ cents per lb.} = 6330.9 \\ \times 3\frac{1}{2} = \$221.58.$$

(4)

$$127 \times 11 = 1397 \text{ lbs.} = \text{gross weight.}$$

$$1397 \times .03 = 41.91 \text{ lbs.} = \text{tare.}$$

$$1355.09 \text{ lbs.} = \text{net at } \$\cdot012 \text{ per lb.} = 1355.09 \\ \times .012 = \$16.26.$$

(5)

$$129 \times .13 = \$16.77.$$

(6)

$$31 \times 207 = 6417 \text{ lbs.} = \text{gross weight.}$$

$$207 \times 2\frac{1}{4} = 465\frac{3}{4} \text{ lbs.} = \text{tare.}$$

$$5951\frac{1}{4} \text{ lbs.} = \text{net at } 5\frac{3}{4} \text{ cents per lb.} = 5951\frac{1}{4} \times \\ 5\frac{3}{4} = \$342.1968.$$

(7)

$$214 \times .47 = \$100.58.$$

(10)

(11)

$$\$17429.80 \times .21 = \$3660.2580. \quad \$2920.16 \times .075 = \$219.012.$$

(12)

(13)

$$\$71342.90 \times .25 = \$17835.725. \quad \$913.73 \times .2 = \$182.746.$$

(14)

$$\$14713.19 \times .33 = \$4855.3527.$$

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(2)

$$\$23900 \div 7142300 = \$0.0033462 = \text{rate per dollar.}$$

$$\$0.0033462 \times 14729.50 = \$49.2878 +. \text{ Ans.}$$

(3)

$$\$100000 \div 5793000 = \$0.017262 = \text{rate per dollar.}$$

$$\$0.017262 \times 18600 = \$321.0732. \text{ Ans.}$$

(4)

$$\$100000 \div 5793000 = \$0.017262 = \text{rate per dollar.}$$

$$\$0.017262 \times 7500 = \$129.465. \text{ Ans.}$$

(5)

$$\$100000 \div 5793000 = \$0.017262 = \text{rate per dollar.}$$

$$\$0.017262 \times 11400 = \$196.7868. \text{ Ans.}$$

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(13)

Here $P = \$723.19$, $r = .067$, and $t = 7.32$.Then $I = Prt = 723.19 \times .067 \times 7.32 = \354.6813036 .

(14)

Here $P = \$857.19$, $r = .065$, and $t = 6\frac{1}{2}$ or 6.5 .Then $A = P(1 + rt) = \$857.19 \times 1.4225 = \1219.352775 .

(15)

Here $t = 11$, and $r = .725$.Then $n = tr + 1 = 11 \times .725 + 1 = 8.975$.

(16)

Here $P = \$654.32$, $I = \$234.56$, and $r = .07$.
$$\text{Then } t = \frac{I}{Pr} = \frac{234.56}{654.32 \times .07} = 5.12112 \text{ or } 5 \text{ years } 1 \text{ m. } 13 \text{ d.}$$

(17)

Here $A = \$1200$, $P = \$700$, and $t = 5$.
$$\text{Then } r = \frac{A - P}{Pt} = \frac{1200 - 700}{700 \times 5} = \frac{1}{7} = \text{rate per unit } \therefore 14\frac{2}{7} =$$

rate per cent.

(18)

Here $n = 4$, and $r = .23$.
$$\text{Then } t = \frac{n-1}{r} = \frac{4-1}{.23} = 13 \text{ years } 15 \text{ days.}$$

(19)

Here $P = \$270$, $I = \$87$, and $r = .07$.
$$\text{Then } t = \frac{I}{Pr} = \frac{87}{270 \times .07} = 4 \text{ years } 7\frac{5}{7} \text{ months.}$$

(20)

Here $P = \$680$, $t = 11\frac{1}{2}$, and $r = .11$.Then $A = P(1 + rt) = 680 \times 2.265 = \1540.20 .

(21)

Here $A = \$2000$, $t = 20$, and $r = .08$.Then $P = \frac{A}{1 + rt} = \frac{2000}{2.6} = \$769.23\frac{1}{3}$.

(22)

Here $n = 21$, and $t = 24$.Then $r = \frac{n-1}{t} = \frac{21-1}{24} = .83\frac{1}{3} = \text{rate per unit. } \therefore 83\frac{1}{3} = \text{rate per cent.}$

(23)

Here $n = 23$, and $r = .16$.Then $t = \frac{n-1}{r} = \frac{23-1}{.16} = 137\frac{1}{2} \text{ years.}$

(24)

Here $P = \$679.18$, $r = .0775$, and $t = 11.73$.Then $I = Prt = 679.18 \times .0775 \times 11.73 = \617.4255 .

(25)

Here $P = \$950$, $A = \$1763.42$, and $t = 10$.Then $r = \frac{A - P}{Pt} = \frac{1763.42 - 950}{950 \times 10} = .08562 = \text{rate per unit}$
 $\therefore 8.562 = \text{rate per cent.}$

(26)

Here $P = \$666$, $A = \$1347.50$, and $r = .06$.

$$\text{Then } t = \frac{A - P}{Pr} = \frac{1347.50 - 666}{666 \times .06} = 17.054+ \text{ years, or 17 years 19 days.}$$

(27)

Here $P = \$273$, $I = \$100$, and $r = .09$.

$$\text{Then } t = \frac{I}{Pr} = \frac{100}{273 \times .09} = 4.07 \text{ years} = 4 \text{ years 25 days.}$$

(28)

Here $P = \$476.30$, $A = \$500$, and $t = 2$.

$$\text{Then } r = \frac{A - P}{Pt} = \frac{500 - 476.30}{476.30 \times 2} = .0248 = \text{rate per unit.}$$

$$\therefore 2\frac{1}{2}\% = \text{rate per cent.}$$

(29)

Here $P = \$749.49$, $I = \$257$, and $t = 7$.

$$\text{Then } r = \frac{I}{Pt} = \frac{257}{749.49 \times 7} = .04898 = \text{rate per unit.}$$

$$\therefore 4.898 = \text{rate per cent.}$$

(30)

Here $A = \$1111.11$, $t = 11$, and $r = .11$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1111.11}{2.21} = \$502.7647.$$

(31)

$P = £167.47$, $r = .11$, and $t = 9$.

$$I = Prt = 167.47 \times .11 \times 9 = £165.7953 = £165 \text{ 15s. } 10\frac{1}{2}\text{d.}$$

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(34)

$$11 \div 2 = 5\frac{1}{2} \text{ cents.}$$

(35)

$$16 \div 2 = 8 \text{ cents} = \$0.08.$$

(36)

$$9 \text{ years and 8 months} = 116 \text{ months, and } 116 \div 2 = 58 \text{ cents} \\ = \$0.58.$$

(37)

$$16 \text{ years and 3 months} = 195 \text{ months, and } 195 \div 2 = 97\frac{1}{2} \text{ cents} \\ = \$0.97\frac{1}{2}.$$

(38)

$$11 \text{ years and 7 months} = 139 \text{ months, and } 139 \div 2 = 69\frac{1}{2} \text{ cents} \\ = \$0.695.$$

(39)

$$12 \text{ years and 5 months} = 149 \text{ months, and } 149 \div 2 = 74\frac{1}{2} \text{ cents} \\ = \$0.745.$$

(40)

$$3 \text{ years and 2 months} = 38 \text{ months, and } 38 \div 2 = 19 \text{ cents} = \\ \text{interest of \$1 for given rate and time.}$$

$$\$0.19 \times 279.40 = \$53.086.$$

(41)

$$6 \text{ years and 7 months} = 79 \text{ months, and } 79 \div 2 = 39\frac{1}{2} \text{ cents} = \\ \text{interest of \$1 for given rate and time.}$$

$$\$0.395 \times 189.70 = \$74.9315.$$

(42)

3 years and 11 months = 47 months, and $47 \div 2 = 23\frac{1}{2}$ cents = interest of \$1 for given rate and time.

$$\$0.235 \times 1463 = \$343.805.$$

(43)

11 years and 1 month = 133 months, and $133 \div 2 = 66\frac{1}{2}$ cents = interest of \$1 for given rate and time.

$$\$0.665 \times 28967.50 = \$19263.3875.$$

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(45)

(46)

$$2 \div 6 = \frac{1}{3} \text{ mill} = \$0.0003.$$

$$7 \div 6 = 1\frac{1}{6} \text{ mills} = \$0.001\frac{1}{6}.$$

(47)

(48)

$$11 \div 6 = 1\frac{5}{6} \text{ mills} = \$0.001\frac{5}{6}. \quad 27 \div 6 = 4\frac{1}{2} \text{ mills} = \$0.004\frac{1}{2}.$$

(49)

$$47 \div 6 = 7\frac{5}{6} \text{ mills} = \$0.007\frac{5}{6}.$$

(50)

$$8 \div 2 = 4 \text{ cents} = \$0.04.$$

$$12 \div 6 = 2 \text{ mills} = \$0.002 \text{ and } \$0.04 + \$0.002 = \$0.042.$$

(51)

$$66 \div 6 = 11 \text{ mills} = \$0.011.$$

(52)

$$2 \text{ years } 2 \text{ m'ths} = 26 \text{ months, and } 26 \div 2 = 13 \text{ cents} = \$0.13.$$

$$19 \div 6 = 3\frac{1}{6} \text{ mills} = \$0.003\frac{1}{6} \text{ and } \$0.13 + \$0.003\frac{1}{6} = \$0.133\frac{1}{6}.$$

(53)

7 years 8 m'ths = 92 months, and $92 \div 2 = 46$ cents = \$0.46.
 $9 \div 6 = 1\frac{1}{2}$ mills = \$0.001 $\frac{1}{2}$ and \$0.46 + \$0.001 $\frac{1}{2}$ = \$0.461 $\frac{1}{2}$.

(54)

17 years 11 months = 215 months, and $215 \div 2 = 107\frac{1}{2}$ cents =
 \$1.075.

$23 \div 6 = 3\frac{5}{6}$ mills = \$0.003 $\frac{5}{6}$, and \$1.075 + \$0.003 $\frac{5}{6}$ = \$1.078 $\frac{5}{6}$.

(55)

12 years 7 months = 151 months, and $151 \div 2 = 75\frac{1}{2}$ cents =
 \$0.755.

$17 \div 6 = 2\frac{5}{6}$ mills = \$0.002 $\frac{5}{6}$, and \$0.755 + \$0.002 $\frac{5}{6}$ = \$0.757 $\frac{5}{6}$.

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(57)

Interest on \$1 for 7 months	= \$0.035
Interest on \$1 for 17 days	= <u>2$\frac{5}{6}$</u>

Therefore interest on \$1 for 7 months 17 days, = \$0.037 $\frac{5}{6}$

Then \$0.037 $\frac{5}{6}$ \times 917.30 = \$34.704516.

(58)

Interest on \$1 for 3 months	= \$0.015
Interest on \$1 for 13 days	= <u>2$\frac{1}{6}$</u>

Therefore interest on \$1 for 3 months 13 days = \$0.017 $\frac{1}{6}$

Then \$0.017 $\frac{1}{6}$ \times 842.50 = \$14.462916.

(59)

Interest on \$1 for 2 years 11 months	= \$0.175
Interest on \$1 for 10 days	= <u>13</u>

Therefore interest on \$1 for 2 yrs. 11 m'ths 10 days = \$0.176 $\frac{1}{3}$

Then \$0.176 $\frac{1}{3}$ \times 573.83 = \$101.3766.

(60)

Interest on \$1 for 6 years 9 months = \$0.405

Interest on \$1 for 19 days = 3 $\frac{1}{6}$ Therefore interest on \$1 for 6 years 9 m'ths 19 days = \$0.408 $\frac{1}{6}$ Then $\$0.408\frac{1}{6} \times 642.30 = \262.16545 .

(61)

Interest on \$1 for 5 years 5 months = \$0.325

Interest on \$1 for 7 days = 1 $\frac{1}{6}$ Therefore interest on \$1 for 5 years 5 months 7 days = \$0.326 $\frac{1}{6}$ Then $\$0.326\frac{1}{6} \times 1427.875 = \465.7252 .

(62)

Interest on \$1 for 4 years 7 months = \$0.275

Interest on \$1 for 16 days = 2 $\frac{2}{3}$ Therefore interest on \$1 for 4 years 7 m'ths 16 days = \$0.277 $\frac{2}{3}$ Then $\$0.277\frac{2}{3} \times 709.63 = 197.040596$.

(63)

Interest on \$1 for 7 years 7 months = \$0.455

Interest on \$1 for 22 days = 3 $\frac{1}{2}$ Therefore interest on \$1 for 7 years 7 m'ths 22 days = \$0.458 $\frac{1}{2}$ Then $\$0.458\frac{1}{2} \times 2463.20 = \$1129.7877 + \$2463.20 = \3592.9877 .

(64)

Interest on \$1 for 9 years 9 months = \$0.585

Interest on \$1 for 9 days = 1 $\frac{1}{2}$ Therefore interest on \$1 for 9 years 9 m'ths 9 days = \$0.586 $\frac{1}{2}$ Then $\$0.586\frac{1}{2} \times 999.99 = \586.494135 .

(65)

$$\begin{array}{rcl} \text{Interest on \$1 for 3 years 4 months} & = & \$0.20 \\ \text{Interest on \$1 for 27 days} & = & \underline{4\frac{1}{2}} \end{array}$$

$$\text{Therefore interest on \$1 for 3 years 4 m'ths 27 days} = \$0.204\frac{1}{2}$$

$$\text{Then } \$0.2045 \times 68.70 = \$14.04915.$$

(66)

$$\begin{array}{rcl} \text{Interest on \$1 for 3 years} & = & \$0.18 \\ \text{Interest on \$1 for 28 days} & = & \underline{4\frac{2}{3}} \end{array}$$

$$\text{Therefore interest on \$1 for 3 years 28 days} = \$0.184\frac{2}{3}$$

$$\text{Then } \$0.184\frac{2}{3} \times 742.63 = \$137.139.$$

(67)

$$\begin{array}{rcl} \text{Interest on \$1 for 7 years 4 months} & = & \$0.44 \\ \text{Interest on \$1 for 11 days} & = & \underline{1\frac{5}{6}} \end{array}$$

$$\text{Therefore interest on \$1 for 7 years 4 m'ths 11 days} = \$0.441\frac{5}{6}$$

$$\text{Then } \$0.441\frac{5}{6} \times 200 = \$88.366 + \$200 = \$288.366.$$

(68)

$$\begin{array}{rcl} \text{Interest on \$1 for 9 years 3 months} & = & \$0.555 \\ \text{Interest on \$1 for 9 days} & = & \underline{1\frac{1}{2}} \end{array}$$

$$\text{Therefore interest on \$1 for 9 years 3 months 9 days} = \$0.556\frac{1}{2}$$

$$\text{Then } \$0.5565 \times 743.63 = \$413.830095 + \$743.63 = \$1157.460095.$$

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(70)

$$\text{Interest on \$1 at 6 per cent. for given time} = \$0.526\frac{2}{3}.$$

$$\begin{array}{l} \text{Interest on \$1234.56 at 6 per cent. for given time} = \$0.526\frac{2}{3} \times \\ 1234.56 = \$650.2016. \end{array}$$

$$\begin{array}{l} \text{Hence interest on \$1234.56 at 7 per cent for given time} = \\ \$650.2016 + \text{one sixth of } \$650.2016 = \$758.5685. \end{array}$$

K

(71)

Interest on \$1 at 6 per cent. for given time = $\$0.126\frac{1}{2}$.

Interest on \$9876.54 at 6 per cent. for given time = $\$0.126\frac{1}{2} \times 9876.54 = \1252.67449 .

Hence interest on \$9876.54 at 3 per cent. for given time = $\$1252.67449 \div 2 = \626.337245 .

(72)

Interest on \$1 at 6 per cent. for given time = $\$0.216\frac{1}{2}$.

Interest on \$715.30 at 6 per cent. for given time = $\$0.216\frac{1}{2} \times 715.30 = \154.98166 .

Hence interest on \$715.30 at 8 per cent. for given time = $\$154.98166 + \text{one third of } \$154.98166 = \$206.6422$.

(73)

Interest on \$1 at 6 per cent. for given time = $\$0.141\frac{1}{2}$.

Interest on \$555.55 at 6 per cent. for given time = $\$0.141\frac{1}{2} \times 555.55 = \78.51773 .

Hence interest on \$555.55 at 12 per cent. for given time = $\$78.51773 \times 2 = \$157.03546 + \$555.55 = \712.58546 .

(74)

Interest on \$1 at 6 per cent. for given time = $\$0.016\frac{1}{2}$.

Interest on \$7766.55 at 6 per cent. for given time = $\$0.016\frac{1}{2} \times 7766.55 = \129.4425 .

Hence interest on \$7766.55 at 5 per cent. for given time = $\$129.4425 - \text{one sixth of } \$129.4425 = \$104.86875$.

Amount = $\$104.86875 + \$7766.55 = \$7871.41875$.

(75)

Interest on \$1 at 6 per cent. for given time = $\$0.521\frac{1}{2}$.

Interest on \$500 at 6 per cent. for given time = $\$0.521\frac{1}{2} \times 500 = \$260.666\frac{1}{2}$.

Hence interest on \$500 at 16 per cent. for given time = $\$260.666\frac{1}{2} \times 2\frac{1}{2} = \$695.111 + \$500 = \1195.111 .

(76)

Interest on \$1 at 6 per cent. for given time = $\$0.206\frac{1}{6}$.

Interest on \$576 at 6 per cent. for given time = $\$0.206\frac{1}{6} \times 576 = \118.752 .

Hence interest on \$576 at 5 per cent. for given time = $\$118.752$
— one sixth of $\$118.752 = \98.96 .

(77)

Interest on \$1 at 6 per cent. for given time = $\$0.151\frac{5}{6}$.

Interest on \$2478.91 at 6 per cent. for given time = $\$0.151\frac{5}{6} \times 2478.91 = \376.38116 .

Hence interest on \$2478.91 at $4\frac{1}{2}$ per cent. for given time =
 $\$376.38116$ — one fourth of $\$376.38116 = \282.285 .

(78)

From May 9th to December 11th = 216 days. Interest on \$1
at 6 per cent. for 216 days = $\$0.036$.

Interest on \$780 at 6 per cent. for 216 days = $\$0.036 \times 780 =$
 $\$28.08$.

(79)

From August 16th 1851 to June 19th 1852 = 308 days.

Interest on \$1 at 6 per cent. for given time = $\$0.051\frac{1}{3}$.

Interest on \$1830.63 at 6 per cent. for given time = $\$0.051\frac{1}{3} \times$
 $1830.63 = \$93.97234$.

Hence interest on \$1830.63 at 7 per cent. for given time =
 $\$93.97234 +$ one sixth of $\$93.97234 = \109.63439 .

(80)

From September 3rd 1858 to January 9th 1859 = 128 days.

Interest on \$1 at 6 per cent. for given time = $\$0.021\frac{1}{3}$.

Interest on \$6200 at 6 per cent. for given time = $\$0.021\frac{1}{3} \times$
 $6200 = \$132.266$.

Amount = $\$132.266 + \$6200 = \$6332.266$.

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(82)

From June	2nd to July	17th there are 45 days.
" July	17th to October 6th	" 81 "
" October	6th to December 11th	" 66 "
" December	11th to March 29th	" 109 "
" March	29th to October 7th	" 192 "

Whole sum \$1217.30 for 45 days = \$54778.50 for 1 day.
 1st endorsement 207.80

Balance \$1009.50 for 81 days = \$81769.50 for 1 day.
 2nd endorsement 209.60

Balance \$799.90 for 66 days = \$52793.40 for 1 day.
 3rd endorsement 320.90

Balance \$479.00 for 109 days = \$52211.00 for 1 day.
 4th endorsement 421.83

Balance \$57.17 for 192 days = \$10976.64 for 1 day.

Whole interest = that of \$252529.04 for 1 day.

Interest on \$252529.04 at 6 per cent. for 1 year = \$15151.7424.

Hence interest for 1 day = $\$15151.7424 \div 365 = \41.5116 .

Then interest due = \$41.5116

Balance on Note = \$57.17

Principal and interest due = \$98.6816

(83)

From 17th June to 5th September there are 80 days.

" 5th September to 7th December	" 93 "
" 7th December to 11th June	" 186 "
" 11th June to 7th February	" 241 "
" 7th February to 19th December	" 315 "
" 19th December to 1st May	" 133 "

Whole sum \$7348.25 for 80 days = \$587860.00 for 1 day.
1st endorsement 2463.80

Balance \$4884.45 for 93 days = \$454253.85 for 1 day.
2nd endorsement 392.20

Balance \$4492.25 for 186 days = \$835558.50 for 1 day.
3rd endorsement 982.20

Balance \$3510.05 for 241 days = \$845922.05 for 1 day.
4th endorsement 2842.90

Balance \$667.15 for 315 days = \$210152.25 for 1 day.
5th endorsement 317.23

Balance \$349.92 for 133 days = \$46539.36 for 1 day.

Whole interest = that of \$2980286.01 for 1 day.

Interest on \$2980286.01 at 8 per cent. for 1 year = \$238422.8808.

Hence interest for 1 day = \$238422.8808 ÷ 365 = \$653.2133.

Then interest due = \$653.2133

Balance on Note = \$349.92

Principal and interest due = \$1003.1333

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(2)

\$1800	Principal.
108	Interest for 1st year.
<hr/>	
\$1908	Amount for 1 year = principal for 2nd year.
114.48	Interest for 2nd year.
<hr/>	
\$2022.48	Amount for 2 years = principal for 3rd year.
121.3488	Interest for 3rd year.
<hr/>	
\$2143.8288	Amount for 3 years = principal for 4th year.
128.629728	Interest for 4th year.
<hr/>	
\$2272.458528	Amount for 4 years = principal for 5th year.
136.347511	Interest for 5th year.
<hr/>	
\$2408.806039	Amount for 5 years.
1800	Given Principal.
<hr/>	
\$608.806 =	Compound interest required.

(3)

\$700	Principal.
49	Interest for 1st half year.
<hr/>	
\$749	Amount for 1 half y. = principal for 2nd half y.
52.43	Interest for 2nd half year.
<hr/>	
\$801.43	Amount for 1 year = principal for 3rd half y.
56.1001	Interest for 3rd half year.
<hr/>	
\$857.5301	Amount for 1½ years = principal for 2nd year.
60.027107	Interest for 2nd year.
<hr/>	
\$917.557207	Amount for 2nd year = principal for 5th half y.
64.229004	Interest for 5th half year.
<hr/>	
\$981.786211	Amount for 2½ years = principal for 3rd year.
68.725034	Interest for 3rd year.
<hr/>	
\$1050.511245	Amount for 3 years = principal for 7th half y.
73.535787	Interest for 7th half year.
<hr/>	
\$1124.047032	Amount for 3½ years.
700	Given Principal.
<hr/>	
\$424.047 =	Compound interest required.

(4)

\$673·40	Principal.
20·202	Interest for 1st quarter.
<hr/>	
\$693·602	Amount for 1 quar. = principal for 1st half y.
20·80806	Interest for 1st half year.
<hr/>	
\$714·41006	Am't for 1 half y. = principal for 3rd quarter.
21·4323018	Interest for 3rd quarter.
<hr/>	
\$735·8423618	Amount for 3 quarters = principal for 1st year.
22·0752708	Interest for 1st year.
<hr/>	
\$757·9176326	Amount for 1 year = principal for 5th quarter.
22·7375289	Interest for 5th quarter.
<hr/>	
\$780·6551615	Am't for 5 quarters = principal for 3rd half y.
23·4196548	Interest for 3rd half year.
<hr/>	
\$804·0748163	Am't for 3 half y. = principal for 7th quarter.
24·1222444	Interest for 7th quarter.
<hr/>	
\$828·1970807	Amount for 7 quarters = principal for 2nd year.
24·8459124	Interest for 2nd year.
<hr/>	
\$853·0429 =	Amount for 2 years required.
673·40	Given Principal.
<hr/>	
\$179·6429 =	Compound Interest required.

(5)

\$860	Principal.
34·4	Interest for 1st half year.
<hr/>	
\$894·4	Amount for 1 half year = principal for 1st year.
35·776	Interest for 1st year.
<hr/>	
\$930·176	Amount for 1 year = principal for 3rd half year.
37·20704	Interest for 3rd half year.
<hr/>	
\$967·38304	Amount for 3 half years = principal for 2nd y.
38·69532	Interest for 2nd year.
<hr/>	
\$1006·07836	Amount for 2 years = principal for 5th half year.
40·24313	Interest for 5th half year.
<hr/>	
\$1046·32149	Amount for 5 half years = principal for 3rd year.
41·85285	Interest for 3rd year.
<hr/>	
\$1088·17434 =	Amount for 3 years required.
860	Given Principal.
<hr/>	
\$228·1743 =	Compound Interest required.

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(8)

By the table the am't of \$1 at 6 per cent. for 11 years = \$1.8983.

Then $\$1.8983 \times 875 = \$1661.0125 = \text{Amount.}$

875	Principal.
-----	------------

 $\$786.0125 = \text{Interest.}$

(9)

By the table the am't of \$1 for the given time and rate = \$2.77247.

Then $\$2.77247 \times 643.98 = \$1785.41523 = \text{Amount.}$

643.98	Principal.
--------	------------

 $\$1141.43523 = \text{Interest.}$

(10)

By the table the am't of \$1 at 6 per cent. for 45 years = \$13.76461.

Then $\$13.76461 \times .01 = \$0.137646 = \text{Amount.}$

.01	Principal.
-----	------------

 $\$.127646 = \text{Interest.}$

(11)

By the table the am't of \$1 for the given time and rate = \$2.28793.

Then $\$2.28793 \times 78.2 = \$178.916 = \text{Amount.}$

78.2	Principal.
------	------------

 $\$100.716 = \text{Interest.}$

(12)

By the table the am't of \$1 for the given rate and time = \$2.40662.

Then $\$2.40662 \times 777.77 = \$1871.7968 = \text{Amount.}$

777.77	Principal.
--------	------------

 $\$1094.0268 = \text{Interest.}$

(13)

$$£44 \text{ 5s. 9d.} = £44.2875.$$

By the table the am't of £1 at 6 per cent. for 11 years = £1.8983.

Then $£1.8983 \times 44.2875 = £84.07096 = £84 \text{ 1 5} = \text{Amount.}$

44 5 9 Principal.

£39 15 8 = Interest.

(14)

$$£32 \text{ 4s. } 9\frac{1}{4}\text{d.} = £32.240625.$$

By the table the amount of £1 for the given time and rate =

£1.26532. Then $£1.26532 \times 32.240625 =$

£40.7947076 = £40 15 10 $\frac{3}{4}$ nearly = Amount.

32 4 9 $\frac{1}{4}$ Principal.

£8 11 1 = Interest.

. —————
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(16)

Amount of \$1 for 7 years at 4 per cent = \$1.31593.

$\$7439.87 \div 1.31593 = \$5653.697.$

(17)

Amount of \$1 at 5 per cent for 20 years = \$2.6533.

$\$9193.90 \div 2.6533 = \$3465.081.$

(18)

$$£595 \text{ 10s. } 2\frac{1}{2}\text{d.} = £595.51.$$

Amount of £1 at 6 per cent for 3 years = £1.19102.

$£595.51 \div 1.19102 = £500.$

(19)

Amount of \$1 at 6 per cent for 7 years = \$1.50363.

$\$7111.11 \div 1.50363 = \$4729.295.$

(20)

$$£268 \text{ 0s. } 4\frac{1}{2}\text{d.} = £268.02.$$

Amount of £1 at 5 per cent for 6 years = £1.3401.

$$£268.02 \div 1.3401 = £200.$$

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(3)

Here $A = \$962$, $r = .04$, and $t = 1$. Whence $1 + rt = 1.04$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{962}{1.04} = \$925.$$

(4)

Here $A = \$2202$, $r = .06$, and $t = 5.75$. Whence $1 + rt = 1.345$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{2202}{1.345} = \$1637.174.$$

(5)

Here $A = \$1003.50$, $r = .06$, and $t = \frac{2}{3}$ year. Whence $1 + rt = 1.04$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1003.50}{1.04} = \$964.9038.$$

(6)

Here $A = \$716$, $r = .08$, and $t = \frac{7}{12}$ year. Whence $1 + rt = 1.04\frac{2}{3}$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{716}{1.04\frac{2}{3}} = \$684.0764.$$

(7)

Here $A = \$1342.50$, $r = .065$, and $t = \frac{25}{73}$ year. Whence $1 + rt = 1.022\frac{19}{73}$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1342.50}{1.022\frac{19}{73}} = \$1313.266.$$

(8)

Here $A = \$2400$, $r = .05$, and $t = \frac{336}{73}$ year. Whence $1 + rt = 1.03\frac{17}{73}$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{2400}{1.03\frac{17}{73}} = \$2324.84.$$

(9)

Here $A = \$2202$, $r = .05$, and $t = .75$ year. Whence $1 + rt = 1.0375$.

$$\$2202 \div 1.0375 = \$2122.40963+ = \text{Present worth.}$$

$$\$2202 - \$2122.40963+ = \$79.59036 = \text{Discount.}$$

(10)

Here $A = \$4360$, $r = .06$, and $t = 1\frac{5}{12}$. Whence $1 + rt = 1.085$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{4360}{1.085} = \$4018.43317.$$

(11)

Here $A = \$1647$, $r = .06$, and $t = 1\frac{1}{12}$ year. Whence $1 + rt = 1.055$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1647}{1.055} = \$1561.13744.$$

(12)

Here $A = \$2000$, $r = .06$, and $t = 3\frac{7}{12}$. Whence $1 + rt = 1.215$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{2000}{1.215} = \$1646.09053.$$

(13)

Here $A = \$2070.90$, $r = .05$, and $t = 1\frac{7}{12}$. Whence $1 + rt = 1.07\frac{1}{4}$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{2070.90}{1.07\frac{1}{4}} = \$1918.9806.$$

$$\$2070 - \$1918.9806 = \$151.919 = \text{Discount required.}$$

(14)

Here $A = \$970.63$, $r = .08$, and $t = 1\frac{1}{2}$ year. Whence $1 + rt = 1.07\frac{1}{2}$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{970.63}{1.07\frac{1}{2}} = \$904.313.$$

(15)

Here in first case $A = \$1512$, $r = .07$, and $t = .5$ year. Whence $1 + rt = 1.035$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1512}{1.035} = \$1460.8695.$$

Also $A = 1512$, $r = .07$, and $t = 1$. Whence $1 + rt = 1.07$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1512}{1.07} = \$1413.0841.$$

$\$1460.8695 + \$1413.0841 = \$2873.9536 = \text{Present worth of whole amount.}$

$$\$3024 - \$2873.9536 = \$150.0464 = \text{Discount required.}$$

(16)

Here in first case $A = \$440$, $r = .08$, and $t = 1.25$. Whence $1 + rt = 1.1$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{440}{1.1} = \$400.$$

In second case $A = \$896$, $r = .08$, and $t = 1.5$. Whence $1 + rt = 1.12$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{896}{1.12} = \$800.$$

$$\$400 + \$800 = \$1200.$$

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(18)

Here the time the note has to run is 2 years, 3 months, 3 days.

Interest of \$1 at 7 per cent for 2 yrs., 3 m., 3 days = $\$0.1580\frac{5}{8}$.

Interest of \$986 at 7 per cent for 2 years, 3 months, 3 days =

$$\$0.1580\frac{5}{8} \times 986 = \$155.8701.$$

(19)

Here the time the note has to run is 103 days = 3 months 13 days.

Interest of \$1 at 8 per cent for 3 months 13 days = $\$0.022\frac{8}{9}$.

Interest of \$640 at 8 per cent for 3 months, 13 days =

$$\$0.022\frac{8}{9} \times 640 = \$14.6488.$$

(20)

Here the time the note has to run is 94 days = 3 months 4 days.

Interest of \$1 at 6 per cent for 3 months 4 days = $\$0.015\frac{2}{3}$.

Interest of \$563.80 at 6 per cent for 3 months 4 days =

$$\$0.015\frac{2}{3} \times 563.80 = \$8.8328 \text{ and } \$563.80 - \$8.8328 = \$554.967.$$

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(22)

Interest on \$1 for 93 days at 7 p. c. = $\$0.0180\frac{5}{8}$, and this taken from \$1 gives a remainder of $\$0.9819\frac{1}{8}$ = present worth of \$1.

$$\text{Then } \$3755 \div 0.9819\frac{1}{8} = \$3824.15.$$

(23)

Interest on \$1 for 6 months 3 days at 5 per cent = $\$0.0254\frac{1}{6}$, and this taken from \$1 gives a remainder $\$0.9745\frac{5}{6}$ = present worth of \$1.

$$\text{Then } \$1147.80 \div 0.9745\frac{5}{6} = \$1177.734.$$

(24)

Interest on \$1 for 48 days at $3\frac{1}{2}$ per cent = $\$0.004\frac{1}{2}$, and this taken from \$1 gives a remainder $\$0.995\frac{1}{2}$ = present worth of \$1.

Then $\$713.90 \div 0.995\frac{1}{2} = \717.2471 .

Page 268.

(4)

$$\begin{array}{r}
 \$200 \times 3 = 600 \\
 150 \times 4 = 600 \\
 250 \times 6 = 1500 \\
 \hline
 600 \quad 600 \quad 2700 (4\frac{1}{2} \text{ months.} \\
 \quad \quad 2400 \\
 \hline
 \quad \quad 300 \} \\
 \quad \quad \hline
 \quad \quad 600 \} = \frac{1}{2}
 \end{array}$$

(5)

$$\begin{array}{r}
 \frac{1}{4} \times 0 = 0 \\
 \frac{1}{4} \times 3 = \frac{3}{4} \\
 \frac{1}{4} \times 6 = 1\frac{1}{2} \\
 \frac{1}{4} \times 9 = 2\frac{1}{4} \\
 \hline
 1 \quad 1) 4\frac{1}{2} \\
 \hline
 4\frac{1}{2} \text{ months.}
 \end{array}$$

(6)

$$\begin{array}{r}
 \$50 \times 2 = 100 \\
 40 \times 5 = 200 \\
 30 \times 7 = 210 \\
 \hline
 120 \quad 120) 510 (4\frac{1}{4} \text{ months.} \\
 \quad \quad 480 \\
 \hline
 \quad \quad 30 \} \\
 \quad \quad \hline
 \quad \quad 120 \} = \frac{1}{4}
 \end{array}$$

(7)

$$\begin{array}{r}
 \$1000 \times 0 = 0 \\
 1500 \times 1 = 1500 \\
 600 \times 3 = 1800 \\
 700 \times 5 = 3500 \\
 1400 \times 7 = 9800 \\
 \hline
 5200 \quad 5200) 16600 (3\frac{5}{6} \text{ months.} \\
 \quad \quad 15600 \\
 \hline
 \quad \quad 1000 \} \\
 \quad \quad \hline
 \quad \quad 5200 \} = \frac{5}{6}
 \end{array}$$

(8)

Six months from 15th January = 15th July, and from 1st July to 15th July there are 14 days.

Six months from 10th February = 10th August, and from 1st July to 10th August there are 40 days.

Six months from 6th March = 6th September, and from 1st July to 6th September there are 67 days.

Six months from 8th June = 8th December, and from 1st July to 8th December there are 160 days.

$$\begin{array}{rcl}
 \$3750 \times 14 & = & 52500 \\
 3000 \times 40 & = & 120000 \\
 2400 \times 67 & = & 160800 \\
 2250 \times 160 & = & 360000 \\
 \hline
 11400 & & 11400)693300(60\frac{31}{8} \text{ days.} \\
 & & 684000 \\
 \hline
 & & 9300 \} \\
 & & \hline
 & & 11400 \} = \frac{31}{8}
 \end{array}$$

Therefore the note must be made payable on the 61st day from the 1st of July, which is the 31st of August.

Page 269.

(2)

Whole stock : A's stock :: whole profit : A's profit.

That is, \$4300 : \$3000 :: \$1117 : $\frac{1117 \times 3000}{4300} = \$779.302+ = \text{A's sh.}$

\$1117 - \$779.302+ = \$337.697 = B's share.

(3)

Whole stock = \$6470 + \$3780 + \$9860 = \$20110.

Whole stock : A's stock :: whole profit : A's profit.

$$\text{That is, } \$20110 : \$6470 :: \$7890 : \frac{7890 \times 6470}{20110} = \$2538.453+ = \text{A's sh.}$$

Again, whole stock : B's stock :: whole profit : B's profit.

$$\text{That is, } \$20110 : \$3780 :: \$7890 : \frac{7890 \times 3780}{20110} = \$1483.053+ = \text{B's sh.}$$

Lastly, whole stock : C's stock :: whole profit : C's profit.

$$\text{That is, } \$20110 : \$9860 :: \$7890 : \frac{7890 \times 9860}{20110} = \$3868.493+ = \text{C's sh.}$$

(4)

Whole stock : B's stock :: whole gain : B's gain.

$$\text{That is, } \$320 : \$120 :: \$80 : \frac{80 \times 120}{320} = \$30 = \text{B's gain.}$$

Again, whole stock : C's stock :: whole gain : C's gain.

$$\text{That is, } \$320 : \$200 :: \$80 : \frac{80 \times 200}{320} = \$50 = \text{C's share.}$$

(5)

Whole stock : B's stock :: whole gain : B's gain.

$$\text{That is, } \$2800 : \$1200 :: \$728 : \frac{728 \times 1200}{2800} = \$312 = \text{B's gain.}$$

Again, whole stock : C's stock :: whole gain : C's gain.

$$\text{That is, } \$2800 : \$1600 :: \$728 : \frac{728 \times 1600}{2800} = \$416 = \text{C's gain.}$$

(6)

Whole stock : B's stock :: whole amount to be divided : B's share.

$$\text{That is, } \$3 : \$2 :: \$100 : \frac{100 \times 2}{3} = \$66 \cdot 66\frac{2}{3} = \text{B's share.}$$

Again, whole st'k : C's st'k :: whole amo't to be divided : C's sh'e.

$$\text{That is, } \$3 : \$1 :: \$100 : \frac{100 \times 1}{3} = \$33 \cdot 33\frac{1}{3} = \text{C's share.}$$

(7)

$$£1400 : £500 :: £1100 : \frac{1100 \times 500}{1400} = £392\frac{6}{7} = \text{B's share.}$$

$$£1100 - £392\frac{6}{7} = £707\frac{1}{7} = \text{C's share.}$$

(8)

$$\begin{array}{c} \text{casks.} \quad \text{casks.} \quad 180 \times 200 \\ 900 : 200 :: 180 : \frac{\quad}{900} = 40 \text{ casks} = \text{B's loss.} \end{array}$$

$$\begin{array}{c} 180 \times 300 \\ 900 : 300 :: 180 : \frac{\quad}{900} = 60 \text{ casks} = \text{C's loss.} \end{array}$$

$$180 - (40 + 60) = 80 \text{ casks} = \text{D's loss.}$$

(9)

$$\$1800 : \$800 :: \$100 : \frac{100 \times 800}{1800} = \$44 \cdot 44\frac{4}{9} = \text{B's share.}$$

$$\$1800 : \$600 :: \$100 : \frac{100 \times 600}{1800} = \$33 \cdot 33\frac{1}{3} = \text{C's share.}$$

$$\$44 \cdot 44\frac{4}{9} + \$33 \cdot 33\frac{1}{3} = \$77 \cdot 77\frac{7}{9}, \text{ and } \$100 - \$77 \cdot 77\frac{7}{9} =$$

$$\$22 \cdot 22\frac{2}{9} = \text{D's share.}$$

(10)

$$6 : 1 :: 120 : \frac{120 \times 1}{6} = 20.$$

$$6 : 2 :: 120 : \frac{120 \times 2}{6} = 40.$$

$$6 : 3 :: 120 : \frac{120 \times 3}{6} = 60.$$

(11)

$$\text{Whole loss} = \$900 - \$540 = \$360.$$

$$8 : 1 :: \$360 : \frac{360}{8} = \$45 = \text{B's loss.}$$

$$8 : 2 :: \$360 : \frac{360 \times 2}{8} = \$90 = \text{C's loss.}$$

$$\$45 + 90 = \$135, \text{ and } \$360 - 135 = \$225 = \text{D's loss.}$$

(12)

$$\$12 : \$6 :: \$1320 : \frac{1320 \times 6}{12} = \$660 = \text{B's gain.}$$

$$\$12 : \$4 :: \$1320 : \frac{1320 \times 4}{12} = \$440 = \text{C's gain.}$$

$$\$12 : \$2 :: \$1320 : \frac{1320 \times 2}{6} = \$220 = \text{D's gain.}$$

(13)

$$£35 + £29 = £64, \text{ and } £110 - £64 = £46 = \text{D's profit.}$$

$$\text{D's profit} : \text{B's profit} :: \text{D's stock} : \text{B's stock.}$$

$$\text{That is, } £46 : £35 :: £1090 : \frac{1090 \times 35}{46} = £829 \text{ 6s. } 11\frac{1}{2}\text{d.} = \text{B's st.}$$

$$\text{Again, D's profit} : \text{C's profit} :: \text{D's stock} : \text{C's stock.}$$

$$\text{That is, } £46 : £29 :: £1090 : \frac{1090 \times 29}{46} = £687 \text{ 3s. } 5\frac{1}{2}\text{d.} = \text{C's st.}$$

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(2)

$$\left. \begin{array}{l} \$357 \times 5 = \$1785 \text{ for one month} \\ 371 \times 7 = 2597 \text{ for one month} \\ 154 \times 11 = 1694 \text{ for one month} \end{array} \right\} = \$6076 \text{ for one month.}$$

$$\$6076 : \$1785 :: \$347 \cdot 20 : \frac{347 \cdot 20 \times 1785}{6076} = \$102.$$

$$\$6076 : \$2597 :: \$347 \cdot 20 : \frac{347 \cdot 20 \times 2597}{6076} = \$148 \cdot 40.$$

$$\$6076 : \$1694 :: \$347 \cdot 20 : \frac{347 \cdot 20 \times 1694}{6076} = \$96 \cdot 80.$$

(3)

$$\left. \begin{array}{l} 40 \times 6 = 240 \text{ for one month} \\ 30 \times 5 = 150 \text{ for one month} \\ 50 \times 1 = 50 \text{ for one month} \end{array} \right\} = 440 \text{ for one month.}$$

$$440 : 240 :: \$160 : \frac{160 \times 240}{440} = \$87 \cdot 27 \frac{3}{11}; \text{ B's share.}$$

$$440 : 150 :: \$160 : \frac{160 \times 150}{440} = \$54 \cdot 54 \frac{6}{11}; \text{ C's share.}$$

$$440 : 50 :: \$160 : \frac{160 \times 50}{440} = \$18 \cdot 18 \frac{2}{11}; \text{ D's share.}$$

(4)

$$\left. \begin{array}{l} £150 \times 6 = £900 \text{ for one month} \\ 200 \times 3 = 600 \text{ for one month} \\ 125 \times 16 = 2000 \text{ for one month} \end{array} \right\} = £3500 \text{ for one month.}$$

$$£3500 : £900 :: £291 \text{ 13s. 4d.} : \frac{£291 \text{ 13s. 4d.} \times 900}{3500} = £75.$$

$$£3500 : £600 :: £291 \text{ 13s. 4d.} : \frac{£291 \text{ 13s. 4d.} \times 600}{3500} = £50.$$

$$£3500 : £2000 :: £291 \text{ 13s. 4d.} : \frac{£291 \text{ 13s. 4d.} \times 2000}{3500} = £166 \text{ 13s. 4d.}$$

(5)

$$\left. \begin{array}{l} \$4000 \times 12 = \$48000 \text{ for one month} \\ 3000 \times 15 = 45000 \text{ for one month} \\ 5000 \times 8 = 40000 \text{ for one month} \end{array} \right\} = \$133000 \text{ for one month.}$$

$$\$133000 : \$48000 :: \$665 : \frac{665 \times 48000}{133000} = \$240 ; \text{B's share.}$$

$$\$133000 : \$45000 :: \$665 : \frac{665 \times 45000}{133000} = \$225 ; \text{C's share.}$$

$$\$133000 : \$40000 :: \$665 : \frac{665 \times 40000}{133000} = \$200 ; \text{D's share.}$$

(6)

$$\left. \begin{array}{l} 56 \times 12 = 672 \text{ for one day} \\ 64 \times 15 = 960 \text{ for one day} \\ 80 \times 18 = 1440 \text{ for one day} \end{array} \right\} = 3072 \text{ for one day.}$$

$$3072 : 672 :: \$320 : \frac{320 \times 672}{3072} = \$70 = \text{rent to be paid by 1st troop.}$$

$$3072 : 960 :: \$320 : \frac{320 \times 960}{3072} = \$100 = \text{ " " " 2nd "}$$

$$3072 : 1440 :: \$320 : \frac{320 \times 1440}{3072} = \$150 = \text{ " " " 3rd "}$$

(8)

$$\text{Sum of profits} = 240 + 800 + 400 = \$1440.$$

$$\text{Whole profit} : \text{A's profit} :: \text{Whole stock for 1 m.} : \text{A's st. for 1 m.}$$

$$\text{That is, } 1440 : 240 :: 34560 : \frac{34560 \times 240}{1440} = 5760 = \text{A's stock}$$

for one month. Hence, since A's stock was in for 6 months,
it will be $\$5760 \div 6 = \960 .

(Continued on next page.)

(8 Continued.)

Whole profit : B's profit :: Whole stock for 1 m. : B's st. for 1 m.

$$1440 : 800 :: 34560 : \frac{34560 \times 800}{1440} = 19200 = \text{B's stock for one}$$

month. And, since B's stock was in for 12 months, $19200 \div 12 = \$1600$ will be his stock.

Whole profit : C's profit :: whole stock for 1 m. : C's st. for 1 m.

$$1440 : 400 :: 34560 : \frac{34560 \times 400}{1440} = \$9600 = \text{C's stock for one}$$

month and hence his stock will be $\$9600 \div 15 = \640 .

(9)

A's profit was \$240 for 6 months = \$40 for 1 month.

B's profit was \$800 for 12 months = \$66 $\frac{2}{3}$ for 1 month.

C's profit was \$400 for 15 months = \$26 $\frac{2}{3}$ for 1 month.

Sum of profits for 1 month = \$133 $\frac{1}{3}$

Whole profit for 1 m. : A's profit for 1 m. :: whole stock : A's st.

$$133\frac{1}{3} : 40 :: 3200 : \frac{3200 \times 40}{133\frac{1}{3}} = \$960 = \text{A's stock.}$$

$$133\frac{1}{3} : 66\frac{2}{3} :: 3200 : \frac{3200 \times 66\frac{2}{3}}{133\frac{1}{3}} = \$1600 = \text{B's stock.}$$

$$133\frac{1}{3} : 26\frac{2}{3} :: 3200 : \frac{3200 \times 26\frac{2}{3}}{133\frac{1}{3}} = \$640 = \text{C's stock.}$$

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(4)

\$0.12 $\frac{1}{2}$ = selling price.

\$0.09 = buying price.

\$0.03 $\frac{1}{2}$ = gain per lb.

$$\$0.03\frac{1}{2} \times 317 = \$11.095.$$

(5)

\$1.20 = selling price.

\$0.87 $\frac{1}{2}$ = buying price.

\$0.32 $\frac{1}{2}$ = gain per bushel.

$$\$0.32\frac{1}{2} \times 2138 = \$694.85.$$

(6)

$\$0.15 \times 317 \times 13 = \$618.15 =$ cost of 13 barrels at $\$0.15$ per lb.
 $\$735 - 618.15 = \116.85 gain.

(7)

$\$3.15 \times 22 \times 17 = \$1178.10 =$ price of 17 kegs at $\$3.15$ per gal,
 $\$0.37\frac{1}{2} \times 1178.1 = \$441.7875 =$ ad valorem duty.
 $\$1178.10 + \$441.7875 + \$26.33 = \$1646.2175 =$ whole cost.
 $\$1646.2175 - \$1625 = \$21.2175$ loss.

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(10)

Here for every \$1 I expend I wish to receive \$1.30, and hence the selling price will be $\$3.25 \times 1.30 = \$4.22\frac{1}{2}$.

(11)

Here for every \$1 I expend I wish to receive \$1.05, and hence the selling price will be $\$1.05 \times 13420 = \14091 .

(12)

Here for every \$1 I expend I desire to receive \$1.15, and hence the selling price will be $\$1.15 \times .11 = \$0.1265 = 12\frac{1}{2}\frac{2}{5}$ cents.

(13)

Here for every \$1 I expend I wish to receive \$1.23, and hence the selling price will be $\$1.23 \times 15.25 = \$18.75\frac{1}{4}$.

(14)

Here for every \$1 I expend I am willing to receive \$0.89, and hence the selling price will be $\$0.89 \times 7890 = \7022.10 .

Page 277.

(16)

Here the whole gain is $\$0.87\frac{1}{2} - \$0.60 = \$0.27\frac{1}{2}$.

That is, $\$0.60$ gain $\$0.27\frac{1}{2}$, and therefore 1 cent gains $\frac{27\frac{1}{2}}{60} =$

$$\frac{55}{120} = \frac{11}{24} \text{ of a cent.}$$

And hence, the gain per cent $= \frac{11}{24} \times 100 = \frac{1100}{24} = 45\frac{5}{6}$ per cent.

(17)

Here the loss on each lb. is 2 cents.

That is, every 13 cents invested gives a loss of 2 cents.

Therefore every cent invested loses $\frac{1}{13}$ of 2 $= \frac{2}{13}$ cents.

And hence, the loss per cent $= \frac{2}{13} \times 100 = \frac{200}{13} = 15\frac{5}{13}$ per c.

(18)

Here the gain on each barrel is $\$1.60$.

That is, every $\$6.20$ invested gives a gain of $\$1.60$.

Therefore every $\$1$ invested gains $\frac{1}{6.20}$ of $160 = \frac{8}{31}$ of a \$.

And hence, the gain per cent $= \frac{8}{31} \times 100 = \frac{800}{31} = 25.8 = 25\frac{4}{5}$ p.c.

(19)

Here the gain on each yard is 35 cents.

That is, every $\$2.75$ invested gives a gain of 35 cents.

Therefore every $\$1$ invested gains $\frac{1}{2.75}$ of 35 $= \frac{35}{2.75} = \frac{7}{5}$ of a dollar.

And hence the gain per cent $= \frac{7}{5} \times 100 = \frac{700}{5} = 140$ p.c.

(20)

Here the gain on every bushel is 9 cents.

That is every 47 cents invested gives a gain of 9 cents.

Therefore every cent invested gains $\frac{1}{47}$ of 9 $= \frac{9}{47}$ cents.

And hence the gain per cent $= \frac{9}{47} \times 100 = \frac{900}{47} = 19\frac{7}{47}$ p.c.

(21)

Here the loss on each lb. is $1\frac{1}{2}$ cents.

That is every 12 cents invested gives a loss of $1\frac{1}{2}$ cents.

Therefore every cent invested gives $\frac{1}{12}$ of $1\frac{1}{2} = \frac{1}{8}$ of a cent.

And hence, the gain per cent $= \frac{1}{8} \times 100 = \frac{100}{8} = 12\frac{1}{2}$ p. c.

(22)

Here the whole gain is $\$127 - \$93 = \$34$.

That is, $\$93$ gain $\$34$, and therefore $\$1$ gains $\frac{34}{93}$ of a dollar.

Hence, gain per cent $= \frac{34}{93} \times 100 = \frac{3400}{93} = 36\frac{52}{93}$ per cent.

(23)

Here the loss is $\$6742.50 - \$6000 = \$742.50$.

That is, $\$6742.50$ lose $\$742.50$, and therefore $\$1$ loses $\frac{742.50}{6742.50}$ of $742.50 = \frac{99}{899}$ of a dollar.

Hence gain per cent $= \frac{99}{899} \times 100 = \frac{9900}{899} = 11\frac{11}{899}$ per cent.

(24)

Here $\$5700 + \$275 + \$1987.32 = \$7962.32 =$ whole sum expended.

Whole gain $= \$8750 - \$7962.32 = \$787.68$.

That is $\$7962.32$ gain $\$787.68$, and therefore $\$1$ gains $\frac{787.68}{7962.32}$ of $787.68 = \frac{9846}{99529}$ of a \$.

Hence gain per cent $= \frac{9846}{99529} \times 100 = \frac{984600}{99529} = 9.89$ or nearly 10 per cent.

(25)

$\$4.25 \times 723 = \$3072.75 =$ price of 723 yds. @ $\$4.25$.

$\$3072.75 \times .07 = \$215.0925 =$ amount for Insurance.

$\$3072.75 \times .22 = \$676.005 =$ amount for ad valorem duty.

Then whole cost $= \$3072.75 + \$215.0925 + \$23.70 + \$2.70 + \$3.16 + \$676.005 = \$3993.4075$.

Whole gain $= \$5270 - \$3993.4075 = \$1276.5925$.

That is, $\$3993.4075$ gains $\$1276.5925 \therefore \1 gains $\frac{1276.5925}{3993.4075}$ of $\$1276.5925 = \frac{510637}{1697363}$ of a \$.

Hence gain per cent $= \frac{510637}{1697363} \times 100 = 31.96749$ or nearly 32 per cent.

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(27)

Loss on \$1 is 4 cents, or for every \$1 paid I receive \$0.96.
Hence cost = $\$24.60 \div 0.96 = \25.625 .

(28)

Loss on \$1 is 10 cents, or for every \$1 paid he receives \$0.90.
Hence cost = $\$2360 \div .90 = \2622.22 .

(29)

Gain on \$1 is 11 cents, or for every \$1 paid he receives \$1.11.
Hence cost = $\$7400 \div 1.11 = \6666.666 .

(30)

Gain on \$1 is 17 cents, or for every \$1 paid he receives \$1.17.
\$117 : \$100 :: \$3789.40 : $\frac{3789.40 \times 100}{117} = \3238.803 Ans.

(31)

Loss on \$1 is 13 cents, or for every \$1 paid I receive \$0.87.
\$87 : \$100 :: \$2740 : $\frac{2740 \times 100}{87} = \3149.425 Ans.

Page 279.

(3)

\$2 gains 50 cents.

Hence \$0.50 ; \$0.10 :: \$2.00 : $\frac{2.00 \times 10}{50} = 40$ cents.

(4)

$$\$2.00 : \$2.80 :: \$2.50 : \frac{2.50 \times 2.80}{2.00} = \$3.50.$$

(5)

8 cents gain 5 cents in 9 months.

$$\text{Hence } 9 \text{ mo's} : 6 \text{ mo's} :: 5 \text{ cents} : \frac{5 \times 6}{9} = 3\frac{1}{3} = \text{gain for 6 mo's.}$$

$$8 \text{ cts.} : 12 \text{ cts.} :: 3\frac{1}{3} : \frac{3\frac{1}{3} \times 12}{8} = 5 \text{ cts. gain on 12 cts. for 6 mo's.}$$

Therefore $12 + 5 = 17 =$ his selling price.

(6)

$$\$1.60 : \$1.85 :: \$.55 : \frac{1.85 \times .55}{1.60} = \$0.6359375 = \text{what L}$$

ought to get in order to sell at the same profit as K.

But L only gets 60 cents, therefore K has the advantage.

$$70 \text{ yds. of cloth at } \$1.85 = \$1.85 \times 70 = \$129.50.$$

$$\$129.50 \div \$.60 = 215\frac{5}{6}.$$

(7)

$$5 \text{ tons of butter at } \$102 = \$102 \times 5 = \$510$$

$$10\frac{1}{2} \text{ tons of tallow at } \$135 = \$135 \times 10\frac{1}{2} = \$1417.50$$

$$\text{Total value} = \$1927.50$$

$$\text{Deduct ready money, } \$600.30$$

$$\underline{\$1327.20}$$

$$\$1327.20 \div \$4.20 = 316 \text{ barrels.}$$

Page 281.

(3)

$$\begin{array}{r}
 7 \text{ oz.} \times 22 = 154 \text{ carats.} \\
 12\frac{1}{2} \text{ " } \times 21 = 262\frac{1}{2} \text{ " } \\
 17 \text{ " } \times 9 = 153 \text{ " } \\
 \hline
 36\frac{1}{2} \qquad 36\frac{1}{2})569\frac{1}{2} \text{ " } \\
 \qquad \qquad 2 \quad 2 \text{ " } \\
 \hline
 73)1239(15\frac{1}{3} \text{ carats.} \\
 \quad 73 \\
 \hline
 \quad 409 \\
 \quad 365 \\
 \hline
 \quad 44
 \end{array}$$

(4)

$$\begin{array}{r}
 2 \text{ gallons @ } 14\text{s.} = 28\text{s.} \\
 1 \text{ " @ } 12\text{s.} = 12\text{s.} \\
 2 \text{ " @ } 9\text{s.} = 18\text{s.} \\
 4 \text{ " @ } 8\text{s.} = 32\text{s.} \\
 \hline
 9 \qquad \qquad 9)90\text{s.} \\
 \hline
 \qquad \qquad 10\text{s.}
 \end{array}$$

(5)

$$\begin{array}{r}
 15 \text{ bushels @ } \$1.20 = \$18.00 \\
 30 \text{ " @ } \$1.50 = \$45.00 \\
 60 \text{ " @ } \$1.10 = \$66.00 \\
 83 \text{ " @ } \$1.75 = \$145.25 \\
 \hline
 188 \qquad \qquad 188)\$274.25(\$1.458 \\
 \qquad \qquad \quad 188 \\
 \qquad \qquad \hline
 \qquad \qquad \quad 86.2 \\
 \qquad \qquad \quad 75.2 \\
 \qquad \qquad \hline
 \qquad \qquad \quad 11.05 \\
 \qquad \qquad \quad 9.40 \\
 \qquad \qquad \hline
 \qquad \qquad \quad 1.650 \\
 \qquad \qquad \quad 1.504 \\
 \qquad \qquad \hline
 \qquad \qquad \quad .146
 \end{array}$$

(6)

$$\begin{array}{rcl}
 12 \text{ lbs. @ } 50 \text{ cents} & = & 600 \text{ cents.} \\
 16 \text{ " @ } 72 \text{ " } & = & 1152 \text{ " } \\
 22 \text{ " @ } 65 \text{ " } & = & 1430 \text{ " } \\
 18 \text{ " @ } 85 \text{ " } & = & 1530 \text{ " } \\
 100 \text{ " @ } 42 \text{ " } & = & 4200 \text{ " } \\
 \hline
 168 & & 8912 \text{ cents (53}\frac{1}{2}\text{ cents.} \\
 & & 840 \\
 & & \hline
 & & 512 \\
 & & 504 \\
 & & \hline
 & & 8 \} = 2\frac{1}{2}. \\
 & & \frac{8}{168}
 \end{array}$$

Page 283.

(11)

$$\begin{array}{c}
 \text{Prices. Differences. Prices.} \\
 125 = \left\{ \begin{array}{l} 160 - 35 \overbrace{\hspace{1cm}} 15 + 110 \\ 140 - 15 \overbrace{\hspace{1cm}} 25 + 100 \end{array} \right\} = 125
 \end{array}$$

$$\begin{array}{c}
 \text{Prices. Differences. Prices.} \\
 125 = \left\{ \begin{array}{l} 160 - 35 \overbrace{\hspace{1cm}} 15 + 110 \\ 140 - 15 \overbrace{\hspace{1cm}} 25 + 100 \end{array} \right\} = 125.
 \end{array}$$

Ans. 35 bush. @ \$1.10, 15 @ \$1.60, 15 @ \$1, and 25 @ \$1.40.
 35 bush. @ \$1.00, 15 @ \$1.40, 15 @ \$1.10, and 25 @ \$1.60.

(12)

	Prices.	Differences.	Prices.	
45 =	60	15	3 + 42	} = 45
	50	5	7 + 38	
			15 + 30	

	Prices.	Differences.	Prices.	
{	60	15	3 + 42	} = 45.
	50	5	7 + 38	
			15 + 30	

Ans. 15 quarts @ 42 cents, 3 @ 60 cents, 5 @ 38 cents, 5 @ 30 cents, and 7 + 15 + 22 @ 50 cents.
 15 quarts @ 38 cents, 3 @ 50 cents, 5 @ 42 cents, 15 @ 38 cents, and 7 + 15 = 22 @ 60 cents.

(13)

	Prices.	Differences.	Prices.		
12½ =	18	5½	½ + 12	} = 12½.	
	17	4½			
	16	3½			
	15	2½			2½ + 10
	14	1½			

Ans. ½ lb. @ 18 cents, ½ @ 17 cents, ½ @ 16 cents, 2½ @ 15 cents, 2½ @ 14 cents, 5½ + 4½ + 3½ = 13½ @ 12 cents, and 2½ + 1½ = 4 @ 10 cents.

(14)

	Prices.	Differences.	Prices.	
10 =	13	3	3 + 7	} = 10
	12	2	5 + 5	

Ans. 3 lbs. @ 7d., 3 @ 13d., 2 @ 5d., and 5 @ 12d.

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(17)

By case I we find that 17 quarts @ 31 cents, 6 @ 16 cents, 6 @ 19 cents, and 6 @ 23 cents will make a mixture worth 25 cents per quart.

Therefore 17 qts. : 87 qts. :: 6 qts. : $\frac{6 \times 87}{17} = 30\frac{1}{2}$ quarts @

16 cents, and as there are 6 lbs. at each of the other prices, the same statement may be used, and the answer is therefore $30\frac{1}{2}$ quarts @ each price.

(18)

To produce a mixture worth 75 cents per bushel, we require 45 bushels @ 80 cents, 5 @ 37 cents, and 5 @ 68 cents.

Therefore 45 bush. : 70 bush. :: 5 bush. : $\frac{5 \times 70}{45} = 7\frac{2}{3}$ bush.

oats @ 37 cents.

45 bush. : 70 bush. :: 5 bush. : $\frac{5 \times 70}{45} = 7\frac{2}{3}$ bush.

barley @ 68 cents.

(19)

To produce a mixture worth 1s. per lb., we require $1\frac{1}{2}$ lbs. @ 16d., $1\frac{1}{2}$ @ 14d., and 6 @ 10 $\frac{1}{2}$ d.

Then $1\frac{1}{2}$ lbs. : 50 lbs. :: $1\frac{1}{2}$ lbs. : 50 lbs. brass @ 14d.

$1\frac{1}{2}$ lbs. : 50 lbs. :: 6 lbs. : 200 lbs. pewter @ 10 $\frac{1}{2}$ d.

(20)

By case I we find that 1 oz. of 20 carats fine, 1 of 21 carats fine and 3 of 23 carats fine, will make a mixture 22 carats fine.

Then 1 oz. : 30 oz. :: 1 oz. : 30 oz. of 21 carats fine.

1 oz. : 30 oz. :: 3 oz. : 90 oz. of 23 carats fine.

Page 285.

(22)

To produce a mixture worth \$1.40 per lb., we require 20 lbs. @ \$1.00, 40 @ \$1.20, 40 @ \$1.60, and 20 @ \$1.80. But all of these added together, will make 120 lbs.

lbs. lbs. lbs. lbs.
 Therefore 120 : 20 :: 168 : $\frac{168 \times 20}{126} = 28$ lbs., the required quantity @ \$1.00.

120 : 40 :: 168 : $\frac{168 \times 40}{126} = 56$ lbs., the required quantity @ \$1.20.

120 : 40 :: 168 : $\frac{168 \times 40}{120} = 56$ lbs., the required quantity @ \$1.60.

120 : 20 :: 168 : $\frac{168 \times 20}{120} = 28$ lbs., the required quantity @ \$1.80.

(23)

To produce a mixture worth 4s. 4d. per lb., we require 10 lbs. @ 5s. and 8 @ 3s. 6d. But these added together make 18 lbs.

lbs. lbs. lbs. lbs.
 Therefore 18 : 10 :: 27 : $\frac{27 \times 10}{18} = 15$ lbs., the required quantity of tea @ 5s.

18 : 8 :: 27 : $\frac{27 \times 8}{18} = 12$ lbs., the required quantity of tea @ 3s. 6d.

(24)

To produce a mixture worth \$2.70 per gallon, we require 20 gallons @ \$2.40, 10 @ \$2.60, 10 @ \$2.80, and 30 @ \$2.90. But all of these added together will make 70 gallons. Therefore

gals. gals. gals. gals.

$$70 : 20 :: 63 : \frac{63 \times 20}{70} = 18 \text{ gallons, the required quantity of brandy @ \$2.40.}$$

$$70 : 10 :: 63 : \frac{63 \times 10}{70} = 9 \text{ gallons, the required quantity of brandy @ \$2.60.}$$

$$70 : 10 :: 63 : \frac{63 \times 10}{70} = 9 \text{ gallons, the required quantity of brandy @ \$2.80.}$$

$$70 : 30 :: 63 : \frac{63 \times 30}{70} = 27 \text{ gallons, the required quantity of brandy @ \$2.90.}$$

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(4)

$$1974.80 \times \frac{3}{4} = £740.55 = £740 \text{ 11s.}$$

(5)

$$765.43 \times \frac{2}{3} = £306.172 = £306 \text{ 3s. } 5\frac{7}{8}\text{d.}$$

(6)

$$8172.19 \times \frac{1}{4} = £2043.0475 = £2043 \text{ 0s. } 11\frac{3}{8}\text{d.}$$

(9)

$$£743 \text{ 18s. 11d.} = £743.94583 \text{ and } 743.94583 \div \frac{3}{10} = \$2479.8194.$$

(10)

$$£119\ 9s.\ 8\frac{1}{4}d. = £119.484375 \text{ and } 119.484375 \div \frac{3}{8} = \$318.625.$$

(11)

$$£473\ 17s.\ 1\frac{1}{2}d. = £473.8572916, \text{ and } 473.8572916 \div \frac{7}{36} = \$2030.816964.$$

Page 290.

(13)

$$1006.90 \div 4.867 = £206.88309 = £206\ 17s.\ 7\frac{3}{4}d.$$

(14)

$$916.87 \div 4.867 = £188.38504 = £188\ 7s.\ 8\frac{1}{4}d.$$

(15)

$$2114.81 \div 4.867 = £434.52023 = £334\ 10s.\ 4\frac{1}{4}d.$$

(17)

$$£2043\ 11s.\ 3d. = £2043.5625 \text{ and } 2043.5625 \times 4.867 = \$9946.01868.$$

(18)

$$£777\ 7s.\ 7d. = £777.37916 \text{ and } 777.37916 \times 4.867 = \$3783.50437.$$

(19)

$$£557\ 19s.\ 5\frac{1}{2}d. = £557.972916 \text{ and } 557.972916 \times 4.867 = \$2715.65418.$$

Page 294.

(4)

$$\$16785.25 \times 5.04 = 84597 \text{ francs } 66 \text{ centimes.}$$

(5)

Commercial value of the marc banco = 35 cents.

Add 1 per cent .35 "

35.35

$$\text{Then } 35.35 \text{ cents} \times 4000 = \$1414.$$

(6)

$$\$35678 \times 1.0225 = \$36480.755.$$

(7)

The par value of 1 ruble = 75 cents.

Deduct 2 per cent 1.5 "

73.5

$$\text{Then } 73.5 \text{ cents} \times 2560 = \$1881.60.$$

(8)

Old commercial par of £1 sterling = \$4.444 = \$4.44444

Add 8 per cent .35555

\$4.79999

$$\text{Then } \$4.79999 \times 800 = \$3839.999 = \$3840.00.$$

Page 295.

(3)

£1 = 420d.

19½d. = 1 franc.

300 francs = 60 ducats.

1 ducat = 360 maravedis.

$x = £1000.$

$$x = \frac{84 \times 220 \times 1 \times 60 \times 360 \times 1000}{19\frac{1}{2} \times 300} = 1564138 \text{ maravedis by cir. ex.}$$

$$42\frac{1}{2}d. : £1000 :: 272 \text{ maravedis} : \frac{272 \times 1000 \times 20 \times 12}{42\frac{1}{2}} =$$

$$\frac{16 \times 272 \times 1000 \times 8 \times 12}{17} = 1536000 \text{ maravedis by direct exchange.}$$

Difference = 1564138 — 1536000 = 28138 maravedis.

34)28138

8)827 reals 20 maravedis.

103 piastres 3 reals 20 maravedis.

(4)

Old commercial par of £1 sterling = \$4.444

To which add 10 per cent. of itself = .4444

Gives price of £1 sterling = \$4.8884

\$4888.40 ÷ \$4.8884 = £1000 = amount of bill he receives if he remits direct to London.

$$\left. \begin{array}{l} \$1 = 515 \text{ centimes.} \\ 2580 \text{ cen.} = £1 \text{ sterling.} \\ x = \$4888.40 \end{array} \right\} x = \frac{515 \times 4888.40}{2580} = £975.78526.$$

= £975 15s. 8¼d.+ = amount of bill he receives if he remits through Paris.

35 cents = 1 marc.

13½ marcs = £1 sterling.

$x = \$4888.40.$

$$x = \frac{4888.40}{.35 \times 13\frac{1}{2}} = \frac{391072}{385} = £1015.77142 = £1015 15s. 5d.+ =$$

amount of bill he receives by remitting through Hamburg.

(5)

$$\begin{array}{lcl}
 18 \text{ cents.} & = & 1 \text{ franc.} \\
 25 \text{ francs.} & = & 240 \text{d.} \\
 180 \text{d.} & = & 3 \text{ milrees.} \\
 5 \text{ milrees.} & = & 18 \text{ marcs ban.} \\
 1200 \text{ marcs ban.} & = & x
 \end{array}
 \left. \vphantom{\begin{array}{lcl} 18 \text{ cents.} \\ 25 \text{ francs.} \\ 180 \text{d.} \\ 5 \text{ milrees.} \\ 1200 \text{ marcs ban.} \end{array}} \right\} x = \frac{18 \times 25 \times 180 \times 1200 \times 5}{240 \times 3 \times 18}$$

$= \$375 =$ circuitous exchange or sum he pays for 1200 marks.

$1200 \times .35 = \$420 =$ direct exchange or sum paid for 1200 marks. $\$420 - \$375 = \$45 =$ gain by circuitous exchange.

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(3)

$$(3)^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243.$$

(4)

$$(20)^{10} = 20 \times 20 \times 20 \times 20 \times 20 \times 20 \times 20 \times 20 \times 20 \times 20 = 10240000000000.$$

(5)

$$(1.05)^6 = 1.05 \times 1.05 \times 1.05 \times 1.05 \times 1.05 \times 1.05 = 1.340095640625.$$

(6)

$$\left(\frac{3}{5}\right)^7 = \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} = \frac{2187}{78125}.$$

(7)

$$\left(\frac{5}{9}\right)^5 = \frac{5}{9} \times \frac{5}{9} \times \frac{5}{9} \times \frac{5}{9} \times \frac{5}{9} = \frac{3125}{59049}.$$

(8)

$$11\frac{2}{3} = 57. \quad \left(\frac{57}{5}\right)^3 = \frac{57}{5} \times 57 \times \frac{57}{5} = \frac{185193}{125} = 1481\frac{68}{125}.$$

Page 299.

(9)

$$4^2 \times 4^4 \times 4^5 \times 4^7 = 4^{2+4+5+7} = 4^{18}.$$

(10)

$$13^{11} \div 13^2 = 13^{11-2} = 13^9.$$

(11)

$$(3^3)^5 = 3^{3 \times 5} = 3^{15}.$$

(12)

$$\begin{aligned} \{(7^4 \times 7^3) \div (7^2 \times 7^2)\}^6 &= \{(7^4 \div 7^2) \times (7^3 \div 7^2)\}^6 = \\ &= \{7^2 \times 7^1\}^6 = (7^3)^6 = 7^{3 \times 6} = 7^{18}. \end{aligned}$$

(13)

$$\begin{aligned} \{(5^3 \times 5^4 \times 5^{11} \times 5^9) \div (5^3 \times 5^2 \times 5^7 \times 5^5)\}^3 &= \\ \{(5^{3+4+11+9}) \div (5^{3+2+7+5})\}^3 &= \{5^{27} \div 5^{17}\}^3 = \\ (5^{27-17})^3 &= (5^{10})^3 = 5^{10 \times 3} = 5^{30}. \end{aligned}$$

Page 304.

(4)

$$\begin{array}{r} \begin{array}{c} \cdot \cdot \cdot \\ 195364 \end{array} (442 \\ 16 \\ \hline 84)353 \\ 336 \\ \hline 882)1764 \\ 1764 \end{array}$$

(5)

$$\begin{array}{r} \begin{array}{c} \cdot \cdot \\ .0676 \end{array} (.26 \\ 4 \\ \hline 46)276 \\ 276 \end{array}$$

(6)

$$\begin{array}{r} \begin{array}{c} \cdot \cdot \cdot \\ 984064 \end{array} (992 \\ 81 \\ \hline 189)1740 \\ 1701 \\ \hline 1982)3964 \\ 3964 \end{array}$$

(7)

$$\begin{array}{r}
 \begin{array}{c} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{array} 5 \cdot 0000000000 (2 \cdot 23606 \\
 \begin{array}{c} 4 \\ - \end{array} \\
 42) 1 \cdot 00 \\
 \quad \cdot 84 \\
 \quad - \\
 443) \cdot 1600 \\
 \quad \cdot 1329 \\
 \quad - \\
 4466) 27100 \\
 \quad 26796 \\
 \quad - \\
 447206) 3040000 \\
 \quad 2683236 \\
 \quad - \\
 \quad 356764
 \end{array}$$

(9)

$$\begin{array}{r}
 \begin{array}{c} \cdot \\ \cdot \\ \cdot \\ \cdot \end{array} 60 \cdot 487129 (7 \cdot 777 \\
 \begin{array}{c} 49 \\ - \end{array} \\
 147) 1148 \\
 \quad 1029 \\
 \quad - \\
 1547) 11971 \\
 \quad 10829 \\
 \quad - \\
 15547) 114229 \\
 \quad 108829 \\
 \quad - \\
 \quad 5400
 \end{array}$$

(11)

$$\begin{array}{r}
 \begin{array}{c} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{array} 0000012321 (0 \cdot 00111 \\
 \begin{array}{c} 1 \\ - \end{array} \\
 21) 23 \\
 \quad 21 \\
 \quad - \\
 221) 221 \\
 \quad 221
 \end{array}$$

(8)

$$\begin{array}{r}
 \begin{array}{c} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{array} 500000000000 (0 \cdot 707106 \\
 \begin{array}{c} 49 \\ - \end{array} \\
 1407) 10000 \\
 \quad 9849 \\
 \quad - \\
 14141) 15100 \\
 \quad 14141 \\
 \quad - \\
 1414206) 9590000 \\
 \quad 8485236 \\
 \quad - \\
 \quad 1104764
 \end{array}$$

(10)

$$\begin{array}{r}
 \begin{array}{c} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{array} 79792266297612001 (282475249 \\
 \begin{array}{c} 4 \\ - \end{array} \\
 48) 397 \\
 \quad 384 \\
 \quad - \\
 562) 1392 \\
 \quad 1124 \\
 \quad - \\
 5644) 26826 \\
 \quad 22576 \\
 \quad - \\
 56487) 425062 \\
 \quad 395409 \\
 \quad - \\
 564945) 2965397 \\
 \quad 2824725 \\
 \quad - \\
 5649502) 14067261 \\
 \quad 11299004 \\
 \quad - \\
 56495044) 276825720 \\
 \quad 225980176 \\
 \quad - \\
 564950489) 5084554401 \\
 \quad 5084554401
 \end{array}$$

(14)

$$\dot{1} = \frac{1}{9} \text{ and } \sqrt{\frac{1}{9}} = \frac{1}{3}.$$

(16)

$$5\frac{1}{7} = 5.142857142857 \text{ and } \sqrt{5.142857142857} = 2.267786.$$

(17)

$$\frac{217}{638} = .4033457230 \text{ and } \sqrt{.4033457230} = .63509.$$

(18)

$$13\frac{1}{2} = 13.2 \text{ and } \sqrt{13.2} = 3.633.$$

————

Page 305.

(20)

$$\begin{array}{r} \dot{1}\dot{1}\dot{3}\dot{3}\dot{3}\dot{1}\dot{1}(2626 \\ 4 \\ - \\ 46)433 \\ 411 \\ \hline 552)2233 \\ 1434 \\ \hline 5546)46611 \\ 46611 \end{array}$$

(21)

$$\begin{array}{r} \dot{3}\dot{3}\dot{2}\dot{3}\dot{3}\dot{3}\dot{4}\dot{4}(4344 \\ 24 \\ - \\ 123)523 \\ 413 \\ \hline 1304)11033 \\ 10024 \\ \hline 13124)100544 \\ 100544 \end{array}$$

(22)

$$\begin{array}{r}
 \begin{array}{r}
 \cdot \cdot \\
 4234 \cdot 101230(43 \cdot 412 \\
 31 \\
 \hline
 133)1134 \cdot \\
 1004 \\
 \hline
 1414)130 \cdot 10 \\
 122 \cdot 21 \\
 \hline
 14231)2 \cdot 3412 \\
 1 \cdot 4231 \\
 \hline
 142322) \cdot 413130 \\
 \cdot 340144 \\
 \hline
 22431
 \end{array}
 \end{array}$$

(23)

$$\begin{array}{r}
 \begin{array}{r}
 \cdot \cdot \cdot \cdot \\
 888888 \cdot 8880(888 \cdot 88 \\
 71 \\
 \hline
 178)1788 \\
 1601 \\
 \hline
 1878)18788 \\
 16801 \\
 \hline
 18878)1887 \cdot 88 \\
 1688 \cdot 01 \\
 \hline
 188878)188 \cdot 8780 \\
 168 \cdot 8801 \\
 \hline
 18 \cdot 8878
 \end{array}
 \end{array}$$

(24)

$$\begin{array}{r}
 \begin{array}{r}
 \cdot \cdot \cdot \cdot \cdot \\
 248664e \ t69(54373 \\
 21 \\
 \hline
 t4)386 \\
 354 \\
 \hline
 t83)3264 \\
 2809 \\
 \hline
 t867)657e \ t \\
 62e \ t1 \\
 \hline
 t8723)281969 \\
 281969 \\
 \hline
 \end{array}
 \end{array}$$

Page 307.

(27)

$$\begin{array}{r}
 100^2 = 10000 \\
 60^2 = 3600 \\
 \hline
 \end{array}$$

$$\text{Difference} = 6400 \text{ and } \sqrt{6400} = 80.$$

(28)

$$50^2 = 2500$$

$$80^2 = 6400$$

$$\text{Sum} = \overline{8900} \text{ and } \sqrt{8900} = 94.34 \text{ nearly.}$$

(29)

$$24^2 = 576 \div 2 = 288 \text{ and } \sqrt{288} = 16.97.$$

(30)

$$36^2 = 1296$$

$$20^2 = 400$$

$$\text{Difference} = \overline{896} \text{ and } \sqrt{896} = 29.933.$$

(31)

$$40^2 = 1600$$

$$14^2 = 196$$

$$\text{Difference} = \overline{1404} \text{ and } \sqrt{1404} = 37.469.$$

$$40^2 = 1600$$

$$26^2 = 676$$

$$\text{Difference} = \overline{924} \text{ and } \sqrt{924} = 30.397.$$

$$37.469 + 30.397 = 67.866 \text{ and } 67.866 \div 3 = 22.622.$$

(32)

$$1760 \text{ sq. yds.} = 15840 \text{ sq. ft. and } \sqrt{15840} = 125.857.$$

(33)

$$\sqrt{141376} = 376.$$

(34)

$$3^2 = 9$$

$$3^2 = 9$$

$$\text{Sum} = \overline{18} \text{ and } \sqrt{18} = 4.24264.$$

(35)

$$16^2 = 256$$

$$12^2 = 144$$

$$\text{Sum} = 400 \text{ and } \sqrt{400} = 20.$$

(36)

$$3^2 + 3^2 + 3^2 = 27 \text{ and } \sqrt{27} = 5.196.$$

(37)

$$\left(\frac{1}{10}\right)^2 = \frac{1}{100} \text{ and } (1)^2 = 1.$$

$$\text{Then } \frac{1}{100} : 1 :: 450 : \frac{450}{\frac{1}{100}} = 45000.$$

(38)

$$1 \text{ sq. acre} = 160 \text{ sq. perches. } 160 \div 3.1416 = 50.929462 \text{ and } \sqrt{50.929462} = 7.136.$$

 Page 311.

(2)

		62712728317(3973
		27
		<hr/>
$3^2 \times 300 =$	2700	35712
$3 \times 9 \times 30 =$	810	
$9^2 =$	81	
	<hr/>	
	3591	32319
	<hr/>	
$39^2 \times 300 =$	456300	3393728
$39 \times 7 \times 30 =$	8190	
$7^2 =$	49	
	<hr/>	
	464539	3251773
	<hr/>	
$397^2 \times 300 =$	47282700	141955317
$397 \times 3 \times 30 =$	35730	
$3^2 =$	9	
	<hr/>	
	47318439	141955317

(3)

	1953125(125
	1
	—
$1^2 \times 300 = 300$	953
$1 \times 2 \times 30 = 60$	
$2^2 = 4$	
	728
	—
$12^2 \times 300 = 43200$	225125
$12 \times 5 \times 30 = 1800$	
$5^2 = 25$	
45025	225125

(4)

	1076890625(1025
	1
	—
$1^2 \times 300 = 300$	76
$10^2 \times 300 = 30000$	76890
$10 \times 2 \times 30 = 600$	
$2^2 = 4$	
	61208
	—
$102^2 \times 300 = 3121200$	15682625
$102 \times 5 \times 30 = 15300$	
$5^2 = 25$	
3136525	15682625

(5)

		$\cdot 69\dot{7}86\dot{4}10\dot{3}(\cdot 887$
		512
		<hr/>
$8^2 \times 300 =$	19200	185864
$8 \times 8 \times 30 =$	1920	
$8^2 =$	64	
	<hr/>	
	21184	169472
	<hr/>	<hr/>
$88^2 \times 300 =$	2323200	16392103
$88 \times 7 \times 30 =$	18480	
$7^2 =$	49	
	<hr/>	
	2341729	16392103

(6)

		$\cdot 102503 \cdot 232(46 \cdot 8$
		64
		<hr/>
$4^2 \times 300 =$	4800	38503
$4 \times 6 \times 30 =$	720	
$6^2 =$	36	
	<hr/>	
	5556	33336
	<hr/>	<hr/>
$46^2 \times 300 =$	634800	5167 \cdot 232
$46 \times 8 \times 30 =$	11040	
$8^2 =$	64	
	<hr/>	
	645904	5167 \cdot 232

(7)

		$\dot{1}79597 \cdot 069288(56 \cdot 42.$
		125
		<hr/>
$5^2 \times 300 =$	7500	54597
$5 \times 6 \times 30 =$	900	
$6^2 =$	36	
	<hr/>	
	8436	50616
	<hr/>	<hr/>
$56^2 \times 300 =$	940800	3981 \cdot 069
$56 \times 4 \times 30 =$	6720	
$4^2 =$	16	
	<hr/>	
	947536	3790 \cdot 144
	<hr/>	<hr/>
$564^2 \times 300 =$	95428800	190 \cdot 925288
$564 \times 2 \times 30 =$	33840	
$2^2 =$	4	
	<hr/>	
	95462644	190 \cdot 925288

(8)

		$\dot{4}83 \cdot 736625(7 \cdot 85.$
		343
		<hr/>
$7^2 \times 300 =$	14700	140 \cdot 736
$7 \times 8 \times 30 =$	1680	
$8^2 =$	64	
	<hr/>	
	16444	131 \cdot 552
	<hr/>	<hr/>
$78^2 \times 300 =$	1825200	9 \cdot 184625
$78 \times 5 \times 30 =$	11700	
$5^2 =$	25	
	<hr/>	
	1836925	9 \cdot 184625

(9)

$$\begin{array}{r}
 8^2 \times 300 = 19200 \\
 8 \times 6 \times 30 = 1440 \\
 6^2 = 36 \\
 \hline
 20676
 \end{array}
 \qquad
 \begin{array}{r}
 \cdot 63605\dot{6}(\cdot 86. \\
 512 \\
 \hline
 124056 \\
 \\
 124056
 \end{array}$$

Page 312.

(12)

$$\sqrt[2]{.19} = .105263157894+ \text{ and } \sqrt[3]{.105263157894} = .4721.$$

(13)

$$\sqrt[3]{.17} = .176470588235+ \text{ and } \sqrt[4]{.176470588235} = .5609.$$

(14)

$$\frac{1}{8} \text{ of } 2\frac{1}{2} = \frac{5}{6} = .833333333+ \text{ and } \sqrt[5]{.833333333} = .941.$$

(15)

$$28\frac{3}{4} = 28.75 \text{ and } \sqrt[3]{28.75} = 3.063.$$

(16)

$$32\frac{8}{11} = 32.\dot{7}\dot{2} \text{ and } \sqrt[4]{32.\dot{7}\dot{2}} = 3.198.$$

Page 313.

(18)

One million = 33233344 senary.

		33233344(244.
		12
		<hr/>
$2^2 = 4 \times 300 =$	2000	21233
$2 \times 30 = 100 \times 4 =$	400	
$4^2 =$	24	
	<hr/>	
	2424	14544
	<hr/>	
$24^2 = 1104 \times 300 =$	332000	2245344
$24 \times 30 = 1200 \times 4 =$	5200	
$4^2 =$	24	
	<hr/>	
	341224	2245344

(19)

		6131271.000000(165.32.
		1
		<hr/>
$1^2 \times 300 =$	300	5131
$1 \times 30 \times 6 =$	220	
$6^2 =$	44	
	<hr/>	
	564	4270
	<hr/>	
$16^2 = 304 \times 300 =$	111400	641271
$16 \times 30 = 520 \times 5 =$	3220	
$5^2 =$	31	
	<hr/>	
	114651	600115
	<hr/>	
$165^2 = 32571 \times 300 =$	12015300	41154.000
$165 \times 30 = 5370 \times 3 =$	20350	
$3^2 =$	11	
	<hr/>	
	12035661	36131.423
	<hr/>	
$1653^2 = 3272071 \times 300 =$	1205625300	3022.355000
$1653 \times 30 = 54010 \times 2 =$	130020	
$2^2 =$	4	
	<hr/>	
	1205755324	2413.732650
		<hr/>
		406.422130

(20)

		$10\dot{2}210\dot{1}2 \cdot 10\dot{2}000000$
		$1 \overline{)112 \cdot 012} = \text{root.}$
$1 \times 1000 =$	1000	2221
$1 \times 1 \times 100 =$	100	
$1^2 =$	1	
	<u>1101</u>	<u>1101</u>
$11^2 = 121 \times 1000 =$	121000	1120012
$11 \times 100 = 1100 \times 2 =$	2200	
$2^2 =$	11	
	<u>200211</u>	<u>1101122</u>
$112^2 = 21021 \times 1000 =$	21021000	11120 \cdot 102
$1120^2 = 2102100 \times 1000 =$	2102100000	11120 \cdot 102000
$1120 \times 100 = 112000 \times 1 =$	112000	
$1^2 =$	1	
	<u>2102212001</u>	<u>2102 \cdot 212001</u>
$11201^2 = 211010101 \times 1000 =$	211010101000	2010 \cdot 112222000
$11201 \times 100 = 1120100 \times 2 =$	10010200	
$2^2 =$	11	
	<u>211020111211</u>	<u>1122 \cdot 111000122</u>
		<u>111 \cdot 001221101</u>

Page 314.

(25)

$$3^3 : 6^3 :: 4 \text{ lbs.} : \text{Ans.} = 32 \text{ lbs.}$$

(26)

$$1^3 : \left(\frac{7}{2}\right)^3 :: \$120 : \text{Ans.} = \$5145.$$

(27)

$$(70)^3 : \left(\frac{623}{6}\right)^3 :: 180 \text{ lbs.} : \text{Ans.}$$

$$343000 : \frac{241804367}{125} :: 180 : \text{Ans.} =$$

$$180 \times \frac{241804367}{125} \times \frac{1}{343000} = 1015.1 \text{ lbs.}$$

(28)

$$973^3 = 921167317$$

$$45^3 = 91125$$

$$62^3 = 238328$$

$$30^3 = 27000$$

$$80^3 = 512000$$

$$20^3 = 8000$$

$$9221167317 - (91125 + 238328 + 27000 + 512000 + 8000) = 920290864 \text{ and } \sqrt[3]{920290864} = 972.69.$$

(29)

8 feet 3 inches = 99 inches, 3 feet = 36 inches, and 2 feet 7 inches = 31 inches.

$$99 \times 36 \times 31 = 110484 \text{ and } \sqrt[3]{110484} = 47.9843.$$

(30)

After the first has wound off her portion, there will remain $\frac{3}{4}$ of the thread.

Then the whole ball : part remaining :: cube of diameter of whole ball : cube of diameter of part remaining.

That is, $1 : \frac{3}{4} :: 3^3 : x^3$, and hence $x = 3 \times \sqrt[3]{\frac{1}{4}} = 3 \times \sqrt[3]{.75} = .90856 \times 3 = 2.72568 =$ diameter of the ball after the first has wound off her portion.

Similarly after the second has wound off her portion, there will remain $\frac{1}{2}$ of the ball, and after the third has taken her portion, $\frac{1}{4}$ of the ball.

Hence $1 : \frac{1}{2} :: 3^3 : x^3$, whence $x = 3 \times \sqrt[3]{\frac{1}{2}} = 3 \times \sqrt[3]{.5} = 3 \times .79370 = 2.38110 =$ diameter after the second has taken her portion.

$1 : \frac{1}{4} :: 3^3 : x^3$, whence $x = 3 \times \sqrt[3]{\frac{1}{4}} = 3 \times \sqrt[3]{.25} = 3 \times .62996 = 1.88988 =$ diameter after the third has taken her portion.

Hence 1st takes off 3	— 2.72568 =	.27432 inches.
2nd “ “	2.72568 — 2.38110 =	.34458 “
3rd “ “	2.38110 — 1.88988 =	.49122 “
4th “ “	remaining	1.88988 “

Page 315.

(1)

$$\sqrt{19987173376} = 141376, \text{ and } \sqrt{141376} = 376.$$

(2)

$$\sqrt[3]{308915776} = 676, \text{ and } \sqrt{676} = 26.$$

(3)

$$\sqrt[3]{40353607} = 343, \text{ and } \sqrt[3]{343} = 7.$$

(4)

$$\sqrt[3]{387420489} = 729, \sqrt[3]{729} = 9, \text{ and } \sqrt{9} = 3.$$

(5)

$$\sqrt[3]{134217728} = 512, \sqrt[3]{512} = 8, \text{ and } \sqrt[3]{8} = 2.$$

 Page 321.

(13)

The mantissa of the logarithm of 8193 (the first four digits) = .913443, and the next following mantissa is .913496.

Then from .913496

Subtract .. .913443

Difference, 53; and 53×217 (remaining digits of given number) = 11501, from which we cut off three digits, since we multiplied by a number of three digits, and since the highest digit cut off is not less than 5, we add unity to the part retained, which gives us 12.

Then mantissa of logarithm of first four digits .913443

Add, 12

Mantissa of logarithm of given number, .913455

To which attach the characteristic 6 and required logarithm = 6.913455.

The mantissa of the logarithm of 7392 (the first four digits) = .868762, and the next following mantissa is .868821.

Then from .868821

Subtract .. .868762

Difference, 59; and 59×45 (remaining digits of given number) = 2655, from which we cut off two digits, since we multiplied by a number of two digits, and since the highest digit cut off is not less than 5, we add unity to the part retained, which gives us 27.

Then mantissa of logarithm of first four digits, .868762

Add, 27

Mantissa of logarithm of given number, .868789

(Continued on next page.)

(13 continued.)

To which attach the characteristic 1 and required logarithm =
1.868789.

The mantissa of the logarithm of 8437 (the first four digits)
= .926188, and the next following mantissa is .926240.

Then from .926240

Subtract .. .926188

Difference, 52 ; and 52×42 (remaining digits of given number) = 2184, from which we cut off two digits, since we multiplied by a number of two digits, and since the highest digit cut off is not less than 5, we add unity to the part retained, which becomes 22.

Then mantissa of logarithm of first four digits .926188

Add, 22

Mantissa of logarithm of given number, .926210

To which attach the characteristic $\bar{1}$ and required logarithm =
 $\bar{1}.926210$.

(14)

The mantissa of the logarithm of 2345 = .370143, and the next following mantissa is .370338.

Then from .370338

Subtract .. .370143

Difference, 185 ; and $185 \times 64 = 11840$, from which we cut off two digits, since we multiplied by a number of two digits, which gives us 118.

Then mantissa of logarithm of 2345 = .370143

Add, 118

Mantissa of logarithm of given number = .370261

To which attach the characteristic $\bar{4}$ and required logarithm =
 $\bar{4}.370261$.

(Continued on next page.)

(14 continued.)

The mantissa of the logarithm of 1007 = .003029, and the next following mantissa is .003461.

Then from .003461

Subtract .. .003029

Difference, 432 ; and $432 \times 013 = 5616$, from which we cut off three digits, since we multiplied by a number of three digits, and since the highest digit cut off is not less than 5, we add unity to the part retained, which gives us 6.

Then mantissa of logarithm of 1007 = .003029

Add, 6

Mantissa of logarithm of given number .003035

To which attach the characteristic $\bar{3}$ and required logarithm = $\bar{3}.003035$.

(15)

Mantissa of logarithm of 5237..... .719083

Difference from column D = 83 ; and $83 \times 6 = 498$

from which we cut off 1 digit and add..... 50

And also attach the characteristic 1, and required

logarithm = 1.719133

Mantissa of logarithm of 1294..... .111934

Difference from column D = 335 ; and $335 \times 76 =$

25460 from which we cut off two digits and add, 255

And also attach the characteristic 2 and required

logarithm = 2.112189

(16)

Mantissa of logarithm of	·0004713	=	·673297
P. P. corresponding to	·00000009	=	83
P. P. " to	·000000008	=	74

Sum, = ·6733874

Therefore required mantissa = ·673387 and required logarithm
= 4·673387.

Mantissa of logarithm of	9136000	=	·960756
P. P. corresponding to	700	=	33
P. P. " to	10	=	5
P. P. " to	2	=	9

Sum, = ·96078959

Therefore required mantissa = ·960790 and required logarithm
= 6·960790.

(17)

Mantissa of logarithm of	4·23400	=	·626751
P. P. corresponding to	20	=	20
P. P. " to	9	=	92

Sum, = ·6267802

Therefore required logarithm is 0·626780.

Mantissa of logarithm of	763·1	=	·882581
P. P. corresponding to	·02	=	11
P. P. " to	·009	=	51
P. P. " to	·0008	=	46
P. P. " to	·00007	=	40

Sum, = ·882597600

Therefore required logarithm is 2·882598.

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(20)

Given logarithm, $\cdot 137139$ Next lower in table, $\cdot 137037 = \log.$ of 1371.Difference $\quad \quad \quad 102$, Tabular difference $= 316$.

Then $1020000 \div 316$ gives 3227 for digits in 5th, 6th, 7th, and 8th places.

Hence the digits of the natural number are 13713227; and since the characteristic is 4, i. e., one less than the number of digits to the left of the decimal point, the required number is $13713 \cdot 227$.

Given logarithm, $\cdot 718134$ Next lower in table, $\cdot 718086 = \log.$ of 5225.Difference, $\quad \quad \quad 48$, Tabular difference $= 83$.

Then $48000 \div 83$ gives 578 for digits in 5th, 6th and 7th places.

Hence the digits of the natural number are 5225578, and since the characteristic is 0, i. e., one less than the number of digits to the left of the decimal point, the required number is $5 \cdot 225578$.

Given logarithm, $\cdot 635421$ Next lower in table, $\cdot 635383 = \log.$ of 4319.Difference, $\quad \quad \quad 38$, Tabular difference $= 101$.

Then $38000 \div 101$ gives 376 for digits in 5th, 6th, and 7th places.

Hence the digits of the natural number are 4319376, and since the characteristic is $\bar{4}$, i. e., one more than the number of ciphers between the decimal point and the first figure to the right, the required number is $\cdot 0004319376$.

(21)

Given $\log. \cdot 921686 = \log. \text{ of } 8350.$

And since the characteristic is 2, i. e., one less than the number of digits to the left of the decimal point, the required number is 835.

Given logarithm, $\cdot 922165$

Next lower in table, $\cdot 922154 = \log. \text{ of } 8359.$

Difference = 11, Tabular difference = 52.

Then $11000 \div 52$ gives 211 for digits in 5th, 6th, and 7th places.

Hence the digits of the natural number are 8359211; and since the characteristic is $\bar{1}$, i. e., one more than the number of ciphers between the decimal point and first figure to the right, the required number is $\cdot 8359211.$

(22)

Given logarithm, $\cdot 407968$

Next lower in table, $\cdot 407901 = \log. \text{ of } 2558.$

Difference, = 67

Highest P.P. not greater than 67 = 51 corresponds to 3
for 5th place.

Highest P.P. not greater than 160 = 153 corresponds to 9
for 6th place.

Highest P.P. not greater than 70 = 68 corresponds to
4 for 7th place.

Therefore digits of required number are 2558394; and since the characteristic is 5, there must be six digits to the left of the decimal point.

Hence required number is 2558394.

(Continued on next page.)

(22 continued.)

Given logarithm,	$\cdot 408386$	
Next lower in table,	$\cdot 408240 = \log. \text{ of } 2560.$	
Difference, =	<u>146</u>	
Highest P.P. not greater than 146 =	136	corresponds to 8
	<u>100</u>	in 5th place.
Highest P.P. not greater than 100 =	85	corresponds to 5
	<u>150</u>	in 6th place.
Highest P.P. not greater than 150 =	136	corresponds to
	<u>140</u>	8 in 7th place.
Highest P.P. not greater than 140 =	136	corresponds to
		8 in 8th place.

Therefore digits of required number are 25608588 ; and since the characteristic is 7, there must be eight digits to the left of the decimal point.

Hence required number is 25608588.

Given logarithm,	$\cdot 416369$	
Next lower in table,	$\cdot 416308 = \log. \text{ of } 2608.$	
Difference, =	<u>61</u>	
Highest P.P. not greater than 61 =	49	corresponds to 3
	<u>12</u>	in 5th place.

Therefore digits of required number are 26083 ; and since the characteristic is $\bar{3}$, there must be two ciphers between the decimal point and first figure.

Hence required number is $\cdot 0026083$.

(23)

Given logarithm,	·877777
Next lower in table,	·877774 = log. of 7547.
Difference =	<u>3</u>
There is no P.P. not greater than 3	0 corresponds to 0 in 5th place.
	<u>30</u>
Highest P.P. not greater than 30 =	29 corresponds to 5 in 6th place.
	<u>10</u>
Highest P.P. not greater than 10 =	6 corresponds to 1 in 7th place.
	<u>40</u>
Highest P.P. not greater than 40 =	35 corresponds to 6 in 8th place.
	<u>50</u>
Highest P.P. not greater than 50 =	46 corresponds to 8 in 9th place.
	<u>4</u>

Therefore digits of required number are 754705168 ; and since the characteristic is 4, there must be five digits to the left of the decimal point.

Hence required number is 75470·5168.

Given logarithm,	·555555
Next lower in table,	·555457 = log. of 3593.
Difference, =	<u>98</u>
Highest P.P. not greater than 98 =	98 corresponds to 8 in 5th place.

Therefore digits of required number are 35938 ; and since the characteristic is 0, there must be one digit to the left of the decimal point.

Hence required number is 3·5938.

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(1)

$$10 - 5.631642 = 4.368358.$$

$$10 - 0.714000 = 9.286000.$$

(2)

$$10 - \bar{3}.123456 = \bar{12}.876544.$$

$$10 - \bar{7}.213149 = 16.786851.$$

(3)

$$10 - 6.124357 = 3.875643 \text{ and } 10 - 2.000837 = 11.999163.$$

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(5)

$$\text{Logarithm of } 61 = 1.785330$$

$$\text{" } 22 = 1.342423$$

$$\text{" } 65 = 1.812913$$

$$\text{Sum} = 4.940666 = \text{logarithm of } 87230.$$

(6)

$$\text{Logarithm of } 52 = 1.716003$$

$$\text{" } 734 = 2.865696$$

$$\text{" } 6 = 0.778151$$

$$\text{Sum} = 5.359850$$

$$5.359835 = \text{logarithm of } 229000$$

$$15 = \quad \quad \quad 7$$

$$\text{Ans. } 229007$$

(7)

$$\begin{array}{rcl}
 \text{Logarithm of } 35.86 & = & 1.554610 \\
 \text{" } 2.1046 & = & 0.323169 \\
 \text{" } .8372 & = & \bar{1}.922829 \\
 \text{" } .00294 & = & \bar{3}.468347 \\
 & & \hline
 \text{Sum} & = & \bar{1}.268955 = \text{logarithm of } .185700 \\
 & & \bar{1}.268812 \\
 & & \hline
 & & 143 = \qquad \qquad \qquad 61 \\
 & & \hline
 & & \text{Ans. } .185761
 \end{array}$$

(8)

$$\begin{array}{rcl}
 \text{Log. of } .00008764 & = & \bar{5}.942702 \\
 \text{" } .86359 & = & \bar{1}.936308 \\
 & & \hline
 \text{Sum} & = & \bar{5}.879010 \\
 & & \bar{5}.878981 = \text{logarithm of } .000075680 \\
 & & \hline
 & & 29 = \qquad \qquad \qquad 5 \\
 & & \hline
 & & \text{Sum } .000075685
 \end{array}$$

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(11)

$$\begin{array}{rcl}
 \text{Logarithm of } .6734 & = & \bar{1}.828273 \\
 \text{" } .0009278 & = & \bar{4}.967454 \\
 & & \hline
 \text{Difference} & = & 2.860819 \\
 & & 2.860817 = \text{logarithm of } 725.8000 \\
 & & \hline
 & & 2 = \qquad \qquad \qquad 33 \\
 & & \hline
 & & \text{Ans. } 725.8033
 \end{array}$$

(12)

$$\text{Logarithm of } 437.89 = 2.641365$$

$$\text{" } 62.735 = 1.797510$$

$$\text{Difference} = \underline{\quad\quad\quad} .843855 = \text{logarithm of } 6.98$$

(13)

$$\text{Logarithm of } 93.217 = 1.969495$$

$$\text{" } .0007132 = \underline{\quad\quad\quad} 4.853211$$

$$\text{Difference} = \underline{\quad\quad\quad} 5.116284$$

$$\underline{\quad\quad\quad} 5.116276 = \text{logarithm of } 130700.0$$

$$\underline{\quad\quad\quad} 8 = \underline{\quad\quad\quad} 2.4$$

$$\text{Ans. } \underline{\quad\quad\quad} 130702.4$$

(14)

$$\text{Logarithm of } 23 = 1.361728$$

$$\text{" } 189 = 2.276462$$

$$\text{" } 2.748 = \underline{\quad\quad\quad} 0.439017$$

$$\text{Sum} = \underline{\quad\quad\quad} 4.077207$$

$$\text{Logarithm of } 9835267 = 6.992786$$

$$\underline{\quad\quad\quad} 4.077207$$

$$\text{Difference} = \underline{\quad\quad\quad} 2.915579$$

$$\underline{\quad\quad\quad} 2.915558 = \text{logarithm of } 823.300$$

$$\underline{\quad\quad\quad} 21 = \underline{\quad\quad\quad} 39$$

$$\text{Ans. } \underline{\quad\quad\quad} 823.339$$

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(17)

$$\text{Logarithm of } 5 = 0.698970.$$

$$\text{Then } 0.698970 \times 5 = 3.494850 = \text{logarithm of } 3125.$$

(18)

Logarithm of $1.073 = .030600$.

Then $.030600 \times 6 = .183600 = \text{logarithm of } 1.5261$.

(19)

Logarithm of $.0279 = \bar{2}.445604$.

Then $\bar{2}.445604 \times 4 = \bar{7}.782416 = \text{logarithm of } .00000060592$.

(20)

Logarithm of $1.111 = .045714$.

Then $.045714 \times 11 = .502854 = \text{logarithm of } 3.1831$.

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(23)

Logarithm of $913426000 = 8.960673$.

$8.960673 \div 7 = 1.2800961 = \text{logarithm of } 19.0588$.

(24)

Logarithm of $1.61342 = .207747$.

$.207747 \div 11 = .01888609 = \text{logarithm of } 1.0444$.

(25)

Logarithm of $.000007139 = \bar{6}.853637 = \bar{10} + 4.853637$.

$(\bar{10} + 4.853637) \div 5 = \bar{2}.970727 = \text{logarithm of } .0934817$.

(26)

Logarithm of $.002147 = \bar{3}.331832 = \bar{7} + 4.331832$.

$(\bar{7} + 4.331832) \div 7 = \bar{1}.6188331 = \text{logarithm of } .41575$.

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(28)

$$14000 = 7 \times 2 \times 1000 \therefore \log. 14000 = (\log. 7) + (\log. 2) + (\log. 1000).$$

$$\text{Log. } 7 = 0.845098$$

$$\text{Log. } 2 = 0.301030$$

$$\text{Log. } 1000 = 3$$

$$\text{Sum, } = 4.146128 = \log. 14000$$

$$4.9 = 7^2 \div 10 \therefore \log. 4.9 = (\log. 7) \times 2 - (\log. 10).$$

$$\text{Log. } 7 = 0.845098 \times 2 = 1.690196$$

$$\text{Log. } 10 = 1$$

$$\text{Difference} = .690196 = \log. 4.9.$$

$$.00196 = 49 \times 4 \div 100000 = 7^2 \times 2^2 \div 100000$$

$$\therefore \log. .00196 = (\log. 7) \times 2 + (\log. 2) \times 2 - (\log. 100000).$$

$$\text{Log. } 7 = 0.845098 \times 2 = 1.690196$$

$$\text{Log. } 2 = 0.301030 \times 2 = 0.602060$$

$$\text{Sum} = 2.292256$$

$$\text{Log. of } 100000 = 5 \text{ and } 2.292256 - 5 = \bar{3}.292256 = \log. \text{ of } .00196.$$

$$\text{Since } 5 = 10 \div 2, \text{ the logarithm of } 5 = \log. 10 - \log. 2 = 1 - 0.301030 = 0.698970.$$

$$1750 = 5^2 \times 7 \times 10 \therefore \log. 1750 = (\log. 5) \times 2 + (\log. 7) + (\log. 10).$$

$$\text{Log. } 5 = 0.698970 \times 2 = 1.397940$$

$$\text{Log. } 7 = .845098$$

$$\text{Log. } 10 = 1$$

$$\text{Sum, } = 3.243038 = \log. \text{ of } 1750.$$

$$1428.571428 = \frac{1}{7} \times 10000 \therefore \log. 1428.571428 = (\log. \frac{1}{7}) + \log. 10000.$$

(Continued on next page.)

(28 continued.)

$$\text{Log. } \frac{1}{7} = (\text{log. } 1) - (\text{log. } 7) = 0 - 0.845098 = \bar{1}.154902$$

$$\text{Log. } 10000 = 4$$

$$\therefore \text{log. of } 1428.571428 = \text{sum} = 3.154902$$

$$\cdot 00000112 = 2^4 \times 7 \div 100000000 \therefore \text{log. } \cdot 00000112 =$$

$$(\text{log. } 2) \times 4 + (\text{log. } 7) - (\text{log. } 100000000).$$

$$\text{Log. } 2 = 0.301030 \times 4 = 1.204120$$

$$\text{Log. } 7 = 0.845098$$

$$\text{Sum} = 2.049218 = \text{and log. } 100000000 = 8$$

$$2.049218 - 8 = \bar{6}.049218 = \text{log. } \cdot 00000112$$

$$3.0625 = 1\frac{3}{8} \therefore \text{log. } 3.0625 = (\text{log. } 49) - (\text{log. } 16) =$$

$$(\text{log. } 7) \times 2 - (\text{log. } 4) \times 4.$$

$$\text{Log. } 7 = 0.845098 \times 2 = 1.690196$$

$$\text{Log. } 2 = 0.301030 \times 4 = 1.204120$$

$$\text{Difference} = 0.486076 = \text{log. of } 3.0625.$$

(29)

$$49\frac{1}{2} = \frac{99}{2} = 3^2 \times 11 \times \frac{1}{2} \therefore \text{log. } 49\frac{1}{2} = (\text{log. } 3) \times 2 + (\text{log. } 11) + (\text{log. } \frac{1}{2}).$$

$$\text{Log. } 3 = 0.477121 \times 2 = 0.954242$$

$$\text{Log. } 11 = 1.041393$$

$$\text{Log. } \frac{1}{2} = \bar{1}.698970$$

$$\text{Sum} = 1.694605 = \text{log. of } 49\frac{1}{2}.$$

$$363 = 11^2 \times 3 \therefore \text{log. } 363 = (\text{log. } 11) \times 2 + (\text{log. } 3).$$

$$\text{Log. } 11 = 1.041393 \times 2 = 2.082786$$

$$\text{Log. } 3 = 0.477121$$

$$\text{Sum} = 2.559906 = \text{log. of } 363.$$

Log. $\cdot 5$ or $\frac{1}{2} = \bar{1}.698970$, and by altering the characteristic we get 0.698970 for log. of 5.

(Continued on next page.)

(29 continued.)

$$4\cdot\ddot{0}9 = 4\frac{1}{11} = \frac{41}{11} = 3^2 \times 5 \div 11 \therefore \log. 4\cdot\ddot{0}9 = (\log. 3) \times 2 + (\log. 5) - (\log. 11).$$

$$\text{Log. } 3 = \cdot 477121 \times 2 = 0\cdot 954242$$

$$\text{Log. } 5 = \cdot 698970$$

$$1\cdot 653212$$

$$\text{Log. } 11 = 1\cdot 041393 \text{ and } 1\cdot 653212 - 1\cdot 041393 = 0\cdot 611819 = \log. \text{ of } 4\cdot\ddot{0}9.$$

$$2\cdot\dot{4} = 2\frac{4}{9} = \frac{22}{9} = 11 \times 2 \div 9 \therefore \log. 2\cdot\dot{4} = (\log. 11) + (\log. 2) - (\log. 3) \times 2.$$

$$\text{Log. } 2 = (\log. 10) - (\log. 5) = 1 - 0\cdot 698970 = 0\cdot 301030.$$

$$\text{Log. } 11 = 1\cdot 041393$$

$$\text{Log. } 2 = 0\cdot 301030$$

$$1\cdot 342423$$

$$\text{Log. } 3 = 0\cdot 477121 \times 2 = 0\cdot 954242 \text{ and } 1\cdot 342423 - 0\cdot 954242 = 0\cdot 388181 = \log. \text{ of } 2\cdot\dot{4}.$$

$$392\cdot\ddot{7}2 = 392\frac{8}{11} = \frac{4320}{11} = 2^4 \times 3^3 \times 10 \div 11 \therefore \log. 392\cdot\ddot{7}2 = (\log. 2) \times 4 + (\log. 3) \times 3 + (\log. 10) - (\log. 11).$$

$$\text{Log. } 2 = 0\cdot 301030 \times 4 = 1\cdot 204120$$

$$\text{Log. } 3 = 0\cdot 477121 \times 3 = 1\cdot 431363$$

$$\text{Log. } 10 = 1$$

$$\text{Sum} = 3\cdot 635483$$

$$\text{Log. } 11 = 1\cdot 041393 \text{ and } 3\cdot 635483 - 1\cdot 041393 = 2\cdot 594090 = \log. \text{ of } 392\cdot\ddot{7}2.$$

$$293333\frac{1}{3} = \frac{880000}{3} = 2^3 \times 11 \times 10000 \div 3 \therefore \log. 293333\frac{1}{3} = (\log. 2) \times 3 + (\log. 11) + (\log. 10000) - (\log. 3).$$

$$\text{Log. } 2 = 0\cdot 301030 \times 3 = 0\cdot 903090$$

$$\text{Log. } 11 = 1\cdot 041393$$

$$\text{Log. } 10000 = 4$$

$$\text{Sum} = 5\cdot 944483$$

(Continued on next page.)

Log. 3 = 0.477121 and $5.944483 - 0.477121 = 5.467362 =$
log. of 293333 $\frac{1}{3}$.

$19.965 = 11^3 \times 5 \times 3 \div 1000 \therefore \log. 19.965 = (\log. 11) \times 3$
 $+ (\log. 5) + (\log. 3) - (\log. 1000).$

$$\text{Log. } 11 = 1.041393 \times 3 = 3.124179$$

$$\text{Log. } 5 = 0.698970$$

$$\text{Log. } 3 = 0.477121$$

$$\text{Sum} = 4.300270$$

Log. 1000 = 3 and $4.300270 - 3 = 1.300270 = \log. \text{ of } 19.965.$

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(6)

Here we have given the first term 4, the number of terms 17,
and the sum of the series 884, to find l , the last term.

$$\text{Then } l = \frac{2s}{n} - a = \frac{884 \times 2}{17} - 4 = 104 - 4 = 100.$$

(7)

Here we have given the first term 21, the last term 497 and
the number of terms 41, to find the common difference.

$$\text{Then } d = \frac{l - a}{n - 1} = \frac{497 - 21}{41 - 1} = \frac{476}{40} = \frac{119}{10} = 11\frac{9}{10}.$$

(8)

Here we have given a , l , and d , to find n , and since $a = 12$,
 $l = 96$, and $d = 6$, we have

$$n = \frac{l - a}{d} + 1 = \frac{96 - 12}{6} + 1 = \frac{84}{6} + 1 = 14 + 1 = 15.$$

(9)

Here we have given l , d , and s , to find n , and since $l = 14$, $d = 1$, and $s = 105$, we have

$$n = \frac{2l + d}{2d} + \sqrt{\left(\frac{2l + d}{2d}\right)^2 - \frac{2s}{d}} = \frac{2 \times 14 + 1}{2 \times 1} + \sqrt{\left(\frac{2 \times 14 + 1}{2 \times 1}\right)^2 - \frac{2 \times 105}{1}} = 14\frac{1}{2} + \sqrt{\left(\frac{29}{2}\right)^2 - 210} = 14\frac{1}{2} + \sqrt{84\frac{1}{4} - 210} = 14\frac{1}{2} + \sqrt{\frac{1}{4}} = 14\frac{1}{2} + \frac{1}{2} = 15.$$

(10)

Here we have given a , d , and s , to find l , and since $a = \frac{2}{3}$, $d = \frac{2}{3}$, and $s = 1180$, we have

$$l = -\frac{1}{2}d + \sqrt{2ds + (a - \frac{1}{2}d)^2} = -\frac{1}{2} \text{ of } \frac{2}{3} + \sqrt{2 \times \frac{2}{3} \times 1180 + (\frac{2}{3} - \frac{1}{2} \times \frac{2}{3})^2} = -\frac{1}{3} + \sqrt{1720 + (\frac{1}{3})^2} = -\frac{1}{3} + \sqrt{1720\frac{1}{9}} = -\frac{1}{3} + \sqrt{14161} = -\frac{1}{3} + \frac{119}{3} = \frac{118}{3} = 39\frac{1}{3}.$$

(11)

Here we have given a , l , and s , to find d , and since $a = 8$, $l = 170$, and $s = 4895$, we have

$$d = \frac{(l + a)(l - a)}{2s - l - a} = \frac{(170 + 8)(170 - 8)}{2 \times 4895 - 170 - 8} = \frac{178 \times 162}{9790 - 178} = \frac{28836}{9612} = 3.$$

(12)

Here we have given a , l , and d , to find n , and since $a = 5$, $l = 27\frac{1}{2}$, and $d = 2\frac{1}{4}$, we have

$$n = \frac{l - a}{d} + 1 = \frac{27\frac{1}{2} - 5}{2\frac{1}{4}} + 1 = \frac{22\frac{1}{2}}{2\frac{1}{4}} + 1 = \frac{45}{2} + 1 = 10 + 1 = 11$$

(13)

Here we have given a , l , and n , to find s , and since $a = 2$, $l = 478$, and $n = 86$, we have

$$s = (a + l) \frac{n}{2} = (2 + 478) \frac{86}{2} = 480 \times 43 = 20640.$$

(14)

Here we have given a , l , and d , to find s , and since $a = 2$, $l = 998$, and $d = 6$, we have

$$s = \frac{(l+a)(l-a)}{2d} + \frac{l+a}{2} = \frac{(998+2)(998-2)}{2 \times 6} + \frac{998+2}{2} =$$

$$\frac{1000 \times 996}{12} + \frac{1000}{2} = 83000 + 500 = 83500.$$

(15)

Here we have given a , n , and d , to find l , and since $a = 5$, $n = 11$, and $d = 2\frac{1}{4}$, we have

$$l = a + (n-1)d = 5 + (11-1)2\frac{1}{4} = 5 + 10 \times 2\frac{1}{4} = 5 + \frac{45}{2} = \frac{55}{2} = 27\frac{1}{2}.$$

(16)

Here we have given l , d , and n , to find s , and since $l = 199$, $d = 11$, and $n = 19$, we have

$$s = \{2l - (n-1)d\} \frac{n}{2} = \{2 \times 199 - (19-1)11\} \frac{19}{2} =$$

$$\{398 - 18 \times 11\} \frac{19}{2} = 200 \times \frac{19}{2} = 1900.$$

(17)

Here we have given s , a , and l , to find n , and since $s = 39840$, $a = 2$, and $l = 478$, we have

$$n = \frac{2s}{l+a} = \frac{2 \times 39840}{478+2} = \frac{79680}{480} = 166.$$

(18)

Here we have given s , l , and a , to find d , and since $s = 83500$, $l = 998$, and $a = 2$, we have

$$d = \frac{(l + a)(l - a)}{2s - l - a} = \frac{(998 + 2)(998 - 2)}{2 \times 83500 - 998 - 2} = \frac{1000 \times 996}{167000 - 1000} = \frac{996000}{166000} = 6.$$

(19)

Here we have given s , a , and d , to find n , and since $s = 260$, $a = 2$, and $d = 2$, we have

$$n = \frac{d - 2a}{2d} + \sqrt{\frac{2s}{d} + \left(\frac{2a - d}{2d}\right)^2} = \frac{2 - 2 \times 2}{2 \times 2} + \sqrt{\frac{2 \times 260}{2} + \left(\frac{2 \times 2 - 2}{2 \times 2}\right)^2} = -\frac{1}{2} + \sqrt{260 + \left(-\frac{1}{2}\right)^2} = -\frac{1}{2} + \sqrt{260\frac{1}{4}} = -\frac{1}{2} + 16.13226 = 15.63226 \text{ days} = 15 \text{ days, } 15 \text{ hours, } 10 \text{ minutes, } 27.264 \text{ seconds.}$$

(20)

Here we have given s , a , and d , to find l , and since $s = 83500$, $a = 2$, and $d = 6$, we have

$$l = -\frac{1}{2}d + \sqrt{2ds + \left(a - \frac{1}{2}d\right)^2} = -\frac{1}{2} \times 6 + \sqrt{2 \times 6 \times 83500 + \left(2 - \frac{1}{2} \times 6\right)^2} = -3 + \sqrt{1002000 + (2 - 3)^2} = -3 + \sqrt{1002001} = -3 + 1001 = 998.$$

(21)

Here we have given s , n , and l , to find a , and since $s = \$1125$, $n = 18$, and $l = 120$, we have

$$a = \frac{2s}{n} - l = \frac{2 \times 1125}{18} - 120 = 125 - 120 = 5.$$

(22)

Here we have given a , l , and n , to find d , and since $a = 5$, $l = 27\frac{1}{2}$, and $n = 11$, we have

$$d = \frac{l - a}{n - 1} = \frac{27\frac{1}{2} - 5}{11 - 1} = \frac{22\frac{1}{2}}{10} = 2\frac{1}{4}.$$

(23)

Here we have a , d , and n , given to find s , and since to deposit one stone he must walk 5 yards, and the distance travelled for each succeeding stone is 5 yards, therefore $a = 5$, $d = 5$, and $n = 220$.

$$\begin{aligned} \text{Then } s &= \left\{ 2a + (n - 1)d \right\} \frac{n}{2} = \left\{ 2 \times 5 + (220 - 1)5 \right\} \frac{220}{2} \\ &= \left\{ 10 + 219 \times 5 \right\} 110 = \\ 1105 \times 110 &= 121550 \text{ yards} = 69\frac{1}{16} \text{ miles.} \end{aligned}$$

(24)

Here we have s , n , and l , given to find a , and since $s = 39840$, $n = 166$, and $l = 478$, we have

$$a = \frac{2s}{n} - l = \frac{2 \times 39840}{166} - 478 = 480 - 478 = 2.$$

(25)

Here we have n , a , and d , given to find s , and since $n = 12$, $a = 4$, and $d = 2$, we have

$$\begin{aligned} s &= \left\{ 2a + (n - 1)d \right\} \frac{n}{2} = \left\{ 2 \times 4 + (12 - 1)2 \right\} \frac{12}{2} = \left\{ 8 + 11 \times 2 \right\} 6 = \\ 30 \times 6 &= 180. \end{aligned}$$

(26)

Here we have given a , l , and n , to find s , and $a = 1$, $l = 24$, and $n = 24$.

$$\text{Then } s = (a + l) \frac{n}{2} = (1 + 24) \frac{24}{2} = 25 \times 12 = 300.$$

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(5)

Here $n = 11$, $a = £1024$, and $r = 1\frac{1}{2}$.Then $l = ar^{n-1} = 1024 \times (\frac{3}{2})^{10} = 1024 \times \frac{59049}{1024} = £59049$.

$$s = \frac{rl - a}{r - 1} = \frac{\frac{3}{2} \times 59049 - 1024}{\frac{3}{2} - 1} = \frac{177147 - 1024}{\frac{1}{2}} = \frac{175099}{\frac{1}{2}} = £175099 = \text{whole fortune.}$$

(6)

Here $a = 7$, $l = 1240029$ and $s = 1860040$.

$$\text{Then } r = \frac{s - a}{s - l} = \frac{1860040 - 7}{1860040 - 1240029} = \frac{1860033}{620011} = 3.$$

(7)

Here $n = 12$, $a = £1$, and $l = £2048$.

$$\text{Then } r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}} = \left(\frac{2048}{1}\right)^{\frac{1}{12-1}} = \sqrt[11]{2048} = 2.$$

$$s = \frac{rl - a}{r - 1} = \frac{2 \times 2048 - 1}{2 - 1} = 4096 - 1 = £4095.$$

(8)

Here $r = \frac{3}{2}$, $n = 8$, and $l = 106\frac{403}{12}$.

$$\text{Then } s = \frac{l(r^n - 1)}{(r-1)r^{n-1}} = \frac{106\frac{403}{12} \times [(\frac{3}{2})^8 - 1]}{(\frac{3}{2} - 1)(\frac{3}{2})^7} = \frac{54675 \times \frac{6305}{256}}{\frac{1}{2} \times \frac{2187}{128}} = \frac{25 \times 6305}{512} = 307\frac{441}{512}.$$

(9)

Here $a = 1$, $n = 7$, and $r = 3$.

$$\text{Then } s = \frac{a(r^n - 1)}{r - 1} = \frac{1 \times (3^7 - 1)}{3 - 1} = \frac{2186}{2} = 1093.$$

(10)

Here $a = 1$, $l = 10077696$, and $n = 10$.

$$\begin{aligned} \text{Then } s &= \frac{l^{\frac{n}{n-1}} - a^{\frac{n}{n-1}}}{l^{\frac{1}{n-1}} - a^{\frac{1}{n-1}}} = \frac{(10077696)^{\frac{10}{10-1}} - 1^{\frac{10}{10-1}}}{(10077696)^{\frac{1}{10-1}} - 1^{\frac{1}{10-1}}} = \\ &= \frac{\sqrt[9]{(10077696)^{10}} - 1}{\sqrt[9]{10077696} - 1} = \frac{\sqrt[3]{(216)^{10}} - 1}{\sqrt[3]{216} - 1} = \frac{6^{10} - 1}{6 - 1} = \frac{60466176 - 1}{5} \\ &= \frac{60466175}{5} = 12093235. \end{aligned}$$

(11)

Here $a = 6$, $l = 3072$, and $s = 6138$.

$$\text{Then } r = \frac{s - a}{s - l} = \frac{6138 - 6}{6138 - 3072} = \frac{6132}{3066} = 2.$$

(12)

Here $r = 2$, $n = 11$, and $s = 20470$.

$$\begin{aligned} \text{Then } l &= \frac{(r-1)sr^{n-1}}{r^n - 1} = \frac{(2-1) \times 20470 \times 2^{10}}{2^{11} - 1} = \frac{20470 \times 1024}{2047} \\ &= 10240. \end{aligned}$$

(13)

Here $a = 1s.$, $n = 12$, and $r = 2$.

$$\text{Then } s = \frac{a(r^n - 1)}{r - 1} = \frac{1 \times (2^{12} - 1)}{2 - 1} = \frac{4095}{1} = 4095s. \\ = £204 \text{ } 15s.$$

(14)

Here $a = 1$ farthing, $r = 2$, and $n = 32$.

$$\text{Then } s = \frac{a(r^n - 1)}{r - 1} = \frac{1 \times (2^{32} - 1)}{2 - 1} = 4294967295 \text{ far.} = \\ £4473924 \text{ } 5s. \text{ } 3\frac{1}{4}d.$$

(15)

Here $a = 4$, $l = 78732$, and $n = 10$.

$$\text{Then } r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}} = \left(\frac{78732}{4}\right)^{\frac{1}{10-1}} = \sqrt[9]{19683} = 3.$$

(16)

Here $a = 5$, $r = 2$, and $n = 7$.

$$\text{Then } l = ar^{n-1} = 5 \times 2^{7-1} = 5 \times 2^6 = 5 \times 64 = 320.$$

(17)

Here $a = 5$, $l = 327680$, and $r = 4$.

$$\text{Then } s = \frac{rl - a}{r - 1} = \frac{(327680 \times 4) - 5}{4 - 1} = \frac{1310715}{3} = 436905.$$

(18)

Here $a = 1$, $r = 2$, and $n = 64$.

$$\text{Then } s = \frac{a(r^n - 1)}{r - 1} = \frac{1 \times (2^{64} - 1)}{2 - 1} = 18446744073709551615 \text{ gr.}$$

$$18446744073709551615 \div (7680 \times 64) = 37529996894754 \text{ bush.}$$

$$\$1.50 \times 37529996894754 = \$56294995342131.$$

(19)

Here $r = 3$, $n = 10$, and $s = 295240$.

$$\text{Then } l = \frac{(r-1)sr^{n-1}}{r^n - 1} = \frac{(3-1) \times 295240 \times 3^9}{3^{10} - 1} = \frac{2 \times 295240 \times 19683}{59048} = 196830.$$

(20)

Here $a = 1$, $l = 2048$, and $n = 12$.

$$\begin{aligned} \text{Then } s &= \frac{l^{\frac{n}{n-1}} - a^{\frac{n}{n-1}}}{l^{\frac{1}{n-1}} - a^{\frac{1}{n-1}}} = \frac{2048^{1\frac{12}{11}} - 1^{1\frac{12}{11}}}{2048^{\frac{1}{11}} - 1^{\frac{1}{11}}} = \\ &= \frac{\sqrt[11]{(2048)^{12}} - 1}{\sqrt[11]{2048} - 1} = \frac{2^{12} - 1}{2 - 1} = 2^{12} - 1 = 4095, \end{aligned}$$

(21)

Here $a = 5$, $r = 4$, and $n = 9$.

$$\text{Then } l = ar^{n-1} = 5 \times 4^{9-1} = 5 \times 4^8 = 5 \times 65536 = 327680.$$

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(24)

Here $a = \frac{2}{7}$, and $r = \frac{3}{5}$.

$$\text{Then } s = \frac{a}{1-r} = \frac{\frac{2}{7}}{1-\frac{3}{5}} = \frac{\frac{2}{7}}{\frac{2}{5}} = \frac{5}{7}.$$

(25)

Here $a = 4$, and $r = \frac{1}{2}$.

$$\text{Then } s = \frac{a}{1-r} = \frac{4}{1-\frac{1}{2}} = \frac{4}{\frac{1}{2}} = 8.$$

(26)

Here $a = \frac{79}{100}$, and $r = \frac{1}{100}$.

$$\text{Then } s = \frac{a}{1-r} = \frac{\frac{79}{100}}{1-\frac{1}{100}} = \frac{\frac{79}{100}}{\frac{99}{100}} = \frac{79}{99}.$$

(27)

Here $a = \frac{1234}{10000}$, and $r = \frac{1}{10000}$.

$$\text{Then } s = \frac{a}{1-r} = \frac{\frac{1234}{10000}}{1-\frac{1}{10000}} = \frac{\frac{1234}{10000}}{\frac{9999}{10000}} = \frac{1234}{9999}.$$

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(3)

Since there are 9 means and 2 extremes the number of terms is 11.

$$\text{Then } d = \frac{l-a}{n-1} = \frac{92-2}{11-1} = \frac{90}{10} = 9.$$

1st term = 2; 2nd = $2 + 9 = 11$; 3rd = $11 + 9 = 20$; 4th = $20 + 9 = 29$; 5th = $29 + 9 = 38$; 6th = $38 + 9 = 47$; and so on.

And series is 2, 11, 20, 29, 38, 47, 56, 65, 74, 83, 92.

(4)

Since there are 4 means and two extremes the number of terms is 6.

$$\text{Then } d = \frac{l-a}{n-1} = \frac{50-7}{6-1} = \frac{43}{5} = 8\frac{3}{5}.$$

1st term = 7; 2nd = $7 + 8\frac{3}{5} = 15\frac{3}{5}$; 3rd = $15\frac{3}{5} + 8\frac{3}{5} = 24\frac{1}{5}$; 4th = $24\frac{1}{5} + 8\frac{3}{5} = 32\frac{4}{5}$; 5th = $32\frac{4}{5} + 8\frac{3}{5} = 41\frac{2}{5}$; and 6th = $41\frac{2}{5} + 8\frac{3}{5} = 50$.

And series is 7, $15\frac{3}{5}$, $24\frac{1}{5}$, $32\frac{4}{5}$, $41\frac{2}{5}$, 50.

(5)

Since there are 8 means and two extremes the number of terms is 10.

$$\text{Then } r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}} = \left(\frac{4096}{4096}\right)^{\frac{1}{10-1}} = \left(\frac{1}{512}\right)^{\frac{1}{9}} = \frac{1}{2}.$$

1st term = 4096; 2nd = $4096 \times \frac{1}{2} = 2048$; 3rd = $2048 \times \frac{1}{2} = 1024$; 4th = $1024 \times \frac{1}{2} = 512$; 5th = $512 \times \frac{1}{2} = 256$, and so on.

And the means are 2048, 1024, 512, 256, 128, 64, 32, and 16.

(6)

Since there are 7 means and two extremes the number of terms is 9.

$$\text{Then } r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}} = \left(\frac{23514624}{14}\right)^{\frac{1}{9-1}} = (1679616)^{\frac{1}{8}} = 6.$$

1st term = 14; 2nd = $14 \times 6 = 84$; 3rd = $84 \times 6 = 504$;
4th = $504 \times 6 = 3024$; 5th = $3024 \times 6 = 18144$, and so on.

And the means are 84, 504, 3024, 18144, 108864, 653184, and 3919104.

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(3)

Assume 4 to be the number of men.

Then $2 \times 4 = 8 =$ number of women.

And $8 \times 3 = 24 =$ number of children.

6d. $\times 4 = 24$ d. = amount received by the men.

4d. $\times 8 = 32$ d. = " " " women.

2d. $\times 24 = 48$ d. = " " " children.

Sum, = 104d., but it should, by question, = 78d.

$$\text{Then } 104 : 78 :: 4 : \frac{78 \times 4}{104} = 3 = \text{number of men.}$$

$3 \times 2 = 6 =$ number of women, and $6 \times 3 = 18 =$ number of children.
[ren.]

(4)

Assume £8 to be the price of the harness.

Then $\text{£}8 \times 2 = 16 =$ price of horse.

And $\text{£}8 + \text{£}16 = \text{£}24 \times 2 = 48 =$ " chaise.

Sum, = £72, but it should by question = £60.

$$\text{Then } \text{£}72 : \text{£}60 :: \text{£}8 : \frac{8 \times 60}{72} = \text{£}6 \ 13 \ 4 = \text{price of harness.}$$

$\text{£}6 \ 13 \ 4 \times 2 = 13 \ 6 \ 8 =$ " horse.

$\text{£}6 \ 13 \ 4 + \text{£}13 \ 6 \ 8 = \text{£}20 \times 2 = 40 \ 0 \ 0 =$ " chaise.

(5)

Assume 20 as C's age.

Then $20 \times 3 = 60 = \text{B's age.}$

And $60 \times 2 = 120 = \text{A's age.}$

Sum = 200, but by question it should = 140.

Then $200 : 140 :: 20 : \frac{20 \times 140}{200} = 14 = \text{C's age.}$

$14 \times 3 = 42 = \text{B's age, and } 42 \times 2 = 84 = \text{A's age.}$

(6)

Assume 100.

One fourth of 100 = 25 and remainder = $100 - 25 = 75$.

One fifth of 75 = 15 and remainder = $75 - 15 = 60$, but it should be by the question = 72.

Then $60 : 72 :: 100 : \frac{100 \times 72}{60} = 120$.

(7)*

A can do the work in 7 days \therefore he will do $\frac{1}{7}$ of it in 1 day.

B " " 5 " \therefore " $\frac{1}{5}$ " "

C " " 6 " \therefore " $\frac{1}{6}$ " "

Then all working together will do $\frac{1}{7} + \frac{1}{5} + \frac{1}{6} = \frac{107}{210}$ in 1 day.

Therefore to do the whole work it will take them $\frac{1}{\frac{107}{210}} = \frac{210}{107} = 1\frac{103}{107}$ days.

(8)*

A and B working together can do it in 10 days \therefore they will do $\frac{1}{10}$ of it in 1 day.

A can do it in 15 days \therefore he will do $\frac{1}{15}$ of it in 1 day.

Therefore $\frac{1}{10} - \frac{1}{15} = \frac{1}{30} = \text{amount done by B in 1 day.}$

Then if he does $\frac{1}{30}$ in 1 day, it will take him 30 days to do the whole.

* The mode of working these questions by position is so simple that they cannot trouble any one; it has therefore been thought advisable to work them by simple analysis.

(9)*

The first pipe empties the whole of it in 1 hour.

The second pipe empties $\frac{1}{2}$ of it in 1 hour.

The third pipe empties $\frac{1}{3}$ of it in 1 hour.

Then all these pipes running together will empty $1 + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$ in 1 hour.

Therefore to empty the cistern it will take $1 \div \frac{11}{6} = \frac{6}{11}$ hours.

(10)

Assume 84

One third of 84 = 28

One sixth of 84 = 14

One seventh of 84 = 12

Sum = 54, but by question it should = 27.

Then $54 : 27 :: 84 : \frac{84 \times 27}{54} = 42$.

(11)*

All 5 mills working together will grind $7 + 5 + 4 + 3 + 1 = 20$ bushels in 1 hour.

Therefore to grind 500 bushels it will take them $500 \div 20 = 25$ hours.

(12)*

One pipe fills $\frac{1}{12}$ of the cistern in 1 hour, and the other empties $\frac{1}{18}$ of it in 1 hour.

Then $\frac{1}{12} - \frac{1}{18} = \frac{1}{36} =$ part of the cistern filled in 1 hour when both are left open.

And if $\frac{1}{36}$ of it is filled in 1 hour, the whole will be filled in

$\frac{1}{\frac{1}{36}} = 36$ hours.

* See note on page 227.

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(6)

Assume 60 for father's age, then 15 = son's.

$\begin{array}{r} 5 \\ \hline 5 \overline{)55} \\ \hline 11 \\ 10 \\ \hline -1 \end{array}$	$\begin{array}{r} 5 \\ \hline 10 \end{array}$
---	---

Assume 100 for father's age, then 25 = son's.

$\begin{array}{r} 5 \\ \hline 5 \overline{)95} \\ \hline 19 \\ 20 \\ \hline +1 \end{array}$	$\begin{array}{r} 5 \\ \hline 20 \end{array}$
---	---

Errors. Assumed numbers.

- 1	×	100	=	100
+ 1	×	60	=	60
<u> </u>				<u> </u>

Sum of errors = 2 Sum of products = 160

Therefore result required = $160 \div 2 = 80$ = father's age, and
 $\frac{1}{4}$ of 80 = 20 = son's age.

(7)

Assume 80

Assume 44

$\begin{array}{r} 34 \\ \hline 46 \\ 3 \\ \hline 138 \\ 80 \\ \hline 58 \\ \frac{1}{4} \text{ of } 80 = 20 \\ \hline + 38 \end{array}$	$\begin{array}{r} 34 \\ \hline 10 \\ 3 \\ \hline 30 \\ 44 \\ \hline - 14 \\ \frac{1}{4} \text{ of } 44 = 11 \\ \hline - 25 \end{array}$
--	---

(Continued on next page.)

(7 continued.)

Errors. Assumed numbers.

$$\begin{array}{rclcl} -25 & \times & 80 & = & 2000 \\ +38 & \times & 44 & = & 1672 \end{array}$$

$$\text{Sum of errors} = 63 \qquad \qquad \qquad 3672$$

$$\text{Therefore result required} = 3672 \div 63 = 58\frac{4}{7}.$$

(9)

Assume 18 and 7

$$\text{One half of 18} = 9 \quad 2 \times 7 = 14$$

14

— 5

Assume 22 and 3

$$\text{One half of 22} = 11 \quad 2 \times 3 = 6$$

6

+ 5

Errors. Assumed numbers.

$$\begin{array}{rclcl} -5 & \times & 22 & = & 110 \\ +5 & \times & 18 & = & 90 \end{array}$$

$$\text{Sum of errors} = 10 \quad \text{Sum of products} = 200$$

$$\text{Then } 200 \div 10 = 20 = \text{one number, and } 25 - 20 = 5 \\ = \text{other number.}$$

(10)

A.	B.
Suppose 8	6
22½	9
<hr/>	12
180	15
132	18
<hr/>	21
8)48	24
<hr/>	27
+ 6	<hr/>
6	132
<hr/>	
36	
72	
<hr/>	
3)36	
<hr/>	
12	

A.	B.
Suppose 6	6
22½	9
<hr/>	12
135	15
81	18
<hr/>	21
6)54	<hr/>
<hr/>	81
+ 9	
8	
<hr/>	
72	

$$9 - 6 = 3 = \text{difference of errors.}$$

(12)

Assume 30.

$$\begin{aligned}\frac{1}{2} \text{ of } 30 &= 15; \frac{1}{4} \text{ of } 30 = 7\frac{1}{2}; \\ \frac{1}{6} \text{ of } 30 &= 5; \text{ and } \frac{1}{6} \text{ of } 30 = 5; \\ 15 \times 7\frac{1}{2} \times 6 \times 5 &= 3375; \\ 3375 - 6998\frac{2}{3} &= -3623\cdot4 = \text{error.}\end{aligned}$$

Assume 60.

$$\begin{aligned}\frac{1}{2} \text{ of } 60 &= 30; \frac{1}{4} \text{ of } 60 = 15; \\ \frac{1}{6} \text{ of } 60 &= 10; \text{ and } \frac{1}{6} \text{ of } 60 = 10. \\ 30 \times 15 \times 12 \times 10 &= 54000. \\ 54000 - 6998\frac{2}{3} &= +47001\cdot6 = \text{error.} \\ 30^4 &= 810000, \text{ and } 60^4 = 12960000 \\ -3623\cdot4 \times 12960000 &= 46959284000 \\ +47001\cdot6 \times 810000 &= 38071296000\end{aligned}$$

$$\text{Sum} = 50625$$

$$\text{Sum} = 85030560000$$

$$85030560000 \div 50625 = 1679616$$

4th root = square root of square root.

$$\sqrt{1679616} = 1296, \text{ and } \sqrt{1296} = 36 = \text{required number.}$$

NOTE.—For reason why we multiply by the 4th powers of the assumed numbers and then take the 4th root of the quotient, see Arith. page 352,

Let x = the number required.

example 11.

It may, however, perhaps be clearer from the following illustration :

$$\begin{aligned}\text{Then } \frac{x}{2} \times \frac{x}{4} \times \frac{x}{5} \times \frac{x}{6} &= \frac{x^4}{240} = 6998\frac{2}{3} \\ \therefore x^4 &= 1679616 \\ \therefore x &= \sqrt[4]{1679616} = 36.\end{aligned}$$

(13)

Suppose A had 9s. at first.

Then $9 + 1 = 10$; $10 \div 2 = 5$; $5 + 1 = 6 =$ what B had at first.

$6 + 1 = 7$, but should $= 9 - 1 = 8$.

$$\text{Error} = 7 - 8 = -1.$$

Suppose A had 11s. at first.

Then $11 + 1 = 12$; $12 \div 2 = 6$; $6 + 1 = 7 =$ what B had at first.

$7 + 1 = 8$, but should $= 11 - 1 = 10$.

$$\text{Error} = 8 - 10 = -2.$$

(Continued on next page.)

(13 continued.)

Errors.

$$- 2 \times 9 = 18$$

$$- 1 \times 11 = 11$$

$$\text{Diff.} = 1 \quad \text{diff.} = 7$$

$7 \div 1 = 7 =$ shillings A had at first.

$7 + 1 = 8$; $8 \div 2 = 4$; $4 + 1 = 5 =$ shillings B had at first.

(14)

Assume 24 and 6.

$$\frac{24}{2} + \frac{24}{3} + \frac{24}{6} = 24.$$

$$\frac{6}{2} + \frac{1}{3} \text{ of } 6 + \frac{6}{4} = 9.$$

$$24 - 9 = + 15 = \text{error.}$$

Assume 20 and 10.

$$\frac{20}{2} + \frac{20}{3} + \frac{20}{6} = 20.$$

$$\frac{10}{2} + \frac{1}{3} \text{ of } 10 + \frac{10}{4} = 15.$$

$$20 - 15 = + 5 = \text{error.}$$

Errors.

$$+ 15 \times 20 = 300$$

$$+ 5 \times 24 = 120$$

$$\text{Diff.} = 10 \quad \text{diff.} = 180$$

$$180 \div 10 = 18 = \text{one number.}$$

$$30 - 18 = 12 = \text{other number.}$$

(15)

Suppose 1st horse to be worth £20.

$$20 + 50 = 70; 70 \div 2 = £35 = \text{value of 2nd horse.}$$

$$35 + 50 = 85, \text{ but it should equal } 60, \text{ i. e. } (20 \times 3).$$

$$\text{Then } 60 - 85 = - 25 = \text{error.}$$

Suppose 1st horse to be worth £60.

$$£60 + £50 = £110; £110 \div 2 = £55 = \text{worth of 2nd horse.}$$

$$55 + 50 = 105, \text{ but it should equal } 180, \text{ i. e. } (60 \times 3).$$

$$180 - 105 = + 75 = \text{error.}$$

Errors.

$$75 \times 20 = 1500$$

$$25 \times 60 = 1500$$

$$\text{Sum} = 100$$

$$\text{Sum} = 3000$$

$$3000 \div 100 = £30 = \text{value of 1st horse.}$$

$$£30 + £50 = £80; £80 \div 2 = £40 = \text{value of 2nd horse.}$$

(16)

Suppose there were 11 beggars.

$$11 \times 4 = 44; 44 + 6 = 50 = \text{number of pence he had.}$$

$$11 \times 6 = 66; 66 - 12 = 54 = \quad " \quad " \quad "$$

$$54 - 50 = + 4 = \text{error.}$$

Suppose there were 12 beggars.

$12 \times 4 = 48$; $48 + 6 = 54 =$ pence he had.

$12 \times 6 = 72$; $72 \div 12 = 60 =$ pence he had.

$$60 - 54 = + 6 = \text{error.}$$

Errors.

$$+ 6 \times 11 = 66$$

$$+ 4 \times 12 = 48$$

Diff. = 2 diff. = 18, and $18 \div 2 = 9$ = number of beggars.

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(7)

Here $P = \$713.29$, $r = .045$, and $t = 14$.

Then $A = P(1+r)^t$, or $\log. A = \log. P + \log. (1+r) \times t$
 $= 2.853267 + .019116 \times 14 = 3.120891 = \log. \text{ of } Ans.$

Hence amount = \$1320.96.

(8)

Here $n = 7$, $r = .015$.

$$\text{Then } t = \frac{\log. n.}{\log. (1 + r)} = \frac{845098}{\cdot 006466} = 130.698 \text{ payments, and}$$

$$130.698 \div 4 = 32.674 \text{ years} = 32 \text{ years } 8 \text{ months } 2 \text{ days.}$$

(9)

Here $A = \$1111.11$, $P = 111.11$, and $r = .08$.

$$\text{Then } t = \frac{\log. A - \log. P}{\log. (1 + r)} = \frac{3.045757 - 2.045753}{.033424} = \frac{1.000004}{.033424} = 29.918 \text{ years} = 29 \text{ years } 11 \text{ months.}$$

(10)

Here $A = \$3333.33$, $P = \$222.22$, and $t = 120$.

$$\text{Then } r = \sqrt[t]{\frac{A}{P}} - 1; \text{ or } \log. (1 + r) = \frac{\log. A - \log. P}{t} =$$

$$\frac{3.522878 - 2.346784}{120} = \frac{1.176095}{120} = .0098007. \text{ Hence } 1 + r$$

$$= 1.0228, r = .0228, \text{ and rate per cent.} = 2\frac{7}{25}.$$

(11)

Here $n = 2$ and $r = .07$.

$$\text{Then } t = \frac{\log. n.}{\log. (1 + r)} = \frac{0.301030}{0.029384} = 10.2446 \text{ years} = 10 \text{ yrs.}$$

2 months 28 days.

(12)

Here $A = \$100$, $r = .0225$, and $t = 28$.

$$\text{Then } P = \frac{A}{(1 + r)^t}, \text{ or } \log P = \log. A - \log. (1 + r) \times t.$$

$$\log. P = 2 - 0.009664 \times 28 = 2 - 0.270592 = 1.729408.$$

$$\text{Hence } P = \$53.63.$$

(13)

Here $P = \$2468.13$, $r = .0375$, and $t = 26$.

$$\text{Then } A = P (1 + r)^t, \text{ or } \log. A = \log. P + \log. (1 + r) \times t.$$

$$\log. A = 3.392368 + 0.015988 \times 26 = 3.392368 + 0.415688 = 3.808056.$$

$$\text{Hence } A = \$6427.705.$$

(14)

Here $A = \$7137.40$, $r = .0425$, and $t = 22$.

$$\text{Then } P = \frac{A}{(1 + r)^t}, \text{ or } \log. P = \log. A - \log. (1 + r) \times t.$$

$$\log. P = 3.853540 - 0.018076 \times 22 = 3.853540 - 0.397672 = 3.455868.$$

$$\text{Hence } P = \$2856.723.$$

(15)

Here $n = 19$, and $r = .0525$.

$$\text{Then } t = \frac{\log. n}{\log. (1 + r)} = \frac{1.278754}{0.022222} = 57.5445 \text{ payments} =$$

28.7225 years = 28 years 9 months 8 days.

Page 360.

(3)

Here $r = .03$, $a = 500$, $A = 8365$.

$$\begin{aligned} \text{Formula IV. } t &= \frac{\sqrt{\left\{ \frac{8rA}{a} + (2-r)^2 \right\}} - (2-r)}{2r} \\ &= \frac{\sqrt{\left\{ \frac{8 \times .03 \times 8365 + (2 - .03)^2}{500} \right\}} - (2 - .03)}{2 \times .03} \\ &= \frac{\sqrt{\left\{ \frac{2007.6}{500} + 3.8809 \right\}} - 1.97}{.06} \\ &= \frac{\sqrt{4.0152 + 3.8809} - 1.97}{.06} = \frac{\sqrt{7.8961} - 1.97}{.06} \\ &= \frac{2.81 - 1.97}{.06} = \frac{.84}{.06} = \frac{84}{6} = 14 \text{ payments} = 7 \text{ years.} \end{aligned}$$

(4)

Here $a = 112.50$, $r = .015$, $t = 44$.

$$\begin{aligned} \text{Formula I. } A &= at \left(1 + \frac{(t-1)r}{2} \right) \\ &= 112.50 \times 44 \left(1 + \frac{(44-1) \times .015}{2} \right) = 4950 \times 1.3225 \\ &= \$6546.375. \end{aligned}$$

(5)

Here $a = 300$, $A = 1680$, and $t = 5$.

$$\begin{aligned} \text{Formula III. } r &= \frac{2(A - at)}{at(t - 1)} = \frac{2(1680 - 300 \times 5)}{300 \times 5(5 - 1)} \\ &= \frac{2(1680 - 1500)}{300 \times 5 \times 4} = \frac{2 \times 180}{6000} = \frac{360}{6000} = .06. \end{aligned}$$

$$\therefore \text{Rate per cent} = .06 \times 100 = 6.$$

(6)

Here $A = 2080$, $r = .04$, and $t = 16$.

$$\begin{aligned} \text{Formula II. } a &= \frac{2A}{t\{2 + (t - 1)r\}} = \frac{2 \times 2080}{16\{2 + (16 - 1) \cdot 04\}} \\ &= \frac{4160}{16 \times (2 + 15 \times .04)} = \frac{4160}{16 \times 2.6} = \frac{4160}{41.6} = \frac{41600}{416} \\ &= \$100 = 1 \text{ payment or rent for half a year, hence yearly} \\ &\quad \text{rent} = \$100 \times 2 = \$200. \end{aligned}$$

Page 366.

(5)

Here $r = .04$, and $v = \$3000$.

$$\text{Then } a = vr = 3000 \times .04 = \$120.$$

(6)

Here $a = 563$, and $v = 11260$

$$\begin{aligned} \text{Then } r &= \frac{a}{v} = \frac{563}{11260} = \frac{1}{20} = .05, \text{ and hence rate} \\ &\quad \text{per cent.} = 5. \end{aligned}$$

(7)

Here $a = 75$, $r = .05$, and $s = 14$.

$$\text{Then } v = \frac{a}{r(1+r)^s} = \frac{75}{.05 \times (1.05)^{14}}$$

$$\begin{aligned} \log. v &= \log. 75 - \log. (1.05) \times 14 \\ &= 1.875061 - (0.021189 \times 14 + \log. .05) \\ &= 1.875061 - (0.296646 + 2.698970). \\ &= 2.879445. \end{aligned}$$

$\therefore v =$ nat. number corresponding to the logarithm 2.879445,
which is \$757.608.

(8)

Here $a = \$90$, $r = .04$, $t = 12$, $s = 7$, and $\therefore s + t = 19$.

$$\begin{aligned} \text{Formula VII. } v &= \frac{a}{r} \left\{ \frac{1}{(1+r)^t} - \frac{1}{(1+r)^{s+t}} \right\} \\ &= \frac{90}{.04} \left\{ \frac{1}{(1.04)^{12}} - \frac{1}{(1.04)^{19}} \right\} = \frac{9000}{4} \left\{ \frac{1}{1.60101} - \frac{1}{2.10682} \right\} \\ &= 2250 \times (.624605 - .474649) = 2250 \times .149956 \\ &= \$337.3988. \end{aligned}$$

(9)

Here $a = 1500$, and $r = .05$.

$$\begin{aligned} \text{Formula VIII. } v &= \frac{a}{r} = \frac{1500}{.05} = \frac{150000}{5} = \$30000 \\ &= 20 \times 1500 \text{ or 20 years' purchase.} \end{aligned}$$

(10)

Here $a = 22$, $v = 308.64366$, and $r = .04$.

$$\begin{aligned} \text{Then Formula VI. } t &= \frac{\log. a - \log. (a - vr)}{\log. (1+r)} \\ &= \frac{\log. 22 - \log. (22 - 308.6436 \times .04)}{\log. (1.04)} \\ &= \frac{1.342423 - \log. (9.65425)}{\log. (1.04)} = \frac{1.342423 - 0.984707}{0.017033} \\ &= \frac{0.357716}{0.017033} = \frac{357716}{1117033} = 21 +. \end{aligned}$$

(11)

Here $a = 154$, $t = 19$, and $r = .05$.

$$\begin{aligned} \text{Formula IV. } v &= \frac{a}{r} \left\{ 1 - \frac{1}{(1+r)^t} \right\} \\ &= \frac{154}{.05} \times \left\{ 1 - \frac{1}{(1.05)^{19}} \right\} = \frac{15400}{5} \times \left\{ 1 - \frac{1}{2.5269} \right\} \\ &= 3080 \times (1 - .39574) = 3080 \times .60426 = \$1861.12 +. \end{aligned}$$

(12)

Here $A = 600$, $t = 40$, and $r = .0375$.

$$\begin{aligned} \text{Formula II. } a &= \frac{Ar}{(1+r)^t - 1} = \frac{600 \times .0375}{(1.0375)^{40} - 1} \\ &= \frac{22.5}{4.36034 - 1} = \frac{22.5}{3.36034} = \frac{2250000}{336034} \\ &= £6.6957 = £6 \text{ 13s. } 10\frac{1}{2}\text{d} +. \end{aligned}$$

(13)

Here $a = 8$, $A = 187.315625$ and $r = .03$.

$$\begin{aligned} \text{Formula III. } t &= \frac{\log(Ar + a) - \log a}{\log(1+r)} \\ &= \frac{\log(187.315625 \times .03 + 8) - \log 8}{\log 1.03} \\ &= \frac{\log(5.61946875 + 8) - \log 8}{\log 1.03} \\ &= \frac{\log 13.61946875 - \log 8}{\log 1.03} = \frac{1.134160 - 0.903090}{0.012837} \\ &= \frac{0.231070}{0.012837} = \frac{231070}{12837} = 18. \end{aligned}$$

(14)

Here $A = 74$, $r = .04$ and $t = 30$

$$\begin{aligned} \text{Formula I. } A &= a \frac{(1+r)^t - 1}{r} = \frac{74 \times \{(1.04)^{30} - 1\}}{.04} \\ &= \frac{74}{.04} \times (3.24332 - 1) = \frac{7400}{4} \times 2.24332 = \$4150.142 \end{aligned}$$

By Table, page 362. Amount of \$1 for 30 years, at 4 per cent,
= \$56.08494

Then $\$56.08494 \times 74 = \4150.28 .

Page 367.

EXAMINATION PAPERS.

FIRST SERIES.

(2)

$\$7580 \times .19 = \1440.20 , and $\$7580 - \$1440.20 = \$6139.80$.
 D is to have one third as much as A, B, and C together, therefore he will have one-fourth of the whole. $\frac{1}{4}$ of $\$6139.80 = \$1534.95 =$ D's share.

$\$6139.80 - \$1534.95 = \$4604.85 =$ amount to be divided among A, B, and C.

B is to have $\$90.90$ more than C.

A is to have $\$111.11 + \$90.90 = 202.01$ " " "

$\$292.91$

$\$4604.85 - \$292.91 = \$4311.94 =$ three times C's share.

$\$4311.94 \div 3 = \$1437.31\frac{1}{3} =$ C's share.

$\$1437.31\frac{1}{3} + \$90.90 = \$1528.21\frac{1}{3} =$ B's share.

$\$1528.21\frac{1}{3} + \$111.11 = \$1639.32\frac{1}{3} =$ A's share.

(3)

A and B working together can do the work in 96 hours, therefore in one hour they will do $\frac{1}{96}$ of it.

A by himself can do the work in 192 hours; therefore in 1 hour he can do $\frac{1}{192}$ of it. $\frac{1}{96} - \frac{1}{192} = \frac{1}{192} =$ part B can do in one hour. Therefore he will require as many hours to finish it as $\frac{1}{192}$ is contained times in the whole, i. e. $1 \div \frac{1}{192} = 192$ hours. Then $192 \div 14 = 13\frac{1}{2}$ days.

(4)

$\pounds 179\ 14s.\ 8\frac{1}{2}d. = \$718.94\frac{7}{8} = \$718.94583.$

$\$718.94583 \div .00000048 = \$71894583333.3 \div 48 =$
 $\$1497803819.4444.$

(5)

77	44..18..30..77..56..27	
30	4..18..80	8..27
36	2..8	4..9

$77 \times 30 \times 36 = 83160 =$ l. c. m.

(6)

Here $n = 20$, and $r = .0525$.

$$\text{Then } t = \frac{n-1}{r} = \frac{20-1}{.0525} = \frac{19}{.0525} = 361.9028 \text{ years} =$$

361 years 10 months 25 days.

(7)

7342163 octenary = 7t0e57 duodenary, and 61351 nonary = 1e454 duodenary.

$$7t0e57 \div 1e454 = 40.38 \text{ duodenary.}$$

(8)

$$783\frac{1}{2} = 3\frac{1}{2} + 10 \times 8 + 10 \times 10 \times 7.$$

lbs.	oz.	dwt.	grs.		lbs.	oz.	dwt.	grs.	
43	3	17	11	$\times 3\frac{1}{2}$	=	151	7	11	$2\frac{1}{2}$
			10						
<hr/> 433	<hr/> 2	<hr/> 14	<hr/> 14	$\times 8$	=	3465	9	16	16
			10						
<hr/> 4332	<hr/> 3	<hr/> 5	<hr/> 20	$\times 7$	=	30325	11	0	20
						<hr/> 33943	<hr/> 4	<hr/> 8	<hr/> 14 $\frac{1}{2}$

(9)

Here $a = 1$, and $r = \frac{1}{2}$.

$$\text{Then } S = \frac{a}{1-r} = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2.$$

(10)

$$\frac{1}{2} \text{ of } \frac{2}{3} \text{ of } 192 \div \frac{2\frac{1}{2}}{3} = 64 \div \frac{\frac{5}{2}}{3} = 64 \div \frac{\frac{5}{6}}{\frac{3}{2}} = 64 \div \frac{\frac{5}{8}}{\frac{1}{16}} = 64 \times \frac{16}{5} = 129\frac{3}{5}.$$

(11)

Logarithm of 129140163 = 8.111061.

 $8.111061 \div 17 = .477121 = \text{logarithm of } 3.$

(12)

Suppose 48

Suppose 36

18

18

6654

84

63— 18— 9

Errors. Assumed numbers.

— 18 $\times 36 = 648$ — 9 $\times 48 = 432$

Difference of errors = 9

9)216 = sum of products.

24

SECOND SERIES.

(13)

B is to have \$69.18 more than C.

A is to have \$69.18 + \$93.40 = \$162.58 " " "

\$231.76

\$897.43 — \$231.76 = \$665.67 = Amount to be divided equally amongst A, B, and C.

\$665.67 $\div 3 = \$221.89 = \text{C's share.}$

\$221.89 + \$69.18 = \$291.07 = B's "

\$291.07 + \$93.40 = \$384.47 = A's "

(14)

7 lbs. wheat	=	9 lbs. rye	}	7 = 9
5 " rye	=	8 " oats		5 = 8
13 " oats	=	21 " buckwheat		13 = 21 7
27 " buckwheat	=	20 " barley		3 27 = 20 4
24 " barley	=	26 " peas		3 24 = 26 2
11 " peas	=	35 " potatoes		11 = 35
x " potatoes	=	16 " wheat		x = 16

4 \times 2 \times 35 \times 16 4480Ans. $\frac{4480}{3 \times 11} = \frac{4480}{33} = 135\frac{5}{33}$

(15)

$$\frac{3}{8} \text{ of } 4\frac{1}{2} \text{ of } 7\frac{1}{2} \text{ of } \frac{9}{19\frac{1}{2}} \text{ of } \frac{5}{9} \text{ of } 3 \text{ oz. 4 drs. 2 scr. 5 grs.} = \frac{3}{8} \text{ of } \frac{3}{8} \text{ of}$$

$$\frac{3}{8} \text{ of } \frac{1}{3} \text{ of } \frac{5}{9} \text{ of } 1725 \text{ grs.} = 10350 \text{ grs.}$$

$$\frac{6}{11} \text{ of } \frac{6}{11} \text{ of } 2\frac{3}{4} \text{ of } \frac{1}{3} \text{ of } 6\frac{1}{2} \text{ times } 7 \text{ lbs. 3 oz.} = \frac{6}{11} \text{ of } \frac{7}{11} \text{ of } \frac{1}{4} \text{ of}$$

$$\text{of } \frac{1}{3} \text{ of } \frac{1}{2} \text{ of } 41760 \text{ grs.} = 62640 \text{ grs.}$$

$$10350 \div 62640 = .165229.$$

(16)

Dissimilar.

Similar.

Similar and Coterminous.

$$623.4279\dot{3} = 623.42793\dot{7}93 = 623.42793793\dot{7}93$$

$$93.426719\ddot{2} = 93.426719\ddot{2} = 93.4267192929\ddot{2}$$

$$\text{Difference} = 530.00121864500$$

(17)

$$\$1.00 - \$0.046 = \$0.954, \text{ and } \$7493 \div 0.954 = \$7854.29.$$

(18)

36 : 20 weeks	} :: 18 men:	
6 : 5 days		
9 : 11 hours		
11 : 24 cellars		
20 : 22 feet long		
16 : 22 feet wide		
5 : 4 feet deep		

$$= \frac{11 \times 22}{9} = 26\frac{2}{3}.$$

(19)

$$\frac{1}{2} \text{ of } \frac{3}{8} \text{ of } \frac{4}{7} = \frac{6}{35}; \text{ and if } \frac{6}{35} \text{ of a certain number} = \frac{7}{35}, \frac{1}{35} = \frac{1}{35},$$

$$\text{and } \frac{3}{35} = \frac{1}{35} \times 35 = 12.$$

$$(\{ [(\{[(12 \times 12\frac{1}{2}) + 31] \times 3\} - 33) \times 300] \div 17 \} \times 9)$$

$$= 81000.$$

(20)

1176		480..768..348..1176
32		20..32..29
145		5..29

$$1176 \times 32 \times 145 = 5456640.$$

(21)

	838)171347(204	
	1676	
	<hr/>	
	3747	
	3352	
	<hr/>	
17598)46090(2	395)838(2	
35196	790	
<hr/>	<hr/>	
10894)17598(1	48)395(8	
10894	384	
<hr/>	<hr/>	
6704)10894	11)48(4	
6704	44	
<hr/>	<hr/>	
4190)6704(1	4)11(2	
4190	8	
<hr/>	<hr/>	
2514)4190(1	3)4(1	
2514	3	
<hr/>	<hr/>	
1676)2514(1	1)3	
1676	3	
<hr/>	<hr/>	
	838)1676(2	
	1676	

As no number greater than unity will divide all of them without a remainder, they have no G. C. M.

(22)

	$\$12000 \times 4 = \48000	
$\$12000 + \$8000 = \$20000$	$\times 2 = \$40000$	
	<hr/>	
	$\$88000 =$ product of A's stock and time.	
	$\$25000 \times 3 = \75000	
$\$25000 - \$10000 = \$15000$	$\times 3 = \$45000$	
	<hr/>	
	$\$120000 =$ product of B's stock and time.	
	$\$35000 \times 2 = \70000	

(Continued on next page.)

(22 continued.)

$$\frac{2}{7} \text{ of } \$35000 = \$10000. \quad \$35000 - \$10000 = \$25000 \times 4 = \$100000$$

$$\$170000$$

= product of C's stock and time.

$\$88000 + \$120000 + \$170000 = \378000 = sum of the products of stocks and times.

$$\text{Then } \$378000 : \$88000 :: \$15000 : \frac{15000 \times 88000}{378000} = \$3492.06$$

= A's share.

$$\$378000 : \$170000 :: \$15000 : \frac{15000 \times 170000}{378000} = \$6746.03$$

= C's share.

$$\$15000 - (\$3492.06 + \$6746.03) = \$4761.91 = \text{B's share.}$$

(23)

A's gain in 5 months = \$125 \therefore his gain for 9 months

$$= 1\frac{1}{5} \times \$125 \dots\dots\dots = \$225$$

B's gain in 6 months = \$125 \therefore his gain for 9 months

$$= 1\frac{1}{2} \times \$125 \dots\dots\dots = \$187\frac{1}{2}$$

$$\text{C's gain in 9 months} \dots\dots\dots = \$125$$

$$\text{Sum} = \$537\frac{1}{2}$$

$$\text{Then } \$537\frac{1}{2} : \$225 :: \$400 : \frac{400 \times 225}{537\frac{1}{2}} = \$167\frac{13}{13} = \text{A's stock.}$$

$$\$537\frac{1}{2} : \$187\frac{1}{2} :: \$400 : \frac{400 \times 187\frac{1}{2}}{537\frac{1}{2}} = \$139\frac{2}{3} = \text{B's stock.}$$

$$\$537\frac{1}{2} : \$125 :: \$400 : \frac{400 \times 125}{537\frac{1}{2}} = \$93\frac{1}{3} = \text{C's stock.}$$

(24)

$\frac{1}{6} + \frac{1}{8} + \frac{1}{10} + \frac{1}{12} = \frac{57}{120} = \frac{19}{40}$ = part of the cistern filled in one hour when the four pipes are left open.

$\frac{1}{6} + \frac{1}{8} + \frac{1}{4} + \frac{1}{3} = \frac{11}{12} = \frac{30}{36}$ = part of the cistern emptied in one hour when the four are left open.

$\frac{30}{36} - \frac{19}{40} = \frac{19}{120}$ = part of the cistern which remains filled after the eight pipes have been left open for one hour. And if $\frac{19}{120}$ of the cistern are emptied in one hour, it will take $1 \div \frac{19}{120} = 2\frac{2}{19}$ hours to empty the whole of it.

THIRD SERIES.

(26)

As often as the first receives 4 the second receives 3, therefore as often as the first receives 6 the second receives $4\frac{1}{2}$. Then $6 + 4\frac{1}{2} + 7 = 17\frac{1}{2}$.

loaves.

$$17\frac{1}{2}:6 :: 2310:\frac{2310 \times 6}{17\frac{1}{2}} = 792 \text{ loaves} = \text{number the first receives,}$$

$$17\frac{1}{2}:4\frac{1}{2} :: 2310:\frac{2310 \times 4\frac{1}{2}}{17\frac{1}{2}} = 594 \quad " = \quad " \quad \text{second} \quad "$$

$$17\frac{1}{2}:7 :: 2310:\frac{2310 \times 7}{17\frac{1}{2}} = 924 \quad " = \quad " \quad \text{third} \quad "$$

(27)

To produce a mixture worth 8 cents a pound, we require
4 lbs. @ 12 cents, 4 @ 4 cents, 1 @ 5 cents, and 3 @ 9 cents.
or 3 lbs. @ 12 cents, 1 @ 4 cents, 4 @ 5 cents, and 4 @ 9 cents,
lbs.lbs. lbs. lbs.lbs. lbs.

$$\begin{array}{ll} \text{Then } 4:72::4:72 \text{ lbs. @ 4 cts.} & \text{or } 3:72::1:24 \text{ lbs. @ 4 cts.} \\ 4:72::1:18 \text{ lbs. @ 5 cts.} & 3:72::4:96 \text{ lbs. @ 5 cts.} \\ 4:72::3:54 \text{ lbs. @ 9 cts.} & 3:72::4:96 \text{ lbs. @ 9 cts.} \end{array}$$

(28)

Here $A = \$4444.44$, $r = .0444$, and $t = 4.3\frac{1}{3}$.

$$\text{Then } P = \frac{A}{1+rt} = \frac{\$4444.44}{1+.0444 \times 4.3\frac{1}{3}} = \frac{\$4444.44}{1.19289\frac{1}{3}} = \$3725.764,$$

(29)

$$\$1.00 - \$0.0225 = \$0.9775. \quad \$23470 \div 0.9775 = \$24010.23.$$

(30)

Here $A = \$7493.47$, $r = .07$, and $t = 8$.

$$\text{Then } P = \frac{A}{1+rt} = \frac{7493.47}{1+.07 \times 8} = \frac{7493.47}{1.56} = \$4803.5064.$$

(31)

 $\$17460 \div 1.03125 = \$16930.909 = \text{sum to be invested.}$
 $16930.909 \div 2.95 = 5739.29 \text{ yds. cloth.}$
 $16930.900 \times .02\frac{1}{2} = \$423.27272 = \text{ad valorem duty.}$
 $\$17460 + \$1347.90 + \$479.40 + \$169.83 + \$423.27272 =$
 $\$19880.40272 = \text{whole cost.}$
 $\$25000 - \$19880.40272 = \$5119.59728 = \text{whole gain.}$
 $\text{Then } \$19880.40272 : \$100 :: \$5119.59728 : \frac{5119.59728 \times 100}{19880.40272} =$
 $27.75 = 27\frac{3}{4} \text{ per cent.}$

(32)

V.		III.		VIII.		XII.
134234	=	21122021	=	12701	=	3281
5		3		8		12
<hr/>		<hr/>		<hr/>		<hr/>
8		7		10		38
5		3		8		12
<hr/>		<hr/>		<hr/>		<hr/>
44		22		87		464
5		3		8		12
<hr/>		<hr/>		<hr/>		<hr/>
222		68		696		5569 den.
5		3		8		
<hr/>		<hr/>		<hr/>		
1113		206		5569 den.		
5		3				
<hr/>		<hr/>				
5569 den.		618				
		3				
		<hr/>				
		1856				
		3				
		<hr/>				
		5569 den.				

(33)

 $\frac{1}{4} \text{ of } 4\frac{1}{2} \text{ of } \frac{9\frac{1}{2}}{\frac{1}{2}8} \text{ of } \frac{1}{16} \text{ of } \frac{1}{9} \text{ of } £43 \text{ } 18\text{s. } 11\frac{1}{2}\text{d. } £43 \text{ } 18\text{s. } 11\frac{1}{2}\text{d.} =$
 $\$175.79\frac{1}{2}.$

(Continued on next page.)

(33 continued.)

$$\frac{2}{7} \text{ of } \frac{2}{3} \text{ of } \frac{3^2}{4} \text{ of } \frac{1}{15} \text{ of } \frac{7}{9} \text{ of } \$175.79\frac{1}{6}, = \frac{3}{7} \text{ of } \frac{9}{2} \text{ of } \frac{15}{1} \text{ of } \frac{1}{15} \text{ of } \frac{7}{9}$$

$$\text{of } \$175.79\frac{1}{6} = \frac{2}{3} \text{ of } \$175.79\frac{1}{6} = \$263.6875.$$

$$3\frac{2}{3} \text{ of } \frac{1}{17\frac{1}{2}} \text{ of } .56 \text{ of } 1.75 \text{ of } 6\frac{1}{2} \text{ times } \$97.18 =$$

$$\frac{3^5}{9} \text{ of } \frac{1}{3^2} \text{ of } \frac{56}{100} \text{ of } \frac{175}{100} \text{ of } 6\frac{1}{2} \text{ times } \$97.18; 6\frac{1}{2} \text{ times } \$97.18$$

$$= \$631.67.$$

$$\frac{85}{9} \text{ of } \frac{2}{35} \text{ of } \frac{56}{100} \text{ of } \frac{175}{100} \text{ of } \$631.67 = \frac{49}{9 \times 25} \text{ of } \$631.67$$

$$= \frac{7}{25} \text{ of } \$631.67.$$

$$\frac{19}{25} \text{ of } \$631.67 = \$137.5636.$$

$$\text{Then } \$263.6875 - \$137.5636 = \$126.1239 = \text{difference.}$$

(34)

$$\frac{1}{13} = 1 \div 13 \therefore \log. \frac{1}{13} = \log. 1 - \log. 13 = 0 - 1.113943$$

$$= \bar{2}.886057.$$

$$19.5 = 3 \times 13 \times 5 \div 10 \therefore \log. 19.5 = \log. 3 + \log. 13$$

$$+ \log. 5 - \log. 10.$$

$$\log. 3 = 0.477121$$

$$\log. 13 = 1.113943$$

$$\log. 5 = \log. 10 - \log. 2 = 1 - 0.301030 \therefore \log. 5 = 0.698970$$

$$\text{Sum} = 2.290043$$

$$\text{From which take } \log. 10 = 1$$

$$\text{Rem.} = 1.290034$$

$$= \log. 19.5.$$

$$1125 = 5^3 \times 3^2 \therefore \log. 1125 = (\log. 5) + 3 + (\log. 3) \times 2.$$

$$\log. 5 = 0.698970 \times 3 = 2.096910$$

$$\log. 3 = 0.477121 \times 2 = 0.954242$$

$$\text{Sum} = 3.051152 = \log. \text{ of } 1125.$$

(Continued on next page.)

(34 continued.)

$$28 \cdot 16 = 28\frac{1}{2} = \frac{16^2}{6} = 13^2 \div 6 \therefore \log. 28 \cdot 16 = (\log. 13) \times 2 \\ - (\log. 2 + \log. 3).$$

$$\log. 13 = 1 \cdot 113943 \times 2 = 2 \cdot 227886 \\ (\log. 2 + \log. 3) = (0 \cdot 301030 + 0 \cdot 477121) = 0 \cdot 778151 \\ \text{Diff.} = \underline{\underline{1 \cdot 449735}}$$

$$= \log. 28 \cdot 16.$$

$$65000 = 13 \times 5 \times 1000 \therefore \log. 65000 = \log. 13 + \log. 5 \\ + \log. 1000.$$

$$\log. 13 = 1 \cdot 113943 \\ \log. 5 = 0 \cdot 698970 \\ \log. 1000 = 3$$

$$\text{Sum} = \underline{\underline{4 \cdot 812913}} = \log. \text{ of } 65000.$$

$$\log. \cdot 0005 = \log. 5 \text{ with characteristic changed to } -4 \\ = \bar{4} \cdot 698970.$$

$$152 \cdot 1 = 3^2 + 13^2 \div 10 \therefore \log. 152 \cdot 1 = (\log. 3) \times 2 \\ + (\log. 13) \times 2 - \log. 10.$$

$$\log. 3 = 0 \cdot 477121 \times 2 = 0 \cdot 954242 \\ \log. 13 = 1 \cdot 113943 \times 2 = 2 \cdot 227886$$

$$\text{Sum} = \underline{\underline{3 \cdot 182128}}$$

$$\text{From which take } \log. 10 = 1$$

$$\text{Diff.} = \underline{\underline{2 \cdot 182128}} = \log. 152 \cdot 1.$$

$$8 \cdot 112 = 2^4 \times 13^2 \times 3 \div 1000 \therefore \log. 8 \cdot 112 = (\log. 2) \times 4 \\ + (\log. 13) \times 2 + \log. 3 - \log. 1000.$$

$$\log. 2 = 0 \cdot 301030 \times 4 = 1 \cdot 204120 \\ \log. 13 = 0 \cdot 113943 \times 2 = 2 \cdot 227886 \\ \log. 3 = \quad \quad \quad 0 \cdot 477211$$

$$\text{Sum} = \underline{\underline{3 \cdot 909217}}$$

$$\text{From which take } \log. 1000 = 3$$

$$\text{Diff.} = \underline{\underline{0 \cdot 909217}} = \log. 8 \cdot 112.$$

(35)

XII.

$t^2 \times 300 = 21000$	871tet·72 (1t8·22
$t \times 8 \times 30 = 1800$	6e4
$8^2 = 54$	179tet
22854	159768
$t 8^2 \times 300 = 2454000$	20352720
$t 8 \times t \times 30 = 22800$	
$t^2 = 84$	
2476884	1 e t 372 e 4
$t 8 t^2 \times 300 = 249961000$	517428000
$t 8 t \times 2 \times 30 = 54500$	
$2^2 = 4$	
2499e5504	4977 t t t 08
	3e8301e4

(36)

$\frac{1}{6} + \frac{1}{12} + \frac{1}{7} + 5$ years = $\frac{11}{8}$ of life time + 5 years = age at birth of son.
 $\frac{28}{8} - (\frac{11}{8} + 5) = \frac{17}{8}$ of his life time — 5 years = time he lived after birth of son.

$\frac{17}{8}$ of father's life time — 5 years — 4 years = age of son = $\frac{1}{2}$ father's age.

$\frac{17}{8}$ of father's life time — 9 years = $\frac{1}{2}$ father's age.

∴ 9 years is the difference between $\frac{17}{8}$ and $\frac{14}{8}$ of father's age.

∴ 9 years is equal to $\frac{3}{8}$ of father's age.

If 9 years is $\frac{3}{8}$ of his age, $\frac{1}{8}$ will be the $\frac{1}{3}$ of 9 which is 3 years.

If $\frac{1}{8}$ is 3 years, $\frac{28}{8}$ or the whole age will be $3 \times 28 = 84$ years.

Or by Position.

Assume 42 for father's age at death, the son's age = 21.

$\frac{1}{6} + \frac{1}{12} + \frac{1}{7} + 5 = \frac{11}{8} + 5$; $\frac{11}{8}$ of 42 = $16\frac{1}{2}$ and $16\frac{1}{2} + 5 = 21\frac{1}{2}$ = age of father when son was born.

∴ he lived after birth of his son $42 - 21\frac{1}{2} = 20\frac{1}{2}$ years.

(Continued on next page.)

(36 continued.)

By the question he lived $21 + 4 = 25$ years.

The error $25 - 20\frac{1}{2} = -4\frac{1}{2}$.

Assume 98 for father's age, then son's age $= \frac{1}{2}$ of 98 = 49.

$\frac{1}{6} + \frac{1}{12} + \frac{1}{7} + 5 = \frac{1}{28} + 5$; $\frac{1}{28}$ of 98 = $38\frac{1}{2}$, and $38\frac{1}{2} + 5 = 43\frac{1}{2}$
= age of father at birth of son.

\therefore he lived after birth of his son $98 - 43\frac{1}{2} = 54\frac{1}{2}$ years.

But by the question he lived $49 + 4$ years = 53 years.

Then $53 - 54\frac{1}{2} = +1\frac{1}{2}$ = error.

Errors.

$$-4\frac{1}{2} \times 98 = 392$$

$$+1\frac{1}{2} \times 42 = 63$$

$$\begin{array}{r} \hline \text{Sum} = 6 \qquad \qquad 504 \end{array}$$

$$504 \div 6 = 84 = \text{father's age.}$$

(37)

m.	fur.	per.	yds.	ft.	in.		fur.	per.	yds.
63	3	7	3	2	7	\div	7	23	$3\frac{1}{2}$
8							40		
<u>507</u>							<u>303</u>		
40							5 $\frac{1}{2}$		
<u>20287</u>							<u>1518$\frac{1}{2}$</u>		
5 $\frac{1}{2}$							151 $\frac{1}{2}$		
<u>101438</u>							<u>1670$\frac{1}{2}$</u>		
10143 $\frac{1}{2}$							3		
<u>111581$\frac{1}{2}$</u>							<u>5010$\frac{1}{2}$</u>		
3							12		
<u>334746$\frac{1}{2}$</u>							<u>60129</u>		
12									
<u>4016965</u>									

(Continued on next page.)

(37 continued.)

60129)4016965(66·80578 times
 360774

409225

360774

484510

481032

347800

300645

471550

420903

506470

481032

(38)

 $6.3 \div .000000274$

274)63000000000(22992700·72992700

548

820

548

2720

2466

2540

2466

740

548

1920

1918

2000

1918

820

548

2720

2466

2540

2466

740

548

1920

1918

200 remainder.

(39)

$$\frac{7}{8} \text{ yds.} : 6\frac{3}{4} \text{ yds.} :: \$1\frac{3}{4} : \frac{8}{7} \times \frac{69}{11} \times 1\frac{3}{4} = 7\frac{175}{106} = \$5.482.$$

(40)

$$I = Prt = \$4237.71 \times .065 \times 1.67 = \$460.0034205.$$

(41)

$$t = \frac{A - P}{Pr} = \frac{\$1000 - \$674.30}{\$674.30 \times .085} = \frac{325.70}{57.3155} = 5.68258 \text{ years} =$$

5 years 8 months 5.7288 days.

(42)

By Table, page 260, the amount of \$1 for 14 payments at 4 per cent is \$1.73168.

Then $\$1.73168 \times 813.71 = \$1409.0853328 = \text{Amount.}$

Subtract 813.71

Difference = 595.3753328 = Interest.

(43)

\$300	×	0	=	0
700	×	4	=	2800
750	×	7	=	5250
850	×	9	=	7650
400	×	13	=	5200
1300	×	19	=	24700
<hr/> 4300)		45600 (10 months 18 $\frac{2}{3}$ days.
			<hr/>	4300
			<hr/>	2600
			<hr/>	30
			<hr/>	78000 = days.
			<hr/>	4300
			<hr/>	35000
			<hr/>	34400
			<hr/>	61800
			<hr/>	431800

(44)

23 per cent of \$4200 = $\frac{23}{100}$ of 4200 = \$966.00, and \$4200 — \$966.00 = \$3234.00. E has half as much as A, B, C, and D together; therefore E has *one-third* of \$3234.00, which is \$1078.00.

Deducting E's share, \$1078, from \$3234, the whole sum to be divided, there remains \$2156 to be divided among A, B, C, and D. Now D gets a certain amount; C gets \$42.11 more than D; B gets \$61.34 ($42.11 + 19.23$) more than D; and A gets \$78.44 ($61.34 + 17.10$) more than D. Together they get, then, *four times* D's share, together with \$42.11 + \$61.34 + \$8.44, or, in other words, four times D's share, together with \$181.89.

That is, four times D's share, together with \$181.89 is equal to \$2156.

Hence \$2156.00 — \$181.89 = \$1974.11 = four times D's share.

Then \$1974.11 $\div 4$ = \$493.5275 = D's share.

Add 42.11

Sum \$535.6375 = C's share.

Add 19.23

Sum \$554.8675 = B's share.

Add 17.10

Sum \$571.9675 = A's share.

(45)

$$P = \frac{A}{1+rt} = \frac{\$3786.80}{1+1.76^*} = \frac{3786.80}{2.76} = \frac{378680}{276} = \$1372.02898 +$$

(46)

$$\{ (3\frac{3}{7} - 2\frac{7}{10}) \times .46 \div \frac{2}{3} \text{ of } .142857 \} \div 8\frac{1}{2} \text{ times } (\frac{1}{2} + \frac{1}{4} + \frac{1}{5} - \frac{33}{2310})$$

$$\{ (.73 \times .12345 \div \frac{679}{150}) + \frac{3}{7} + 9\frac{3}{5} + 17\frac{4}{11} \} \div 27.4922077$$

$$= \{ (3\frac{3}{70} - 2\frac{49}{10}) \times \frac{46}{99} \div \frac{2}{3} \text{ of } \frac{1}{7} \} \div \frac{17}{2} \times (\frac{35}{10} + \frac{10}{70} + \frac{14}{10} - \frac{33}{2310})$$

$$\{ (\frac{66}{90} \times \frac{12345}{99000} \div \frac{679}{150}) + \frac{3}{7} + 9\frac{3}{5} + 17\frac{4}{11} \} \div 27.4922077$$

(Continued on next page.)

$$* rt = .16 \times 11 = 1.76.$$

(46 continued.)

$$\begin{aligned}
 &= \frac{(\frac{51}{70} \times \frac{16}{99} \times \frac{5}{2} \times \frac{7}{7}) \times \frac{2}{17} \times \frac{2310}{1610}}{\{(\frac{11}{15} \times \frac{679}{8500} \times \frac{750}{679}) + 27\frac{151}{388}\} \div 27\cdot4922077} \\
 &= \frac{\frac{391}{66} \times \frac{2}{17} \times \frac{2310}{1610}}{(\frac{1}{10} + 27\frac{151}{388}) \div 27\cdot4922077} = \frac{1}{27\frac{79}{70} \div 27\cdot4922077} \\
 &= \frac{1}{27\cdot4922077 \div 27\cdot4922077} = \frac{1}{1} = 1
 \end{aligned}$$

(47)

312312302 quaternary = 224690 decimal scale.

2312132 quaternary = 11678 decimal scale.

$$\text{Sum} = 236368$$

4234 quinary = 569 decimal, and $569 \times 23011 = 13093259$. $236368 \times 13093259 = 3094827443312$. $555 + 444 + 333 + 222 + 111$ senary = 2553 senary = 645 decimal. $3094827443312 - 645 = 3094827442667$.

6542 septenary = 2333 decimal.

 $3094827442667 \div 2333 = 1326544124\frac{1375}{2333}$ den.

X.

VIII.

$$1326544124 = 11704272374$$

X.

VIII.

$$1375 = 2537$$

X.

VIII.

$$2333 = 4435$$

X.

VIII.

$$\therefore 1326544124\frac{1375}{2333} = 11704272374\frac{1537}{1437}$$

(48)

$$\cdot 1 = \frac{1}{10} \text{ and } (\frac{1}{10})^2 = \frac{1}{100} = \cdot 01$$

$$\cdot 1 = \frac{1}{2} \text{ and } (\frac{1}{2})^2 = \frac{1}{4} = \cdot 012345679.$$

FIFTH SERIES.

(50)

Assume 27 | 2..9..16..27..48 and 81; strike out 2, 9 and 16,
 16 3 since they are con-
 tained as factors in
 the others.

The l. c. m. $= 27 \times 16 \times 3 = 1296$.

(51)

$$t = \frac{\log. n}{\log. (1+r)} = \frac{\log. 7}{\log. (1.06)} = \frac{0.845098}{0.025306} = 33.395 \text{ years.}$$

(52)

20 miles = 1267200 inches ; and 14 ft. 10 in. = 178 inches.
 $1267200 \div 178 = 7119\frac{9}{8}$ times.

(53)

$1749600 = 2^5 \times 3^7 \times 5^2$; increasing each index by unity and multiplying, we have $6 \times 8 \times 3 = 144$.

(54)

$$\begin{aligned} \frac{2}{3} \text{ of } \frac{96}{\frac{5}{6}} \div \frac{\frac{1}{2} \text{ of } 7}{\frac{3\frac{1}{4}}{2}} &= \frac{2}{3} \times \frac{\frac{96}{1}}{\frac{5}{6}} \div \frac{\frac{7}{2}}{\frac{3\frac{1}{4}}{2}} = \frac{2}{3} \times \frac{576}{5} \div \frac{\frac{7}{2}}{\frac{13}{8}} \\ &= \frac{2}{3} \times \frac{576}{5} \div \frac{28}{13} = \frac{2}{3} \times \frac{576}{5} \times \frac{13}{28} = 35\frac{3}{5}. \end{aligned}$$

(55)

A can do the whole work in 12 days, therefore he can do $\frac{1}{12}$ in 1 day. A and B together can do the work in 5 days, therefore they can do $\frac{1}{5}$ in 1 day. Therefore B can do $\frac{1}{5} - \frac{1}{12} = \frac{7}{60}$ in 1 day, and he will require as many times 1 day to do the whole work as $\frac{7}{60}$ is contained times in 1, i. e. $1 \div \frac{7}{60} = \frac{60}{7} = 8\frac{4}{7}$ days.

(56)

$$P = \frac{A}{(1+r)^t}; \log. P = \log. A - \log. (1+r) \times t = \log. 8899.77 \\ - \log. (1.06) \times 22 = 3.949378 - 0.025306 \times 22 \\ = 3.949378 - 0.556732 = 3.392646, \text{ and } \log. 3.392646 \\ = \$2469.71.$$

By Table, page 260, amount of \$1 at 6 per cent. for 22 payments = 3.60354.

Then $\$8899.77 \div 3.60354 = \2469.73 nearly.

(57)

Let the 1st number be 2. Then $2 \times 2 = 4$

$$1\frac{1}{2} \times 3 = 4$$

$$10 - (2 + 1\frac{1}{2}) = 10 - 3\frac{1}{2} = 6\frac{1}{2} + 4 = 26\frac{1}{2}, \text{ but it should equal 4.}$$

Therefore $26\frac{1}{2} - 4 = + 22\frac{1}{2} = \text{error.}$

Let $1\frac{1}{2}$ be the 1st number; then $1\frac{1}{2} \times 2 = 3$

$$1 \times 3 = 3$$

$$10 - (1\frac{1}{2} + 1) = 10 - 2\frac{1}{2} = 7\frac{1}{2} \times 4 = 30, \text{ but it should } = 3.$$

Therefore $30 - 3 = + 27 = \text{error.}$

Errors.

$$+ 27 \times 2 = 54$$

$$+ 22\frac{1}{2} \times 1\frac{1}{2} = 44$$

Diff. = $4\frac{1}{3}$ diff. = 20, and $20 \div 4\frac{1}{3} = 4\frac{8}{13} = \text{1st number.}$

$$4\frac{8}{13} \times 2 = 9\frac{3}{13} = \text{1st product.}$$

Second number = $9\frac{3}{13} \div 3 = 3\frac{1}{13} \times 3 = 9\frac{3}{13} = \text{2nd product.}$

$$10 - 7\frac{9}{13} = 2\frac{4}{13} \times 4 = 9\frac{3}{13} = \text{3rd product.}$$

(58)

Suppose A has 40; then B has $110 - 40 = 70$, and C has $130 - 70 = 60$.

A and C together have $40 + 60 = 100$, but it should be 120.

Therefore $100 - 120 = -20 = \text{error.}$

Suppose A has 80; then B has $110 - 80 = 30$, and C has $130 - 30 = 100$.

A and C together have $80 + 100 = 180$, but they should have 120.

Therefore $180 - 120 = + 60 = \text{error.}$

(Continued on next page.)

(53 continued.)

Errors.

$$+ 60 \times 40 = 2400$$

$$- 20 \times 80 = 1600$$

$$\text{Sum} = 80 \qquad \text{Sum} = 4000$$

$4000 \div 80 = 50 =$ number A has.

Then B has $110 - 50 = 60$, and C has $130 - 60 = 70$.

$$50 + 60 + 70$$

$$\frac{\quad}{3} = 60 = \text{each man's share when equally divided.}$$

(59)

$$\text{Formula I, p. 333. } l = a + (n - 1) d = 7 + (47 - 1) \times 4 \\ = 7 + 46 \times 4 = 7 + 184 = 191.$$

$$\text{Formula VI, p. 333. } s = \left\{ 2a + (n - 1) d \right\} \frac{n}{2} \\ = \left\{ 2 \times 7 + (93 - 1) \times 4 \right\} \frac{93}{2} = \left\{ 14 + (92 \times 4) \right\} \frac{93}{2} \\ = (14 + 368) \times \frac{93}{2} = \frac{382 \times 93}{2} = 17763.$$

(60)

$$t = \frac{\log. n}{\log. (1 + r)} = \frac{\log. 21}{\log. (1.07)} = \frac{1.322219}{0.029384} = 44.997 \text{ years.}$$

SIXTH SERIES.

(61)

B gets \$196.87 more than C, and A gets \$387 + \$196.87 = \$583.87 more than C, therefore together they get *three* times C's share, together with \$196.87 + \$583.87, i. e. three times C's share, together with \$780.74; but together they get \$3700.

Therefore \$3700 = three times C's share, together with \$780.74, or \$3700 - \$780.74 = \$2919.26 = three times C's share.

Hence \$2919.26 \div 3 = \$973.08 $\frac{2}{3}$ = C's share.

$$\text{Add } 196.87$$

$$\text{Sum} = \$1169.95\frac{2}{3} = \text{B's share.}$$

$$\text{Add } 387.00$$

$$\text{Sum} = \$1556.95\frac{2}{3} = \text{C's share.}$$

(62)

$$5716 = 2^2 \times 1429$$

$$1 \quad \dots \quad 2 \quad \dots \quad 4$$

$$1 \quad \dots \quad 1429$$

$$1 \quad \dots \quad 2 \quad \dots \quad 4 \quad \dots \quad 1429 \quad \dots \quad 2858 \quad \dots \quad 5716$$

(63)

$$\left\{ (17\frac{7}{12} - 10\frac{59}{60}) - (.4 + \frac{1}{6} + .9 - \frac{1}{2}) \right\} \div (.8378 \div \frac{1}{2} \text{ of } 31)$$

$$.6322632 \times \frac{1}{2} \text{ of } 9\frac{1}{4} \div (\frac{1}{8} \text{ of } 4\frac{1}{9} \text{ of } \frac{1}{11} \text{ of } 85\frac{1}{37} \div 101)$$

$$6\frac{3}{8} - 1 \div (\frac{8370}{9990} \times \frac{2}{31})$$

$$= \frac{\frac{6322}{9999} \times \frac{1}{2} \times \frac{37}{4} \div (\frac{1}{8} \times \frac{37}{9} \times \frac{1}{11} \times \frac{3161}{37} \times \frac{1}{101})}{5\frac{3}{8} \times \frac{9990}{8370} \times \frac{3}{2}}$$

$$= \frac{\frac{6322}{9999} \times \frac{1}{2} \times \frac{37}{4} \times \frac{5}{1} \times \frac{9}{37} \times \frac{11}{1} \times \frac{37}{3161} \times \frac{101}{1}}{\frac{28}{8} \times \frac{37}{11} \times \frac{3}{2}}$$

$$= \frac{\frac{3161}{101} \times \frac{37}{4} \times \frac{5}{1} \times \frac{1}{37} \times \frac{37}{3161} \times \frac{101}{1}}{\frac{14 \times 37}{5} \quad \frac{14 \times 37}{5}}$$

$$= \frac{5}{\frac{1}{4} \times \frac{5}{1} \times \frac{37}{1}} = \frac{5}{5 \times 37} = \frac{56}{28} = 2\frac{6}{7}.$$

4

(64)

Each child gets 1 child's share, \therefore 17 children get 17 shares.

Each woman gets *three* times a child's share, \therefore 4 women get 12 shares.

Each man gets *six* times a child's share, \therefore 3 men get 18 shares.

And together they get 47 times a child's share.

Therefore $\$7200 \div 47 = \$153.19\frac{7}{47} =$ a child's share.

$$\$153.19\frac{7}{47} \times 3 = \$459.57\frac{21}{47}.$$

$$\$153.19\frac{7}{47} \times 6 = \$919.14\frac{42}{47}.$$

(65)

$254000 = 2^3 \times 5^2 \times 127$. Adding unity to each index and multiplying the results, we get $4 \times 3 \times 2 = 24$.

(66)

$$\frac{2}{3} \text{ of } 4\frac{1}{2} \text{ of } \frac{9\frac{3}{4}}{\frac{1}{14}} \text{ of } \frac{1}{6} \text{ of } £3 \text{ } 16\text{s. } 1\frac{1}{2}\text{d.} = \frac{2}{3} \times \frac{9}{2} \times \frac{66 \times 14}{7 \times 11} \times \frac{1}{6} \\ \times \$15 \cdot 39\frac{1}{6} = 6 \text{ times } \$15 \cdot 39\frac{1}{6} = \$92 \cdot 35.$$

$$\frac{3}{11} \text{ of } 4\frac{3}{8} \text{ of } \frac{19\frac{1}{2}}{3\frac{1}{4}} \text{ of } \frac{95}{117} \text{ of } \frac{1\frac{1}{3}}{2\frac{1}{3}} \text{ of } \cdot 85 \text{ of } \frac{1}{42\frac{1}{2}} \text{ of } \$1783$$

$$= \frac{3}{11} \times \frac{23}{5} \times \frac{39}{\frac{19}{4}} \times \frac{95}{117} \times \frac{1\frac{1}{3}}{2\frac{1}{3}} \times \frac{85}{100} \times \frac{2}{88} \text{ of } \$1783.$$

$$= \frac{3}{11} \times \frac{23}{5} \times \frac{78}{19} \times \frac{95}{117} \times \frac{11}{23} \times \frac{85}{100} \times \frac{2}{85} \times \frac{1783}{1}$$

$$= \$17 \cdot 83 \times 4 = \$71 \cdot 32. \quad \$92 \cdot 35 - \$71 \cdot 32 = \$21 \cdot 03.$$

(67)

$$\left. \begin{array}{l} 7 : 13 = 7 \div 13 = \cdot 538 \\ 9 : 16 = 9 \div 16 = \cdot 562 \\ 8 : 15 = 8 \div 15 = \cdot 533 \\ 10 : 19 = 10 \div 19 = \cdot 526 \end{array} \right\} \text{Therefore } 9 : 16 \text{ is the} \\ \text{greatest, and } 10 : 19 \text{ is} \\ \text{the least.}$$

$$\text{Compound ratio} = \frac{7}{13} \times \frac{3}{16} \times \frac{8}{15} \times \frac{5}{19} \times \frac{21}{247} = 21 : 247.$$

(68)

$$67 \cdot 432 = 67 \frac{432}{1000} = \frac{66758}{990} \text{ and } 7 \cdot 9036 = 7 \frac{9036}{1000} = \frac{78957}{990} \\ \frac{66758}{990} \div \frac{78957}{990} = \frac{66758}{990} \times \frac{990}{78957} = \frac{7410138}{868527} = 8.5318452.$$

(69)

9 per. 9 yds. 7 ft. 120 in. = 365628 inches

 $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{2}{7}$ of 35 acres 2 roods = $\frac{3}{35}$ of 35 acres 2 roods = $\frac{3}{35}$ of 222678720 inches

$$\frac{365628}{\frac{3}{35} \text{ of } 222678720} = \frac{2559396}{133607232} = 0.019156118.$$

(70)

Dissimilar.

Similar.

17.0342

17.03424242

27.06357

27.06357575

98.123456

98.123456456

829.6423

829.642342342

986.1234298

986.1234298429

9.876342

9.876342876342

813.9864234567

813.9864234567

Similar and Coterminous.

17.0342424242424242

27.0635757575757575

98.123456456456456456

829.642342342342342

986.123429842984298429

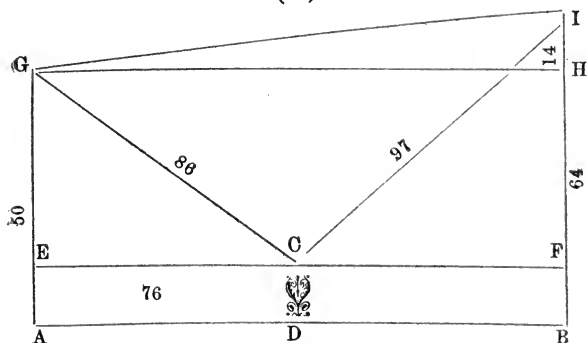
9.876342876342876342

813.986423456745674567

4 carried

2781.849813156689829957

(71)



$$EG = \sqrt{86^2 - 76^2} = \sqrt{1620} = 40.249 \text{ feet}$$

Height of Statue $CD = AG - EG = 50 - 40.249 = 9.751 \text{ ft.} = BF$

$$FI = BI - BF = 64 - 9.751 = 54.249 \text{ feet}$$

$$CF = \sqrt{CI^2 - FI^2} = \sqrt{97^2 - 54.249^2} = \sqrt{6466.045999} = 80.411 \text{ feet}$$

$$GH = EF = EC + CF = 76 + 80.411 = 156.411 \text{ feet and}$$

$$HI = 64 - 50 = 14 \text{ feet}$$

$$GI = \sqrt{GH^2 + HI^2} = \sqrt{156.411^2 + 14^2} = \sqrt{24660.400921} = 157.036 \text{ feet.}$$

(72)

The mixture = spirits + water = $\frac{1}{2}$ of mixture + 25 gal. + $\frac{1}{3}$ of mixture - 5 gal. = $\frac{1}{2} + \frac{1}{3} + 20 \text{ gal.} = \frac{5}{6} + 20 \text{ gal.}$ Then 20 gal. = $\frac{1}{6}$ of the mixture, and therefore the mixture contained $6 \times 20 = 120 \text{ gal.}$

$$\begin{aligned} \text{Then } \frac{1}{2} \text{ of } 120 &= 60 + 25 = 85 \text{ gal.} = \text{spirits} \\ \frac{1}{3} \text{ of } 120 &= 40 - 5 = 35 \text{ gal.} = \text{water} \end{aligned}$$

SEVENTH SERIES.

(73)

$$\begin{array}{r}
 \begin{array}{c} \cdot \cdot \cdot \cdot \cdot \\ 401241 \cdot 3424 \end{array} (422 \cdot 32 \\
 31 \\
 \hline
 132 \quad) \quad 412 \\
 \quad \quad 314 \\
 \quad \quad \hline
 1342 \quad) \quad 4341 \\
 \quad \quad 3234 \\
 \quad \quad \hline
 13443 \quad) \quad 110234 \\
 \quad \quad 101434 \\
 \quad \quad \hline
 140012 \quad) \quad 330024 \\
 \quad \quad \quad 330024.
 \end{array}$$

(74)

Suppose father's age = 60, the son's age now = $60 \div 5 = 12$,
 and son's age four years ago = $12 - 4 = 8$. But the son's
 age four years ago should, by the question, have been $60 \div$
 $7 = 8\frac{4}{7}$.

Therefore $8 - 8\frac{4}{7} = -\frac{4}{7} = \text{error}$.

Suppose father's age = 35; then son's age now = $35 \div 5 = 7$,
 and age four years ago = $7 - 4 = 3$.

But son's age four years ago should, by question, have been 35
 $\div 7 = 5$.

Therefore $3 - 5 = -2 = \text{error}$.

Errors.

$$-2 \times 60 = 120$$

$$-\frac{4}{7} \times 35 = 20$$

$$\text{diff. } 1\frac{3}{7} \quad \text{diff} = 100$$

$$100 \div 1\frac{3}{7} = 70 = \text{father's and son's age} = 70 \div 5 = 14.$$

(79)

Here $a = 1$, $d = 6$, $n = 101$,

$$s = \left\{ 2a + (n-1)d \right\} \frac{n}{2} = \left\{ 2 \times 1 + (101-1) \times 6 \right\} \frac{101}{2}$$

$$= (2 + 600) \frac{101}{2} = \frac{602 \times 101}{2} = 30401.$$

(80)

$$\begin{array}{cccccccc} 4 & & 5 & & & & & \\ 19 & 11 & 35 & 11\frac{3}{4} & 8 & 4\frac{1}{2} & 11\frac{3}{4} \times 4 \times 5 & 228\frac{1}{2} \\ \hline \frac{19}{7} \times \frac{11}{56} \times \frac{35}{121} \times \frac{11\frac{3}{4}}{29} \times \frac{8}{4\frac{3}{4}} \times \frac{4\frac{1}{2}}{3} = \frac{11\frac{3}{4} \times 4 \times 5}{7 \times 7 \times 11 \times 3} = \frac{228\frac{1}{2}}{1617} = 228\frac{1}{2} : 1617. \end{array}$$

(82)

$$\left(\left\{ (9\frac{1}{2} + 4\frac{1}{2} + 3\frac{1}{2} - 16\frac{3}{4}) \times .54 \right\} \div 1\frac{1}{2} \right) \times 35 \text{ times } .142857$$

$$\frac{\left\{ .97 \times .24378 \times (1\frac{1}{4} \times 4\frac{1}{2}) \right\} \times (4\frac{3}{4} - 2\frac{1}{4})}{\left\{ (16\frac{3}{4} - 16\frac{1}{2}) \times \frac{5}{4} \right\} \div 1\frac{1}{2}} \times 35 \times \frac{1}{7}$$

$$= \frac{\frac{88}{90} \times \frac{24378}{9900} \times \frac{12}{44} \times \frac{1800}{451} \times (4\frac{3}{4} - 2\frac{1}{4})}{\frac{121}{44} \times \frac{6}{1800} \times \frac{7}{44} \times \frac{35}{1} \times \frac{1}{7}} = \frac{\frac{1}{2}}{\frac{381}{187}} = \frac{187}{381}$$

(83)

Suppose the *hour* hand moves over 4 minutes, then since the minute hand moves 12 times as fast, it will have travelled over 48 minutes. But in order to overtake the hour hand, the minute hand must traverse the entire circle, 60 minutes, plus the 4 minutes we have supposed the hour hand to have moved forward, *i. e.* 64 minutes. Then 48 should equal 64, for we should find the same number by each process; $48 - 64 = -16$ error.

Suppose hour hand moves over 6 minutes, the minute hand moves over $6 \times 12 = 72$ minutes. But minute hand moves over $60 + 6 = 66$ minutes.

Then $72 - 66 = +6$ error.

(Continued on next page.)

(83 continued.)

Errors.

$$- 16 \times 6 = 96$$

$$+ 6 \times 4 = 24$$

$$\text{Sum } 22 \quad \text{Sum } 120$$

$120 \div 22 = 5\frac{5}{11}$ min. = minutes passed over by the hour hand,
 hence space passed over by the minute hand = $5\frac{5}{11} \times 12$
 = $65\frac{5}{11}$ min. = 1 hour $5\frac{5}{11}$ min. = time.

(84)

$$\text{Log. } 5 = \log. 10 - \log. 2 = 1 - 0.301030 = 0.698970$$

$$3850000 = 5 \times 7 \times 11 \times 10000.$$

$$\therefore \text{Log. } 3850000 = \log. 5 + \log. 7 + \log. 11 + \log. 10000$$

$$= 0.698970 + 0.845098 + 1.041393 + 4 = 6.585461.$$

$$3181.81 = 31.81 \times 100 = 31\frac{9}{11} \times 100 = 3\frac{50}{11} \times 100.$$

$$\therefore \text{Log. } 3181.81 = \log. 5 + \log. 7 + \log. 1000 - \log. 11$$

$$= 0.698970 + 0.845098 + 3 - 1.041393 = 3.502675$$

$$.0000154 = 2 \times 7 \times 11 \div 10000000.$$

$$\therefore \text{Log. } .0000154 = \log. 2 + \log. 7 + \log. 11 - \log. 10000000$$

$$= 0.301030 + 0.845098 + 1.041393 - 7 = 5.187521.$$

$$\text{Log. } \frac{1}{77} = \log. 1 - (\log. 7 + \log. 11) = 0 - (0.845098$$

$$+ 1.041393, = 0 - 1.886491 = 2.113509.$$

$$1.571428 = 1\frac{4}{7} = \frac{11}{7}.$$

$$\text{Log. } 1.571428 = \log. 11 - \log. 7 = 1.041393 - 0.845098$$

$$= 0.196295$$

$$93.17 = 9317 \div 100 = 11^3 \times 7 \div 100.$$

$$\therefore \text{Log. } 9317 = 3 \text{ times } \log. 11 + \log. 7 - \log. 100 = 1.041393$$

$$\times 3 + 0.845098 - 2 = 1.969277.$$

EIGHTH SERIES.

(85)

$$\text{Simple Interest} = Prt = \$700 \times .045 \times 3 = \$94.50.$$

$$\text{Amount Compound Interest} = P(1+r)^t = \$700 \times (1.045)^3$$

$$= \$700 \times 1.14116 = \$798.814 - \$700 = \$98.814 = \text{Comp.}$$

$$\text{Int.}$$

$$\$98.814 - \$94.50 = \$4.314.$$

(86)

X's gain = $\frac{1}{12}$, and Z's = $\frac{1}{2}$; \therefore Y's gain = $1 - (\frac{1}{12} + \frac{1}{2})$
 $= 1 - \frac{7}{12} = \frac{5}{12}$.

X's gain is $\frac{1}{12}$ for 3 months, therefore for 1 month it is $\frac{1}{36}$.

Y's gain is $\frac{5}{12}$ for 9 months, " " " $\frac{5}{108}$.

Z's gain is $\frac{1}{2}$ for 4 months, " " " $\frac{1}{8}$.

$\frac{1}{8} : \frac{1}{36} :: \$3024 : \$3024 \times \frac{1}{36} \times \frac{8}{1} = \$672 = \text{X's stock.}$

$\frac{1}{8} : \frac{5}{108} :: \$3024 : \$3024 \times \frac{5}{108} \times \frac{8}{1} = \$1120 = \text{Y's stock.}$

(87)

$\frac{3}{8} \times \sqrt{1\frac{7}{9}} \div (1\frac{1}{2})^3 = \frac{3}{8} \times \sqrt{\frac{16}{9}} \div (\frac{3}{2})^3 = \frac{3}{8} \times \frac{4}{3} \times \frac{8}{27} = \frac{4}{27}$.

(88)

$$4^2 = 16 \times 300 = 4800$$

$$4 \times 3 = 12 \times 30 = 360$$

$$3^2 = 9$$

$$5169$$

$$43^2 = 1849 \times 300 = 554700$$

$$43 \times 2 = 86 \times 30 = 2580$$

$$2^2 = 4$$

$$557284$$

$$432^2 = 186624 \times 300 = 55987200$$

$$432 \times 1 = 432 \times 30 = 12960$$

$$1^2 = 1$$

$$56000161$$

80677568161 (4321 cube rt.)

64

16677

15507

1170568

1114568

56000161

56000161

(89)

$$7 = \left\{ 8 - 1 \begin{array}{l} \nearrow 3 + 4 \\ \searrow 1 + 6 \end{array} \right\} = 7$$

4 lbs. at 8d.

1 lb. at 4d.

1 lb. at 6d.

} Make a mixture of 6 lbs. at 7d.

$$6 : 112 :: 4 : \frac{112 \times 4}{6} = 74\frac{2}{3} \text{ at 8d.}$$

(Continued on next page.)

(89 continued.)

$$6 : 112 :: 1 : \frac{112 \times 1}{6} = 18\frac{2}{3} \text{ at 4d.}$$

$$6 : 112 :: 1 : \frac{112 \times 1}{6} = 18\frac{2}{3} \text{ at 6d.}$$

(90)

Assume 40 as the sum of the three numbers.

Since $1\text{st} + 2\text{nd} + 3\text{rd} = 40$,

And $1\text{st} + \frac{1}{2}(2\text{nd} + 3\text{rd}) = 34 \therefore \frac{1}{2}(2\text{nd} + 3\text{rd}) = 6 \dots \therefore 2\text{nd} + 3\text{rd} = 12$

And $2\text{nd} + \frac{1}{3}(1\text{st} + 3\text{rd}) = 34 \therefore \frac{2}{3}(1\text{st} + 3\text{rd}) = 6 \dots \therefore 1\text{st} + 3\text{rd} = 9$

And $3\text{rd} + \frac{1}{4}(1\text{st} + 2\text{nd}) = 34 \therefore \frac{3}{4}(1\text{st} + 2\text{nd}) = 6 \dots \therefore 1\text{st} + 2\text{nd} = 8$

$$\begin{array}{r} \text{Adding, } 2 \times (1\text{st} + 2\text{nd} + 3\text{rd}) = 29 \\ \therefore 1\text{st} + 2\text{nd} + 3\text{rd} = 14\frac{1}{2}. \end{array}$$

But the sum should equal 40.

$$\text{Hence } 14\frac{1}{2} - 40 = -25\frac{1}{2}.$$

Assume 48 as the sum of the three numbers.

Since $1\text{st} + 2\text{nd} + 3\text{rd} = 48$.

And $1\text{st} + \frac{1}{2}(2\text{nd} + 3\text{rd}) = 34 \therefore \frac{1}{2}(2\text{nd} + 3\text{rd}) = 14 \dots \therefore 2\text{nd} + 3\text{rd} = 28$

And $2\text{nd} + \frac{1}{3}(1\text{st} + 3\text{rd}) = 34 \therefore \frac{2}{3}(1\text{st} + 3\text{rd}) = 14 \dots \therefore 1\text{st} + 3\text{rd} = 21$

And $3\text{rd} + \frac{1}{4}(1\text{st} + 2\text{nd}) = 34 \therefore \frac{3}{4}(1\text{st} + 2\text{nd}) = 14 \dots \therefore 1\text{st} + 2\text{nd} = 18\frac{2}{3}$

$$\begin{array}{r} \text{Adding, } 2 \times (1\text{st} + 2\text{nd} + 3\text{rd}) = 67\frac{2}{3} \\ \therefore 1\text{st} + 2\text{nd} + 3\text{rd} = 33\frac{1}{3}. \end{array}$$

But the sum should equal 48.

$$\text{Hence } 33\frac{1}{3} - 48 = -14\frac{2}{3} = \text{error.}$$

Errors.

$$-25\frac{1}{2} \times 48 = 1224$$

$$-14\frac{2}{3} \times 40 = 566\frac{2}{3}$$

$$\text{Diff.} = 11\frac{1}{3} \qquad \text{Diff.} = 657\frac{1}{3}$$

$$657\frac{1}{3} \div 11\frac{1}{3} = 58 = \text{the sum of the three numbers.}$$

(Continued on next page.)

(90 nontinued.)

$$1\text{st} + \frac{1}{2} (2\text{nd} + 3\text{rd}) = 34 \therefore \frac{1}{2} (2\text{nd} + 3\text{rd}) = 58 - 34 = 24$$

$$\therefore 2\text{nd} + 3\text{rd} = 48.$$

$$2\text{nd} + \frac{1}{3} (1\text{st} + 3\text{rd}) = 34 \therefore \frac{2}{3} (1\text{st} + 3\text{rd}) = 58 - 34 = 24$$

$$\therefore 1\text{st} + 3\text{rd} = 36.$$

$$1\text{st} + 2\text{nd} + 3\text{rd} = 58, \text{ and } 2\text{nd} + 3\text{rd} = 48 \therefore 1\text{st} = 10.$$

$$1\text{st} + 2\text{nd} + 3\text{rd} = 58, \text{ and } 1\text{st} + 3\text{rd} = 36 \therefore 2\text{nd} = 22.$$

$$2\text{nd} + 3\text{rd} = 48, \text{ and } 2\text{nd} = 22 \therefore 3\text{rd} = 26.$$

(91)

4 means + 2 extremes = 6 terms.

$$\text{Formula IX, p. 333. } d = \frac{l - a}{n - 1} = \frac{40 - 1}{6 - 1} = \frac{39}{5} = 7\frac{4}{5}.$$

$$1, 8\frac{4}{5}, 16\frac{8}{5}, 24\frac{12}{5}, 32\frac{16}{5}, 40.$$

(92)

$$s = 1860040, l = 1240029, \text{ and } r = 3.$$

$$\text{Formula XI, p. 340. } a = rl - (r - 1)s = 1240029 \times 3$$

$$- 2 \times 1860040 = 3720087 - 3720080 = 7.$$

(93)

$$6 \text{ apples} + 7 \text{ pears cost } 33 \text{ pence } \therefore 2 \text{ apples} + 2\frac{1}{2} \text{ pears cost } 11 \text{ pence.}$$

$$10 \text{ apples} + 8 \text{ pears cost } 44 \text{ pence } \therefore 2 \text{ apples} + 1\frac{2}{5} \text{ pears cost } 8\frac{4}{5} \text{ pence.}$$

$$\text{Subtract, and } 2\frac{1}{2} - 1\frac{2}{5} \text{ pears cost } 11\text{d.} - 8\frac{4}{5}\text{d.}$$

$$\text{That is, } \frac{1}{10} \text{ of a pear costs } 2\frac{1}{5}\text{d.}$$

$$\text{If } \frac{1}{10} \text{ cost } \frac{1}{5}\text{d., } \frac{1}{15} \text{ will cost } \frac{1}{15} \text{ of } \frac{1}{5}\text{d., which is } \frac{1}{75}\text{d.}$$

$$\text{If } \frac{1}{10} \text{ cost } \frac{1}{5}\text{d., } \frac{1}{5} \text{ will cost } \frac{1}{5} \times \frac{1}{5}\text{d.} = 3\text{d.}$$

$$6 \text{ apples} + 7 \text{ pears cost } 33 \text{ pence, and } 7 \text{ pears cost } 21\text{d.} \therefore 6 \text{ apples cost } 12\text{d. and } 1 \text{ apple costs } 2\text{d.}$$

(94)

$$\frac{1}{2} \times \frac{3}{4} \times \frac{5}{9} \times \frac{\frac{57}{2}}{\frac{6}{1}} \times \frac{2}{3} \times \frac{4}{5} \times \frac{3}{4}$$

$$= \frac{1}{2} \times \frac{3}{4} \times \frac{5}{9} \times \frac{57}{12} \times \frac{2}{3} \times \frac{4}{5} \times \frac{3}{4} = \frac{19}{2 \times 4 \times 3 \times 2} = \frac{19}{48}.$$

(95)

\$10 = $\frac{2}{3}$ of 2nd rem. — \$20 \therefore $\frac{2}{3}$ of 2nd rem. = \$30 \therefore 2nd rem. = \$40.

\$40 = $\frac{1}{5}$ of 1st rem. — \$30 \therefore $\frac{1}{5}$ of 1st rem. = \$70 \therefore 1st rem. = \$87.50.

\$87.50 = $\frac{1}{2}$ of original sum — \$50 \therefore $\frac{1}{2}$ of original sum = \$137.50 \therefore original sum = \$137.50 \times 2 = \$275.

(96)

$a = 60$, $n = 17$, and $d = 4$.

$$\begin{aligned} \text{Formula VI, p. 333. } s &= \left\{ 2a + (n-1)d \right\} \frac{n}{2} \\ &= \left\{ 2 \times 60 + (17-1) \times 4 \right\} \frac{17}{2} = (120 + 64) \times \frac{17}{2} \\ &= \frac{184 \times 17}{2} = \$1564 = \text{sum received for 17 years.} \end{aligned}$$

Formula I, p. 333. $l = a + (n-1)d = 60 + (17-1) \times 4 = 60 + 64 = \$124 = \text{wages for 17th year.}$

NINTH SERIES.

(98)

£749 16s. 5½d. = £749.823958; £1 Sterling = \$4.867
 £749.823958 \times 4.867 = \$3649.3932.

(99)

$$\begin{array}{r} 2 \overline{)177408} \\ 2 \overline{)88704} \\ 2 \overline{)44352} \qquad 2 \overline{)1386} \\ 2 \overline{)22176} \qquad 3 \overline{)693} \\ 2 \overline{)11088} \qquad 3 \overline{)231} \\ 2 \overline{)5544} \qquad 7 \overline{)77} \\ 2 \overline{)2772} \qquad 11 \end{array}$$

$$2^8 \times 3^2 \times 7 \times 11.$$

(100)

$$\text{Formula III, page 354, } r = \sqrt[t]{\frac{A}{P}} - 1 \therefore r + 1 = \sqrt[t]{\frac{A}{P}}$$

$$\text{Log. } (r + 1) = (\log. A - \log. P) \div t$$

$$\begin{aligned} \text{That is, log. } (r + 1) &= (\log. 11111 \cdot 11 - \log. 704) \div 11 \\ &= (4 \cdot 045757 - 2 \cdot 847573) \div 11 \\ &= 1 \cdot 198184 \div 11 = 0 \cdot 108925 \end{aligned}$$

Therefore $r + 1$ = natural number corresponding to the logarithm $0 \cdot 108925$ which is $1 \cdot 285$.

Since $r + 1 = 1 \cdot 285$, $r = \cdot 285$ = rate per unit and rate per cent. $= \cdot 285 \times 100 = 28\frac{1}{2}$.

(101)

If 9 be $\frac{1}{13}$, $\frac{1}{13}$ or the whole will equal $9 \times 13 = 117$.

(102)

$$3 \text{ gal.} + 4 \text{ gal.} + 7 \text{ gal.} = 14 \text{ gal.}$$

$$\text{Hence } 14 \text{ gal.} : 292 \text{ gal.} :: 3 \text{ gal.} : \frac{292 \times 3}{14} = 62\frac{1}{2} \text{ of 1st kind.}$$

$$14 \text{ gal.} : 292 \text{ gal.} :: 4 \text{ gal.} : \frac{292 \times 4}{14} = 83\frac{2}{7} \text{ gal. of 2d "}$$

$$14 \text{ gal.} : 292 \text{ gal.} :: 7 \text{ gal.} : \frac{292 \times 7}{14} = 146 \text{ gal. of 3d}$$

(103)

$$£\frac{1}{2} + £\frac{1}{3} + £\frac{1}{4} + £\frac{1}{5} = £1\frac{17}{60}$$

$$\begin{aligned} \text{Then } £1\frac{17}{60} : £500 :: £\frac{1}{2} : £500 \times \frac{1}{2} \times \frac{60}{17} &= \frac{£15000}{17} \\ &= £194 \text{ 16s. } 14\frac{2}{17}\text{d.} \end{aligned}$$

$$\begin{aligned} £1\frac{17}{60} : £500 :: £\frac{1}{3} : £500 \times \frac{1}{3} \times \frac{60}{17} &= \frac{£10000}{17} \\ &= £129 \text{ 17s. } 4\frac{4}{17}\text{d.} \end{aligned}$$

$$\begin{aligned} £1\frac{17}{60} : £500 :: £\frac{1}{4} : £500 \times \frac{1}{4} \times \frac{60}{17} &= \frac{£7500}{17} \\ &= £97 \text{ 8s. } 0\frac{4}{17}\text{d.} \end{aligned}$$

$$\begin{aligned} £1\frac{17}{60} : £500 :: £\frac{1}{5} : £500 \times \frac{1}{5} \times \frac{60}{17} &= \frac{£6000}{17} \\ &= £97 \text{ 18s. } 5\frac{2}{17}\text{d.} \end{aligned}$$

(104)

By Table, page 363, present value of annuity of \$1 at 6 per cent. for 23 payments = \$12·30338.

Hence present value of \$100 = \$12·30338 × 100 = \$1230·338·

By Formula V, page 361, $v = \frac{a}{r} \left\{ 1 - \frac{1}{(1+r)^t} \right\}$

$$= \frac{100}{\cdot 06} \times \left(1 - \frac{1}{(1\cdot 06)^{23}} \right)^* = \frac{10000}{6} \times (1 - 0\cdot 25583)$$

$$= \frac{10000}{6} \times 0\cdot 74417 = \frac{7441\cdot 7}{6} = \$1240\cdot 28.$$

(105)

Since each loses 1 hour per day for 24 days, the whole hours lost = 24×25 .

Also, 5 men working 1 hour per day for 12 days make up $5 \times 12 \times 1 = 60$ hours.

Hence they will each have to work as many hours per day as 60 hours is contained times in 24×25 hours *i. e.* $\frac{24 \times 25}{60} = 10$ hours.

(106)

$$a = 5, s = 161 \text{ and } d = 6$$

Then Formula II, p. 333. $l = -\frac{1}{2}d + \sqrt{2d6 \div (a - \frac{1}{2}d)^2} = -\frac{1}{2} \text{ of } 6 + \sqrt{2 \times 6 \times 161 + (5 - \frac{1}{2} \text{ of } 6)^2} = -3 + \sqrt{1932 + 4} = -3 + \sqrt{1936} = -3 + 44 = 41 \text{ years.}$

(107)

$$6^3 : 10^3 :: 1 \text{ day} : \frac{10^3 \times 1}{6^3} = \frac{1000}{216} = 4\cdot 629 \text{ days.}$$

$$\begin{aligned} * \text{ Log. } \frac{1}{(1\cdot 06)^{23}} &= \log. 1 - \log. 1\cdot 06 \times 23 = 0 - 0\cdot 025306 \times 23 \\ &= 0 - 0\cdot 592038 = \bar{1}\cdot 407962 \end{aligned}$$

$$\therefore \frac{1}{(1\cdot 06)^{23}} = \text{natural number corresponding to the logarithm}$$

$\bar{1}\cdot 407962$, which is $0\cdot 25583$.

(108)

For 12 months he was to receive £8 and a suit of clothes; for 7 months he received £2 13s. 4d. and the suit of clothes; \therefore for 5 months he would have received the difference between £8 and £2 13s. 4d., which is £5 6s. 8d.

Hence for 1 month he would have received £5 6s. 8d. \div 5, which is £1 1s. 4d., and hence his wages for the year would have been, in money alone, £1 1s. 4d. \times 12 *i. e.* £12 16s.

Therefore the suit of clothes was valued at £12 16s. — £8 = £4 16s.

TENTH SERIES.

(109)

$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{13}{12}$; if $\frac{13}{12}$ of a number = 48, $\frac{1}{12}$ will = $48 \div 13 = 3\frac{9}{13}$. If $3\frac{9}{13} = \frac{1}{12}$, $\frac{13}{12}$, or the whole number = $3\frac{9}{13} \times 12 = 44\frac{4}{13}$.

(110)

$$6^3 : 8^3 :: 600 : \frac{600 \times 8^3}{6^3} = \frac{600 \times 512}{216} = 1422\cdot2 \text{ lbs.}$$

(See Art. 33, sec. X.)

(111)

Part of ball remaining after 1st has taken off her share = $\frac{3}{4}$

Then whole ball : remainder $::$ cube of diameter of whole : cube of diameter of remainder

$$1 : \frac{3}{4} :: 5^3 : x^3 \text{ hence } x = \sqrt[3]{\frac{3}{4} \times 125} = \sqrt[3]{\frac{375}{4}} = \sqrt[3]{93\cdot75} = 4\cdot542$$

\therefore Part taken off by 1st = 5 in. — 4·542 in. = 0·458 in.

After 2nd had taken off her portion $\frac{1}{2}$ of the ball remained.

$$1 : \frac{1}{2} :: 5^3 : x^3, \text{ hence } x = \sqrt[3]{\frac{1}{2} \times 125} = \sqrt[3]{\frac{125}{2}} = \sqrt[3]{62\cdot5} = 3\cdot968 \text{ in.}$$

\therefore Part taken off by 2nd = 4·542 — 3·968 = 0·574 in.

After 3rd had taken off her share there remained $\frac{1}{4}$ of the ball.

$$1 : \frac{1}{4} :: 5^3 : x^3, \text{ hence } x = \sqrt[3]{\frac{1}{4} \times 125} = \sqrt[3]{31\cdot25} = 3\cdot149 \text{ in.}$$

\therefore Part taken off by 3rd = 3·968 — 3·149 = 0·819 inches

Remainder = 3·149 = part taken off by 4th.

(112)

$$71213 \cdot 43 \div 12 \cdot 342 = 71213430 \div 12342$$

$$12342) 71213430 (5570 \cdot 238552$$

62831

72724

62831

87833

87525

3070.0

2468.4

500.50

371.36

118 130

111.067

7 0520

6.2831

.65780

.62831

.028480

.024684

.003685

IX.

$$\dot{5}5\dot{7}0 \cdot \dot{2}3855\dot{2} (71 \cdot 118 = \text{sq. rt.}$$

54

$$151) 170$$

151

$$1521) 18 \cdot 23$$

15.21

$$15221) 3 \cdot 0285$$

1.5221

$$152228) 1 \ 406452$$

1.360051

.036411

NOTE.—Unless the quotient is carried out to *six places* of decimals, i. e., twice as many as are required in the root, the last figure in the root will be 7 or 6.

(113)

$$\text{1st } \left\{ \begin{array}{l} \$60 \times 48 = \$2880 \text{ for 1 month} \\ \$800 \times 43 = 34400 \text{ for 1 month} \\ \$1500 \times 4 = 6000 \text{ for 1 month} \\ \text{Sum} = \$43280 \end{array} \right\} = \$43280 \text{ for 1 month.}$$

$$\text{2nd } \left\{ \begin{array}{l} \$600 \times 48 = \$28800 \text{ for 1 month} \\ \$1800 \times 42 = 75600 \text{ for 1 month} \\ \text{Sum} = \$104400 \end{array} \right\} = \$104400 \text{ for 1 month.}$$

(Continued on next page.)

(113 continued.)

$$\begin{array}{lcl}
 \text{3rd} & \left\{ \begin{array}{l} \$400 \times 48 = \$19200 \\ \$500 \times 42 = 21000 \\ \$500 \times 36 = 18000 \\ \$500 \times 30 = 15000 \\ \$500 \times 24 = 12000 \\ \$500 \times 18 = 9000 \\ \$500 \times 12 = 6000 \\ \$500 \times 6 = 3000 \\ \hline \text{Sum} = \$103200 \end{array} \right. & = \$103200 \text{ for 1 month.} \\
 \\
 \text{4th} & \left\{ \begin{array}{l} \$900 \times 40 = \$36000 \\ \$900 \times 34 = 30600 \\ \$900 \times 28 = 25200 \\ \$900 \times 22 = 19800 \\ \$900 \times 16 = 14400 \\ \$900 \times 10 = 9000 \\ \$900 \times 4 = 3600 \\ \hline \text{Sum} = \$138600 \end{array} \right. & = \$138600 \text{ for 1 month.}
 \end{array}$$

\$43280

104400

103200

138600

\$389480 for one month.

\$20000 - \$1825 = \$18175 = sum to be divided among the four.

\$389480 : \$18175 :: \$43280 : \$2019.651 = share of 1st.

\$389480 : \$18175 :: \$104400 : \$4871.803 = " 2nd.

\$389480 : \$18175 :: \$103200 : \$4815.805 = " 3rd.

\$389480 : \$18175 :: \$138600 : \$6467.739 = " 4th.

(114)

$$\begin{array}{l}
 \text{Simple Interest, formula IX, p. 248. } t = \frac{n-1}{r} = \frac{16-1}{.05} = \frac{15}{.05} \\
 = \frac{1500}{5} = 300 \text{ years.}
 \end{array}$$

$$\begin{array}{l}
 \text{Compound Interest, formula V, p. 354. } t = \frac{\log. n}{\log. (1+r)} \\
 = \frac{\log. 16}{\log. 1.05} = \frac{1.204120}{0.021189} = \frac{1204120}{21189} = 56.827 \text{ years.}
 \end{array}$$

(115)

For every \$1 the first gave, the second gave \$3 and the third \$6. $\$1 + \$3 + \$6 = \10 .

Hence the 1st gave \$1, the second \$3, and the third \$6 as often as \$10 is contained times in \$9202, which is $920\frac{1}{5}$ times.

$$\$1 \times 920\frac{1}{5} = \$920.20 = \text{payment of 1st person.}$$

$$\$3 \times 920\frac{1}{5} = \$2760.60 = \quad \quad \quad \text{"} \quad \quad \text{2nd} \quad \text{"}$$

$$\$6 \times 920\frac{1}{5} = \$5521.20 = \quad \quad \quad \text{"} \quad \quad \text{3rd} \quad \text{"}$$

(116)

$$25 + 22 = 47 = \text{whole number of men.}$$

$$165 \div 47 = 3\frac{24}{47} = \text{acres cleared by each man.}$$

$$3\frac{24}{47} \times 22 = 77\frac{11}{47} \text{ acres} = \text{acres cleared by company of 22 men.}$$

$$165 \text{ acres} - 77\frac{11}{47} \text{ acres} = 87\frac{36}{47} \text{ acres} = \text{acres cleared by company of 25 men.}$$

1st company contains 3 more men than 2nd company and receives \$86 more.

Therefore \$86 pays 3 men. Hence each man gets $\$86 \div 3 = \$28.66\frac{2}{3}$.

Each man clears $3\frac{24}{47}$ acres, and receives $\$28.66\frac{2}{3}$ for it; therefore cost of 1 acre $= \$28.66\frac{2}{3} \div 3\frac{24}{47} = \$8\frac{82}{95}$.

(117)

$$15^2 = 225; 346 - 225 = 121 = \text{square of the less.}$$

$$\text{Hence less} = \sqrt{121} = 11.$$

(118)

Formula V, page 248, $A = P(1 + rt) = \$1200 \times 1.95 = \2340.00 .

(119)

$$\begin{array}{l|l} 24 : 496 & \\ 9 : 11 & \\ 7 : 4 & \\ 465 : 537\frac{1}{2} & \\ 3\frac{2}{3} : 5\frac{2}{3} & \\ 2\frac{1}{3} : 3\frac{1}{3} & \end{array} \quad \therefore 5\frac{1}{2} : x$$

(Continued on next page.)

(119 continued.)

$$\begin{array}{cccccccccccc}
 & & & & 3 & & & & & & & & & \\
 & & & & 9 & & & & & & & & & \\
 81 & & & & 27 & 2 & & & & & & & & \\
 62 & & & & 185 & 4 & & & & & & & & \\
 496 & 11 & 4 & 675 & 28 & 7 & 11 & 1 & 1 & 1 & 1 & & & \\
 \hline 1 & 1 & 1 & 2 & 5 & 2 & 2 & 24 & 9 & 7 & 465 & & & \\
 & & & & & & & 3 & 3 & & 28 & & & \\
 & & & & & & & & & & 31 & & & \\
 & & 8 & 8 & & & & & & & & & & \\
 \times \frac{11}{11} \times \frac{8}{7} = 11 \times 4 \times 3 = 132 \text{ days.} & & & & & & & & & & & & &
 \end{array}$$

(120)

$ \begin{array}{rcl} A + B + C & = & \frac{47}{60} \\ B + C + D & = & \frac{37}{60} \\ A + C + D & = & \frac{42}{60} \\ A + B + D & = & \frac{45}{60} \\ \hline 3A + 3B + 3C + 3D & = & \frac{171}{60} \\ \therefore A + B + C + D & = & \frac{57}{60} \end{array} $	$ \begin{array}{rcl} A + B + C + D & = & \frac{57}{60} \\ A + B + C & = & \frac{47}{60} \\ \hline \therefore D & = & \frac{10}{60} = \frac{1}{6} \\ \\ A + B + C + D & = & \frac{57}{60} \\ B + C + D & = & \frac{37}{60} \\ \hline \therefore A & = & \frac{20}{60} = \frac{1}{3} \\ \\ A + B + C + D & = & \frac{57}{60} \\ A + C + D & = & \frac{42}{60} \\ \hline \therefore B & = & \frac{15}{60} = \frac{1}{4} \\ \\ A + B + C + D & = & \frac{57}{60} \\ A + B + D & = & \frac{45}{60} \\ \hline \therefore C & = & \frac{12}{60} = \frac{1}{5} \end{array} $
--	--

$\frac{57}{60} : \frac{60}{60} :: \frac{1}{6} : \frac{1}{6} \times \frac{60}{60} \times \frac{60}{57} = \frac{10}{57} = D\text{'s true share which is therefore} = \frac{10}{57} \text{ of } \$6213 = \$1090.$

$\frac{57}{60} : \frac{60}{60} :: \frac{1}{3} : \frac{1}{3} \times \frac{60}{60} \times \frac{60}{57} = \frac{20}{57} = A\text{'s true share which is therefore} = \frac{20}{57} \text{ of } \$6213 = \$2180.$

$\frac{57}{60} : \frac{60}{60} :: \frac{1}{4} : \frac{1}{4} \times \frac{60}{60} \times \frac{60}{57} = \frac{15}{57} = B\text{'s true share which is therefore} = \frac{15}{57} \text{ of } \$6213 = \$1635.$

$\frac{57}{60} : \frac{60}{60} :: \frac{1}{5} : \frac{1}{5} \times \frac{60}{60} \times \frac{60}{57} = \frac{12}{57} = C\text{'s true share which is therefore} = \frac{12}{57} \text{ of } \$6213 = \$1308.$

ELEVENTH SERIES.

(121)

$$\begin{aligned} \cdot\dot{7} &= \frac{7}{9}; \cdot\ddot{83} = \frac{83}{99}; \cdot\dot{7}2\dot{7} = \frac{727}{999}; \cdot\dot{9}132\dot{5} = \frac{91325}{99990} = \frac{91315}{99990} = \frac{45858}{49995}. \\ 8\cdot671347 &= 8\frac{671347}{1000000} = 8\frac{671380}{999900} = 8\frac{1188}{6666}. \end{aligned}$$

(122)

$$713 \text{ unden.} = 816 \text{ den.}; 291 \text{ unden.} = 342 \text{ den.}; 3t1 \text{ unden.} = 474 \text{ den.}$$

291

$$\text{Then } 713 \frac{291}{3t1} \text{ unden.} = 861\frac{342}{474} \text{ den.} = 861\frac{57}{9} \text{ den.}$$

3t1

$$12123 \text{ quat.} = 411 \text{ den.}; 11223 \text{ quat.} = 363 \text{ den.}; 100000 \text{ quat.} = 1024 \text{ den.}$$

$$\text{Then } 12123 \frac{11223}{100000} = 411\frac{363}{1024} \text{ den.}$$

(123)

$$\begin{aligned} 3\frac{3}{8} \text{ of } 2\frac{1}{8} \text{ of } 7\frac{1}{20} \text{ of } £1 &= \frac{27}{8} \text{ of } \frac{11}{8} \text{ of } \frac{151}{20} \text{ of } £1 \\ &= £\frac{44817}{800} \dots\dots\dots = £56 \quad 1 \quad 2\frac{1}{10} \\ 9\frac{7}{8} \text{ of } 3\frac{3}{8} \text{ of } 1s. &= \frac{66}{7} \text{ of } 3\frac{5}{8} \text{ of } 1s. = \frac{110}{3}s. \dots\dots = \quad 1 \quad 16 \quad 8 \\ 8\frac{1}{4} \text{ of } 4\frac{1}{8} \text{ of } 1d. &= \frac{13}{4} \text{ of } 3\frac{3}{8} \text{ of } 1d. = \frac{1089}{32}d. \dots = \quad 0 \quad 2 \quad 10\frac{1}{8} \\ \text{Sum} &= £58 \quad 0 \quad 8\frac{21}{160} \end{aligned}$$

$$\frac{11}{12} \text{ of } \frac{5}{14} \text{ of } \frac{3}{8} \text{ of } 3\frac{1}{2}d. = \frac{11}{12} \times \frac{5}{14} \times \frac{3}{8} \times \frac{7}{2} = \frac{55}{128}d.$$

$$£58 \text{ 0s. } 8\frac{21}{160}d. = \frac{2228501}{160}d.$$

$$\frac{2228501}{160} \div \frac{55}{128} = \frac{2228501}{160} \times \frac{128}{55} = \frac{202591}{6} \times \frac{1}{6} = \frac{810364}{288} = 32414\cdot56.$$

(124)

$$\begin{array}{l|l} 24 : 90 & \\ 2\frac{1}{2} : 4\frac{1}{5} & \\ 12\frac{1}{2} : 9\frac{1}{3} & \therefore 139\frac{1}{2} : x \\ 4\frac{7}{8} : 4\frac{1}{2} & \\ 3\frac{1}{8} : 2\frac{1}{2} & \end{array}$$

(Continued on next page.)

(124 continued.)

$$\begin{array}{cccccccccccc}
 & 9 & & & & & & & & & & & & \\
 18 & 7 & & 3 & & 43 & & & & & & & \\
 90 & 21 & 29 & 9 & 5 & 559 & 1 & 2 & 2 & 8 & & & \\
 \hline
 1 & 5 & 3 & 2 & 2 & 4 & 24 & 5 & 25 & 39 & & & \\
 & & & & & & 8 & & 6 & 13 & & & \\
 & 5 & 9 \times 7 \times 29 \times 43 & 78561 & & & & & & & & & \\
 \times \frac{5}{18} = \frac{2 \times 4 \times 5 \times 4}{161} = \frac{160}{161} = 491\frac{1}{160}.
 \end{array}$$

(125)

\$182 is $\frac{91}{100}$ of buying price $\therefore \$182 \div 91 = \$2 = \frac{1}{100}$ of buying price \therefore buying price $= \$2 \times 100 = \200 .

To realize a profit of 7 per cent., he must receive \$1.07 for every \$1 the goods cost; but they cost him \$200, therefore he must sell for $\$1.07 \times 200 = \214 .

(126)

$$\text{Simple Interest } t = \frac{n-1}{r} = \frac{11\frac{1}{2}-1}{.06} = \frac{10.5}{.06} = \frac{1050}{6} = 175 \text{ years.}$$

$$\begin{aligned}
 \text{Compound Interest } t &= \frac{\log. n}{\log. (1+r)} = \frac{\log. 11\frac{1}{2}}{\log. 1.06} = \frac{1.060698}{0.025306} \\
 &= \frac{1060698}{25306} = 41.914 \text{ years.}
 \end{aligned}$$

(127)

An acre contains 4 roods $= 160$ sq. perches.

$\therefore 160 \div 15\frac{1}{2} = 10\frac{1}{3}\frac{1}{2}$ perches $=$ length.

(128)

35 yards $= 32$ metres $\therefore 1$ yd. $= \frac{32}{35}$ of a metre.

$$\begin{array}{ccccccc}
 69\frac{1}{2} \text{ miles} & = & 69\frac{1}{2} \times \frac{1760}{1} \text{ yards} & = & 69\frac{1}{2} \times \frac{1760}{1} \times \frac{32}{35} \text{ metres} \\
 & & 217 & 80 & & & \\
 & & 1519 & 1760 & 32 & & \\
 & = & \frac{22}{22} \times \frac{1}{1} \times \frac{32}{35} & = & 217 \times 16 \times 32 & = & 111104 \text{ metres.} \\
 & & & & 7 & &
 \end{array}$$

(129)

7 means + 2 extremes = 9 terms.

Formula XIII, p. 340. $r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}} = \left(\frac{19683}{3}\right)^{\frac{1}{8}} = (6561)^{\frac{1}{8}} = 3$

Hence means are 9, 27, 81, 243, 729, 2187, and 6561.

(130)

Formula XXI, p. 344. $s = \frac{a}{1-r} = \frac{7}{1-\frac{1}{4}} = \frac{7}{\frac{3}{4}} = \frac{28}{3} = 9\frac{1}{3}$.

(131)

Part remaining after 1st has received his share = $\frac{3}{4}$.

$1 : \frac{3}{4} :: 60^2 : x^2$; whence $x = \sqrt{3600 \times \frac{3}{4}} = \sqrt{900 \times 3}$
 $= 30\sqrt{3} = 1.732 \times 30 = 51.96$ inches.

Hence 1st ground off $60 - 51.96 = 8.04$ inches.

Part remaining after 2nd had taken off his share = $\frac{1}{4}$.

$1 : \frac{1}{4} :: 60^2 : x^2$; whence $x = \sqrt{3600 \times \frac{1}{4}} = 30\sqrt{2}$
 $= 1.4142 \times 30 = 42.426$.

Hence 2nd ground off $51.96 - 42.426 = 9.534$ inches.

Part remaining after the 3rd had taken off his share = $\frac{1}{4}$.

$1 : \frac{1}{4} :: 6^2 : x^2$; whence $x = \sqrt{3600 \times \frac{1}{4}} = \sqrt{900} = 30$ inches.

Hence 3rd ground off $42.426 - 30$ inches = 12.426 inches,
 and the 4th ground off remaining 30 inches.

(132)

1 guinea = 21s.

1 half-guinea = 10½s.

1 crown = 5s.

1 half-crown = 2½s.

1 shilling = 1s.

Sixpence = ½s.

100 guineas = 2100 shillings.

$2100 \div 40\frac{1}{2} = 51$ times and remainder, 69 half-shillings.

69 half-shil. = $\frac{69}{2}$ s. = £40 = 140.

Sum = 40½s.

TWELFTH SERIES.

(133)

$$\frac{3}{11} \text{ of } \frac{2}{9} \text{ of } \frac{4}{17} = \frac{8}{561}; \quad \frac{2\frac{1}{2}}{4\frac{1}{4}} \text{ of } \frac{2}{5} = \frac{10}{17} \text{ of } \frac{2}{5} = \frac{4}{17}.$$

$$\frac{8}{561} : \frac{4}{17} :: \$12\frac{4}{33} : \$12\frac{4}{33} \times \frac{4}{17} \times \frac{561}{8} = \frac{\$200}{33} \times \frac{4}{17} \times \frac{561}{8} \\ = \$200.$$

(134)

$$\text{By Formula III, page 354, } r = \sqrt[t]{\frac{A}{P}} - 1 \therefore r + 1 = \sqrt[t]{\frac{A}{P}}$$

$$\therefore \text{Log. } (1 + r) = (\log. A - \log. P) \div t \\ = (\log. 1679 \cdot 40 - \log. 700 \cdot 90) \div 5 \\ = (3 \cdot 225154 - 2 \cdot 845656) \div 5. \\ = 0 \cdot 379498 \div 5 = 0 \cdot 075894.$$

$\therefore 1 + r = \text{nat. num. corresponding to the logarithm } 0 \cdot 074894$
which is $1 \cdot 19$, $\therefore r = \cdot 19 = \text{rate per unit, and hence rate per cent.} = 19$.

(135)

Having paid 10 per cent. he had 90 per cent. remaining.

$$\frac{90}{100} \text{ or } \frac{9}{10} \text{ of his salary} = \$1250, \therefore \frac{1}{10} = \frac{1250}{9} = \$138\frac{8}{9}.$$

$$\text{If } \$138\frac{8}{9} = \frac{1}{10}, \text{ the whole} = \$138\frac{8}{9} \times 10 = \$1388 \cdot 888.$$

(136)

21 children receive 21 times a child's share

21 women " 42 " "

21 men " 63 " "

— Together they receive 126 " "

$$£3 \ 13s. \ 6d. \div 126 = 7d. = \text{a child's share.}$$

$$7d. \times 2 = 1s. \ 2d. = \text{a woman's share.}$$

$$7d. + 1s. \ 2d. = 1s. \ 9d. = \text{a man's share.}$$

(137)

A gets 1 time A's share.

B " 1 " A's "

C " 2 " A's "

D " 4 " A's "

Together they get 8 times A's share.

$\$200 \div 8 = \$25 = \text{A's share}; \$25 = \text{B's share.}$

$\$25 + \$25 = \$50 = \text{C's share}; \$25 + \$25 + \$50 = \$100$
 $= \text{D's share.}$

(138)

$$\sqrt{\frac{2}{3}} = \frac{1}{3} \sqrt{18} = \frac{1}{3} \text{ of } 2.62074 = .87358$$

$$\sqrt{\frac{3}{4}} = \frac{1}{2} \sqrt{6} = \frac{1}{2} \text{ of } 2.44948 = .81649$$

$$\text{Difference} = .05709$$

(139)

$\frac{3872}{92807}$ when each term is divided by 121, becomes $\frac{32}{767}$.

$$17\frac{5}{12} + \frac{1}{15} + 144\frac{11}{11} = 161 + \frac{5}{12} + \frac{1}{15} + \frac{11}{11} = 161 + \frac{75}{120} + \frac{8}{120} + \frac{110}{120}$$

$$+ \frac{220}{120} = 161 + \frac{507}{120} = 161 + 4\frac{27}{40} = 162\frac{27}{40} = 162\frac{29}{40}.$$

$$2\frac{13}{35} - \frac{17}{25} = 2\frac{65}{175} - \frac{119}{175} = 1\frac{140}{175} - \frac{119}{175} = 1\frac{21}{35}.$$

$$\begin{array}{cccccc} & & & 5 & & \\ 3 & 6 & 4 & 15 & 21 & 54 \\ - \text{ of } - & \text{ of } - & \text{ of } - & \text{ of } - & \text{ of } - & = - \\ 4 & 7 & 15 & 11 & 23 & 253 \end{array}$$

$$6347 \div 2\frac{3}{4} = \frac{6347}{1} \div \frac{1}{4} = \frac{6347}{1} \times \frac{4}{1} = 2308.$$

(140)

884736	(96 = cube root.
729	
155736	
155736	
25956	
95951161	

$$9^2 = 81 \times 300 = 24300$$

$$9 \times 6 = 54 \times 50 = 1620$$

$$6^2 = 36$$

$$95951161 = 95951 \cdot 2576.$$

(Continued on next page.)

(140 continued.)

 $\dot{9}\dot{5}\dot{9}\dot{5}\dot{1}\cdot\dot{2}\dot{5}\dot{7}\dot{6}$ ($309\cdot76 =$ square root.

9

609) 5951

5481

618·7)470·25

433·09

619·46)37·1676

37·1676

 $\dot{3}0\dot{9}\cdot\dot{7}\dot{6}$ ($17\cdot6=17\frac{2}{3}=$ fourth root.

1

29 | 209

189

34·6 | 2076

2076

(141)

250

300

400

500

$$1450:250::\$520:\frac{\$520 \times 250}{1450} = \$89\frac{1}{9} = \text{contrib. on 1st village.}$$

$$1450:300::\$520:\frac{\$520 \times 300}{1450} = \$107\frac{1}{9} = \text{ " 2d "}$$

$$1450:400::\$520:\frac{\$520 \times 400}{1450} = \$143\frac{1}{9} = \text{ " 3d "}$$

$$1450:500::\$520:\frac{\$520 \times 500}{1450} = \$179\frac{1}{9} = \text{ " 4th "}$$

(142)

By Table on p. 362, the amount of \$1 for 34 payments at 3 per cent. = \$57·73018.

$$\$57\cdot73018 \times 260 = \$15009\cdot84.$$

$$\text{By Formula I, page 361, } A = \frac{a \{ (1+r)^t - 1 \}}{2}$$

$$= \frac{a}{r} \{ (1+r)^t - 1 \} = \frac{260}{.03} \{ (1.03)^{34} - 1 \}$$

$$= \frac{26000}{3} \times (2\cdot731855 - 1) = \frac{26000 \times 1\cdot731855}{3} = \$15009\cdot41.$$

(143)

$$\text{By Formula IX, p. 333, } d = \frac{l-a}{n-1} = \frac{79-2}{6-1} = \frac{77}{5} = 15\frac{2}{5}.$$

Hence the series is 2, $17\frac{2}{5}$, $32\frac{4}{5}$, $48\frac{1}{5}$, $63\frac{3}{5}$, and 79.

$$\text{Formula I, p. 333. } l = a + (n-1)d = 3 + (9-1) \times 4 = 3 + 8 \times 4 = 3 + 32 = 35.$$

$$\begin{aligned} \text{Formula VI, p. 333. } s &= \left\{ 2a + (n-1)d \right\} \frac{n}{2} \\ &= \left\{ 2 \times 3 + (207-1) \times 4 \right\} \frac{207}{2} = (6 + 206 \times 4) \frac{207}{2} \\ &= (6 + 824) \times \frac{207}{2} = \frac{830 \times 207}{2} = 85905. \end{aligned}$$

(144)

B travels 4 miles per day faster than A, and will therefore gain the circumference of the island in $7\frac{3}{4} = 18\frac{1}{4}$ days.

C travels 10 miles per day faster than A, and will therefore gain the whole circumference of the island in $7\frac{3}{10} = 7\frac{3}{10}$ days.

Now B cannot be with A except at the end of $18\frac{1}{4}$ days, or twice $18\frac{1}{4}$ days, or three times $18\frac{1}{4}$ days, or some other multiple of $18\frac{1}{4}$ days.

Similarly C cannot be with A except at the end of $7\frac{3}{10}$ days, or of some other multiple of $7\frac{3}{10}$ days.

Therefore C and B will both be with A for the first time after the lapse of a number of days expressed by the least common multiple of $18\frac{1}{4}$ and $7\frac{3}{10}$.

The greatest common factor of $18\frac{1}{4}$ and $7\frac{3}{10}$ is $3\frac{1}{20}$.

$$\text{Hence the l. c. m. of } 7\frac{3}{10} \text{ and } 18\frac{1}{4} \text{ is } \frac{7\frac{3}{10} \times 18\frac{1}{4}}{3\frac{1}{20}} = 36\frac{1}{2} = \text{number}$$

of days when A, B, and C will first be together,

ARITHMETICAL RECREATIONS.

1. The third of $6 = 2$, and the fourth of $20 = 5$.

Then if 2 becomes 3, what should 5 become? Evidently $7\frac{1}{2}$. *Ans.*

or

$$\left. \begin{array}{l} 6 : 20 \\ \frac{1}{3} : \frac{1}{4} \end{array} \right\} :: 3 : x = \frac{3 \times 20 \times \frac{1}{4}}{6 \times \frac{1}{3}} = 7\frac{1}{2}.$$

2. The half of $5 = 2\frac{1}{2}$; then if 7 becomes $2\frac{1}{2}$, what will 11 become?

$$\frac{2\frac{1}{2} \times 11}{7} = \frac{5\frac{5}{2}}{14}. \text{ Lastly, what part of 9 is } \frac{5\frac{5}{2}}{14}?$$

$$\frac{\frac{5\frac{5}{2}}{14}}{\frac{9}{1}} = \frac{5\frac{5}{2}}{126}. \text{ *Ans.*}$$

or

$$\left. \begin{array}{l} 9 : 5 \\ 7 : 11 \end{array} \right\} :: \frac{1}{2} : x = \frac{\frac{1}{2} \times 5 \times 11}{9 \times 7} = \frac{5\frac{5}{2}}{63} = \frac{5\frac{5}{2}}{126}. \text{ *Ans.*}$$

3. $99\frac{2}{9}$.

4. $\frac{1}{3}$ of 2d. = $\frac{2}{3}$ d. Then $\frac{2}{3}$ d. is what part of 3d.? *Ans.* $\frac{2}{9}$.

5. $1\frac{1}{2}$ d. for a herring and a half is at the rate of 1d. per herring; hence 11 herrings will cost 11d.

6. 12 apples = 21 pears = 7 cents.

If 12 apples cost 7 cents, what will 100 apples cost?

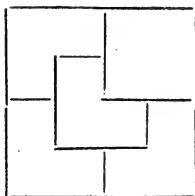
$$12 : 100 :: 7 : \frac{100 \times 7}{12} = 58\frac{1}{3} \text{ cents.}$$

7. If 5 is $\frac{2}{7}$ of a certain number, $\frac{1}{7}$ will be $\frac{1}{2}$ of 5, which is $\frac{5}{2}$.

If $\frac{5}{2}$ is $\frac{1}{7}$ of a certain number, the whole number will be $\frac{5}{2} \times 7 = \frac{35}{2} = 12\frac{3}{2}$. *Ans.*

8. The hurdles are arranged so as to form a rectangular enclosure having 49 hurdles on each side and one on each end. Two additional hurdles will give two hurdles to each end, and will thus double the size of the enclosure.

9. The mode of dividing the plot may be learned from the following figure:—



10. $33\frac{3}{4}$.

11. XIII; rub out the lower half, and there remains the expression VIII = 8.

12. 1st Step: Fill the 3-gallon cask and empty it into the 5-gallon cask.

2nd Step: Again fill the 3-gallon cask out of the 8-gallon cask.

3rd Step: Fill up the 5-gallon cask out of the 3-gallon cask. This will leave one gallon in the latter.

4th Step: Empty the 5-gallon cask into the 8-gallon cask.

5th Step: Pour the one gallon out of the 3-gallon cask into the 5-gallon cask.

6th Step: Fill the 3-gallon cask out of the 8-gallon cask and empty it into the 5-gallon cask.

The following diagrams show this more clearly :

1st Step.



2nd Step.



3rd Step.



4th Step.



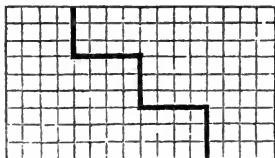
5th Step.



6th Step.



13. The heavy line in the accompanying figure shows how the board is to be cut.



14.

8	1	6
3	5	7
4	9	2

15. Weigh out 7 lbs. as often as possible and there will remain 2 lbs.; add two four pounds and one seven pounds to this, and the sum will be 17 lbs., the share of one.
Weigh 7 lbs. as often as possible out of the remaining 34 lbs. and there will remain 6 lbs., to which add 7 lbs. and 4 lbs., and the sum will be 17 lbs., the share of the second.
The remaining 17 lbs. will be the share of the third.
16. The hurdles are, in the first case, placed 12 on a side and one on each end, and then they inclose a space represented by 12 squares whose area is, by the question, 40 square yards. If two hurdles be taken away there will remain 24, and if these be placed in the form of a square, each side containing 6 hurdles, they will enclose a space represented by 36 squares of the same size as the former. Hence they now inclose three times as much space as before, *i. e.* three times 40 square yards, or 120 square yards.
17. He takes the goose to the remote bank and leaves it there, returning, he next carries over the fox, which he leaves, but takes the goose back with him. He now leaves the goose on the first bank, and carries over the oats which he allows to remain on the remote bank with the fox and returns for the goose.
18. The following diagrams exhibit the solution of this problem ;

I.	II.	III.	IV.	V.	VI.																																																						
<table><tr><td>3</td><td>3</td><td>3</td></tr><tr><td>3</td><td>P</td><td>3</td></tr><tr><td>3</td><td>3</td><td>3</td></tr></table>	3	3	3	3	P	3	3	3	3	<table><tr><td>4</td><td>1</td><td>4</td></tr><tr><td>1</td><td>P</td><td>1</td></tr><tr><td>4</td><td>1</td><td>4</td></tr></table>	4	1	4	1	P	1	4	1	4	<table><tr><td>2</td><td>5</td><td>2</td></tr><tr><td>5</td><td>P</td><td>5</td></tr><tr><td>2</td><td>5</td><td>2</td></tr></table>	2	5	2	5	P	5	2	5	2	<table><tr><td>1</td><td>7</td><td>1</td></tr><tr><td>7</td><td>P</td><td>7</td></tr><tr><td>1</td><td>7</td><td>1</td></tr></table>	1	7	1	7	P	7	1	7	1	<table><tr><td>0</td><td>9</td><td>0</td></tr><tr><td>9</td><td>P</td><td>9</td></tr><tr><td>0</td><td>9</td><td>0</td></tr></table>	0	9	0	9	P	9	0	9	0	<table><tr><td>5</td><td>0</td><td>4</td></tr><tr><td>0</td><td>P</td><td>0</td></tr><tr><td>4</td><td>0</td><td>5</td></tr></table>	5	0	4	0	P	0	4	0	5
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24	20	28	32	36	18																																																						

19. Represent the three husbands by A, B, and C, and their wives respectively by the letters *a*, *b*, and *c*.

I. A and *a* go over, *a* remains while A takes back the boat.

II. *b* and *c* go over and remain while *a* takes back the boat.

III. B and C go over, B remains while C and *c* take back the boat.

IV. A and C go over and remain while *b* takes back the boat.

V. *a* and *b* go over and remain while C takes back the boat.

VI. C. and *c* go over.

20.

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

RULE FOR FILLING MAGIC SQUARES OF ODD NUMBER OF CELLS.

Begin in centre cell of top horizontal row by placing 1 in it; ascend diagonally to the right, and where this carries us beyond the square, transport the next number to the cell at the remote end of the vertical or horizontal band to which it belongs. When in ascending we come to a cell already filled we place the number in the cell next below the cell last filled. The following is a square of 7 cells in a side filled after this method :

30	39	48	1	10	19	28
38	47	7	9	18	27	29
46	6	8	17	26	35	37
5	14	16	25	34	36	45
13	15	24	33	42	44	4
21	23	32	41	43	3	12
22	31	40	49	2	11	20

21. Half-a-dozen dozen = $6 \times 12 = 72$.

Six dozen dozen = $6 \times 12 \times 12 = 864$.

$864 - 72 = 792$. *Ans.*

22. The following shows the mode of performing this.
It will be observed that the two side-counters are
merely moved one counter higher when the other
two are taken away.

23. This problem admits of the following two solutions :

1ST SOLUTION.

Persons.	Full bottles.	Hf.-full bottles.	Empty bottles.
1st	2	3	2
2nd	2	3	2
3rd	3	1	3
	<hr/> 7	<hr/> 7	<hr/> 7

Each person has $3\frac{1}{2}$ bottles of wine and 7 bottles.

2ND SOLUTION.

1st	3	1	3
2nd	3	1	3
3rd	1	5	1
	<hr/> 7	<hr/> 7	<hr/> 7

Each person, as before, has 7 bottles and $3\frac{1}{2}$ bottles of wine.

24. There were in all 8 bottles of wine, of which each drank $\frac{1}{4}$, which is $2\frac{1}{2}$. The third person, therefore, drank $\frac{1}{4}$ of a bottle belonging to him who had but 3 bottles, and $\frac{7}{4}$ of a bottle belonging to him who owned the 5 bottles. Hence the latter should have *seven* times as much of the money as the former, or, in other words, the latter gets 7 shillings and the former 1 shilling.

25. This problem is merely to find some number between 5 and 100 which is exactly divisible by 2 and by 3, but which divided by 5 leaves a remainder 3.

The only numbers between 50 and 100 that are divisible by both 2 and 3 are 54, 60, 66, 72, 78, 84, 90, and 96, and by inspection the only one of these which gives a remainder 3 when divided by 5 is 78; therefore the basket contained 78 eggs.

26. *Ans.* 1 lb., 3 lbs., 9 lbs., and 27 lbs.

For 1 lb. = 1 lb.; 2 lbs. = 3 lbs. — 1 lb., i. e. 3 lbs in one scale and 1 lb. in the other; 3 lbs. = 3 lbs.; 4 lbs. = 3 lbs. + 1 lb.; 5 lbs. = 9 lbs. — (3 lbs. + 1 lb.); 6 lbs. = 9 lbs. — 3 lbs.; 7 lbs. = 9 lbs. + 1 lb. — 3 lbs.; 8 lbs. = 9 lbs. — 1 lb.; 9 lbs. = 9 lbs.; 10 lbs. = 9 + 1 lb.; 11 lbs. = 9 lbs. + 3 lbs. — 1 lb.; 13 lbs. = 9 lbs. + 3 lbs. + 1 lb.; 14 lbs. = 27 lbs. — (9 lbs. + 3 lbs. + 1 lb.); 15 lbs. = 27 lbs. — (9 lbs. + 3 lbs.); 16 lbs. = 27 lbs. + 1 lb. — (9 lbs. + 3 lbs.); 17 lbs. = 27 lbs. — (9 lbs. + 1 lb.); 18 lbs. = 27 lbs. — 9 lbs.; &c., &c.

27. In order to fill seven out of the eight points, it is merely requisite to remember that the second counter must be carried to the point from which the first *started*, the third to the point from which the second started, &c.

Thus if the first counter is carried from 1 to 4 and there deposited; the second must be taken from 6 to 1 and there deposited; the third from 3 to 6; the fourth from 8 to 3; the fifth from 5 to 8; the sixth from 2 to 5; and the seventh either from 7 to 2 or from 2 to 7.

28. The mouth fills the reservoir in 6 hours, therefore it fills $\frac{1}{6}$ in 1 hour; the right eye fills it in 48 hours, therefore it fills

$\frac{1}{8}$ in 1 hour; the left eye fills it in 72 hours, therefore it fills $\frac{1}{72}$ in 1 hour; the foot fills it in 96 hours, therefore it fills $\frac{1}{96}$ in 1 hour. Hence together they fill $\frac{1}{6} + \frac{1}{8} + \frac{1}{72} + \frac{1}{96} = \frac{61}{288}$ in 1 hour, and to fill the reservoir they require $1 \div \frac{61}{288} = \frac{288}{61} = 4$ hours 43 min. $16\frac{4}{61}$ sec.

29. The person who thinks of the numbers must proceed as follows: He must multiply the 1st by 2 and add 5 to the product; he must next multiply this sum by 5 and add the second number to the product; he must next multiply this result by 10 and add the third number to the product; lastly, he must subtract 250 and name the remainder.

The three digits of the remainder will be the three numbers thought of, and will be in the order in which they were thought of.

The reason is obvious: let $a = 1$ st, $b = 2$ nd, and $c = 3$ rd number thought of.

$$a \times 2 + 5 = 2a + 5.$$

$$(2a + 5) \times 5 + b = 10a + b + 25.$$

$$(10a + b + 25) \times 10 + c = 100a + 10b + c + 250.$$

$$(100a + 10b + c + 250) - 250 = 100a + 10b + c = a \text{ in hundreds' place, } b \text{ in tens' place, and } c \text{ in units' place.}$$

30. Since each man possesses 63 square rods of land more than his son, we must form three pairs of numbers, such that the difference of their squares shall be 63.

The difference of the squares of two numbers is equal to their sum multiplied by their difference, and hence 63 must be divided into two factors in three distinct ways, thus:

$$63 = 63 \times 1 = 21 \times 3 = 9 \times 7.$$

If sum = 63 and difference = 1, the numbers are 32 and 31.

If sum = 21 and difference = 3, the numbers are 12 and 9.

If sum = 9 and difference = 7, the numbers are 8 and 1.

Hence the squares of Jones, Brown, and Smith, are respectively 32 rods, 12 rods, and 8 rods on the side, and the son's squares are respectively 31, 9, and 1 yards on the side.

Jones' piece was 23 rods longer on each side than Tom's, and since the difference between 32 and 9 is 23, we may conclude that Jones' square was 32 rods to the side, and Tom's 9 rods on a side.

Brown's piece was 11 rods longer on a side than Harry's, and since if the above numbers 12 and 1 have 11 for their difference, we may conclude that Brown's piece was 12 rods on a side, and Harry's piece 1 rod.

Hence Tom was Brown's son, Harry was Smith's son, and Ned was Jones' son.

31. The mode of arranging the crew may be remembered by attention to the vowels in the following line :

Populeam virgam mater regina ferebat.

The vowels refer to the crew as follows, $a = 1$, $e = 2$, $i = 3$, $o = 4$, and $u = 5$.

We begin with 4 whites because the first vowel is o , next $u = 5$ blacks, next $e = 2$ whites, next $a = 1$ black, next $i = 3$ whites, next $a = 1$ black, next $e = 2$ whites, next $e = 2$ blacks, &c., as follows, o standing for a white and $+$ for a black.

oooo++++oo+ooo+o++++oo++++o++++oo+

32. You select the multiplier or the multiplicand, such that the sum of its digits shall be exactly divisible by nine. Hence upon the principle of the proof by casting out the nines, the product has the sum of its digits exactly divisible by nine. By subtracting the sum of the digits of the remainder from the next higher multiple of 9 you determine the digit crossed out.

Thus suppose you select 117, and he takes for multiplicand 21613. Then $21613 \times 117 = 2528721$. Now suppose he crosses out the 7; upon reading you the remaining digits 252821, you find that their sum $= 20$, which taken from 27 the next higher multiple of 9 leaves 7 the digit he crossed out.

If he crosses out a 0 or a 9, you cannot determine which, but in all other cases you can tell the exact figure.

33. You write the second, fourth, sixth, &c. lines in such a manner as to make the sum of the first pair, the sum of the second pair, &c. an exact number of 9's. Then having settled the number of pairs, you get the answer by multiplying by that number a row of 9's containing as many digits as there are to be figures in the line.

Thus suppose you agree to write 5 lines each, and that each line is to contain 5 digits, or not more than 5 digits. Then $99999 \times 5 = 499995$ will be the answer. This is shown as follows:

Suppose he writes 41113	}		=	99999	}	
You write 58886	}					
Suppose he writes 61451	}					
You write 38548	}					
Suppose he writes 6500	}					
You write 93499	}					
Suppose he writes 1	}					
You write 99998	}					
Suppose he writes 99999	}					
You write 00000	}					
<div style="border-top: 1px solid black; width: 100px; margin: 0 auto;"></div>						
Sum = 499995						

THE END.

LT1001.511.522 R4 1861 c.2.

